מכון ויצמן למדע

THE WEIZMANN INSTITUTE OF SCIENCE

FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Machine Learning and Statistics Seminar

Room 1 ,Ziskind Building
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Group Symmetric Robust Covariance Estimation

Abstract:

Covariance matrix estimation is essential in many areas of modern Statistics and Machine Learning including Graphical Models, Classification/Discriminant Analysis, Principal Component Analysis, and many others. Classical statistics suggests using Sample Covariance Matrix (SCM) which is a Maximum Likelihood Estimator (MLE) in the Gaussian populations. Real world data, however, usually exhibits heavy-tailed behavior and/or contains outliers, making the SCM non-efficient or even useless. This problem and many similar ones gave rise to the Robust Statistics field in early 60s, where the main goal was to develop estimators stable under reasonable deviations from the basic Gaussian assumptions. One of the most prominent robust covariance matrix estimators was introduced and thoroughly studied by D. Tyler in the mid-80s. This important representative of the family of M-estimators can be defined as an MLE of a certain population. The problem of robust covariance estimation becomes even more involved in the high-dimensional scenario, where the number of samples n is of the order of the dimension p, or even less. In such cases, prior knowledge, often referred to as structure, is utilized to decrease the number of degrees of freedom and make the estimation possible. Unlike the Gaussian setting, in Tyler's case even imposition of linear structure becomes challenging due to the non-convexity of the negative log-likelihood. Recently, Tyler's target function was shown to become convex under a certain change of metric (geodesic convexity), which stimulated further investigation of the estimator.

In this work, we focus on the so-called group symmetry structure, which essentially means that the true covariance matrix commutes with a group of unitary matrices. In engineering applications such structures appear due to the natural symmetries of the physical processes; examples include circulant, perHermitian, proper quaternion matrices, etc. Group symmetric constraints are linear, and thus convex in the regular Euclidean metric. We show that they are also convex in the geodesic metric. These properties allow us to develop symmetric versions of the SCM and Tyler's estimator and build a general framework for their performance analysis. The classical results claim that at least n =

p and n = p+1 samples in general position are necessary to ensure the existence and uniqueness of the SCM and Tyler's estimator, respectively. We significantly improve the sample complexity requirements for both estimators under the symmetry structure and show that in some cases even 1 or 2 samples are enough to guarantee the existence and uniqueness regardless of the ambient dimension.