How Many Linear Equations are Required to Solve a Linear Program?

Abstract:

Linear Programming is the backbone of algorithm design and combinatorial optimization. The fastest known algorithms for solving linear programs, both in theory and practice, are based on Interior-Point Methods: The core idea behind IPMs is to iteratively use Newton approximation to reduce a linear program to a sequence of ~\sqrt{n} linear systems. Whether this number of iterations can be improved is one of the major open problems in convex optimization.

A long line of work has shown that \sqrt{n} is indeed optimal for the special class of *self-concordant* Barrier IPMs [Nestrov-Nemirovski94], but for general IPMs very little is known.

We propose a purely information-theoretic query model for studying the rate of convergence of IPMs, via linear-system queries: Each iteration of the algorithm can adaptively specify an arbitrary diagonal matrix H (an ellipsoid) and a vector v, and the oracle returns the least-squares minimizer of the linear system \arg\min_x \|Ax - v\|_H. We show this model captures all known (deterministic) IPMs. Our main result is an \Omega(\sqrt{n}) lower bound on the iterations of any deterministic algorithm in this model, albeit for an (exponentially) ill-conditioned LP. In this talk I will describe this progress, assuming no prior knowledge on IPMs.

Based on Joint work with Shunhua Jiang, Rasmus Kyng and Ming Ding.