Abstract:

We consider a class of sparse random matrices of the form $A_n = (\xi_{i,j}\delta_{i,j})_{i,j=1}^n$, where $\{\xi_{i,j}\}$ are i.i.d. centered random variables, and $\{\delta_{i,j}\}$ are i.i.d. Bernoulli random variables taking value 1 with probability $p_n$, and prove a quantitative estimate on the smallest singular value for $p_n = \Omega\left(\frac{\log n}{n}\right)$, under a suitable assumption on the spectral norm of the matrices. This establishes the invertibility of a large class of sparse matrices. We also find quantitative estimates on the smallest singular value of the adjacency matrix of a directed Erdős-Rényi graph whenever its edge connectivity probability is above the critical threshold $\Omega\left(\frac{\log n}{n}\right)$. This is joint work with Mark Rudelson.