



THE WEIZMANN INSTITUTE OF SCIENCE  
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Algebraic Geometry and Representation Theory Seminar

Room 290C ,Ziskind Building  
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at 11:15

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## Generalized Harish-Chandra functors for general linear groups over finite local rings

### Abstract:

Let  $K$  be a commutative ring. Consider the groups  $GL_n(K)$ . Bernstein and Zelevinsky have studied the representations of the general linear groups in case the ring  $K$  is a finite field. Instead of studying the representations of  $GL_n(K)$  for each  $n$  separately, they have studied all the representations of all the groups  $GL_n(K)$  simultaneously. They considered on  $R := \sum_{n \geq 0} R_n(GL_n(K))$  structures called parabolic (or Harish-Chandra) induction and restriction, and showed that they enrich  $R$  with a structure of a so called positive self adjoint Hopf algebra (or PSH algebra). They use this structure to reduce the study of representations of the groups  $GL_n(K)$  to the following two tasks:

1. Study a special family of representations of  $GL_n(K)$ , called cuspidal representations. These are representations which do not arise as direct summands of parabolic induction of smaller representations.
2. Study representations of the symmetric groups. These representation also has a nice combinatorial description, using partitions.

In this talk I will discuss the study of representations of  $GL_n(K)$  where  $K$  is a finite quotient of a discrete valuation ring (such as  $\mathbb{Z}/p^r$  or  $k[[x]]/x^r$ , where  $k$  is a finite field). One reason to study such representation is that all continuous complex representations of the groups  $GL_n(\mathbb{Z}_p)$  and  $GL_n(k[[x]])$  (where  $\mathbb{Z}_p$  denotes the  $p$ -adic integers) arise from these finite quotients. I will explain why the natural generalization of the Harish-Chandra functors do not furnish a PSH algebra in this case, and how is this related to the Bruhat decomposition and Gauss elimination. In order to overcome this issue we have constructed a generalization of the Harish-Chandra functors. I will explain this generalization, describe some of the new functors properties, and explain how can they be applied to studying complex representations.

The talk will be based on a joint work with Tyrone Crisp and Uri Onn.