Let $B$ denote the range of the Brownian motion in $\mathbb{R}^d$. For a deterministic Borel a measure $\nu$ we wish to find a random measure $\mu$ such that the support of $\mu$ is contained in $B$ and the expectation of $\mu$ is $\nu$. We discuss when exactly can there be such a random measure and construct in those cases. We establish a formula for the expectation of the double integral with respect to $\mu$, which is a strong tool for the geometric measure theory of the Brownian path.