Improved lower and upper bounds on the Hadwiger-Debrunner numbers

Abstract:

A family of sets $F$ is said to satisfy the $(p,q)$-property if among any $p$ sets in $F$, some $q$ have a non-empty intersection. Hadwiger and Debrunner (1957) conjectured that for any $p > q > d$ there exists a constant $c = c_d(p,q)$, such that any family of compact convex sets in $\mathbb{R}^d$ that satisfies the $(p,q)$-property, can be pierced by at most $c$ points. The classical Helly's Theorem is equivalent to the fact that $c_d(p,p)=1$ ($p > d$).

In a celebrated result from 1992, Alon and Kleitman proved the conjecture. However, obtaining sharp bounds on the minimal such $c_d(p,q)$, called 'the Hadwiger-Debrunner numbers', is still a major open problem in combinatorial geometry.

In this talk we present improved upper and lower bounds on the Hadwiger-Debrunner numbers, the latter using the hypergraph container method.

Based on joint works with Shakhar Smorodinsky and Gabor Tardos.