
THE WEIZMANN INSTITUTE OF SCIENCE
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE
Algebraic Geometry and Representation Theory Seminar

Room 290C ,Ziskind Building
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Generalized Harish-Chandra functors for general linear groups over finite local rings

Abstract:

Let K be a commutative ring. Consider the groups $GL_n(K)$. Bernstein and Zelevinsky have studied the representations of the general linear groups in case the ring K is a finite field. Instead of studying the representations of $GL_n(K)$ for each n separately, they have studied all the representations of all the groups $GL_n(K)$ simultaneously. They considered on $R := \sum_{n \geq 0} nR(GL_n(K))$ structures called parabolic (or Harish-Chandra) induction and restriction, and showed that they enrich R with a structure of a so called positive self adjoint Hopf algebra (or PSH algebra). They use this structure to reduce the study of representations of the groups $GL_n(K)$ to the following two tasks:

1. Study a special family of representations of $GL_n(K)$, called cuspidal representations. These are representations which do not arise as direct summands of parabolic induction of smaller representations.
2. Study representations of the symmetric groups. These representation also has a nice combinatorial description, using partitions.

In this talk I will discuss the study of representations of $GL_n(K)$ where K is a finite quotient of a discrete valuation ring (such as \mathbb{Z}/p^r or $k[[x]]/x^r$, where k is a finite field). One reason to study such representation is that all continuous complex representations of the groups $GL_n(\mathbb{Z}_p)$ and $GL_n(k[[x]])$ (where \mathbb{Z}_p denotes the p -adic integers) arise from these finite quotients. I will explain why the natural generalization of the Harish-Chandra functors do not furnish a PSH algebra in this case, and how is this related to the Bruhat decomposition and Gauss elimination. In order to overcome this issue we have constructed a generalization of the Harish-Chandra functors. I will explain this generalization, describe some of the new functors properties, and explain how can they be applied to studying complex representations.

The talk will be based on a joint work with Tyrone Crisp and Uri Onn.