Grassmann Graphs, Vertex Cover and 2-to-2 games

Abstract:

We discuss recent progress on hardness of 2-to-2 games, with applications to the inapproximability of the Vertex-Cover problem.

A 2-to-2 game (which is a variant of Khot's well known unique games), is defined by a graph where there is a variable in each node, and a constraint of a specific structure defined on each edge. While in unique games each edge constraint must be a one-to-one correspondence -- i.e. for each assignment to one node there is exactly one assignment to the other node that satisfies the constraint -- in 2-to-2 games the correspondence has a "two-to-two" structure. The goal is to distinguish between instances in which almost all of the edge constraints can be satisfied, and instances in which almost none of them can be satisfied simultaneously.

We present a new combinatorial hypothesis regarding Grassmann graphs, and show that it implies that 2-to-2 games are NP-hard *in a certain sense*. As a consequence, the hypothesis implies that it is NP-hard to distinguish between graphs that have an independent set of fractional size \((1-1/\sqrt{2})\), and graphs with no independent sets of any constant fractional size. This easily implies that it is NP-hard to approximate the Vertex Cover problem within a factor \(\sqrt{2} - o(1)\).

The talk is mostly based on a joint work with Subhash Khot and Muli Safra, nevertheless, we will also mention results from a more recent extension, which is a joint work with Irit Dinur, Subhash Khot, Guy Kindler and Muli Safra.