מכון ויצמן למדע

THE WEIZMANN INSTITUTE OF SCIENCE

FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Algebraic Geometry and Representation Theory Seminar

Room 261 ,Ziskind Building
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at 12:30

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Generalized RSK

Abstract:

The goal of my talk (based on joint work with Dima Grigoriev, Anatol Kirillov, and Gleb Koshevoy) is to generalize the celebrated Robinson-Schensted-Knuth (RSK) bijection between the set of matrices with nonnegative integer entries, and the set of the planar partitions.

Namely, for any pair of injective valuations on an integral domain we construct a canonical bijection K, which we call the generalized RSK, between the images of the valuations, i.e., between certain ordered abelian monoids.

Given a semisimple or Kac-Moody group, for each reduced word $ii=(i_1,...,i_m)$ for a Weyl group element we produce a pair of injective valuations on $C[x_1,...,x_m]$ and argue that the corresponding bijection $K=K_{ii}$, which maps the lattice points of the positive octant onto the lattice points of a convex polyhedral cone in R^m, is the most natural generalization of the classical RSK and, moreover, K_{ii} can be viewed as a bijection between Lusztig and Kashiwara parametrizations of the dual canonical basis in the corresponding quantum Schubert cell.

Generalized RSKs are abundant in "nature", for instance, any pair of polynomial maps phi,psi:C^m->C^m with dense images determines a pair of injective valuations on $C[x_1,...,x_n]$ and thus defines a generalized RSK bijection $K_{\{phi,psi\}}$ between two sub-monoids of Z_+^m.

When phi and psi are birational isomorphisms, we expect that $K_{\{phi,psi\}}$ has a geometric "mirror image", i.e., that there is a rational function f on C^m whose poles complement the image of phi and psi so that the tropicalization of the composition $psi^{-1}phi$ along f equals to $K_{\{phi,psi\}}$. We refer to such a geometric data as a (generalized) geometric RSK, and view f as a "super-potential". This fully applies to each ii-RSK situation, and we find a super-potential $f=f_{ii}$ which helps to compute K_{ii}.

While each K_{ii} has a "crystal" flavor, its geometric (and mirror) counterpart f_{ii} emerges from the

cluster twist of the relevant double Bruhat cell studied by Andrei Zelevinsky, David Kazhdan, and myself.