
THE WEIZMANN INSTITUTE OF SCIENCE
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE
Algebraic Geometry and Representation Theory Seminar

on Tuesday, May 30, 2017 at 11:15

De Picciotto Building, Room 25
Siddhartha Sahi Rutgers University, USA

Multivariate Hypergeometric functions with a parameter

Abstract:

The theory of hypergeometric functions with matrix argument was developed in the 1950s by S. Bochner for Hermitian matrices, and by C. Herz for symmetric matrices. This theory admits a common generalization to the setting of symmetric cones, which is discussed in the book by Faraut-Koranyi. It also has applications to the study of non-central distributions in statistics and to the theory of random matrices.

In the 1980s, I.G. Macdonald introduced a one parameter family of multivariate hypergeometric functions, which, for special values of the parameter, are the **radial** parts of the matrix hypergeometric functions. He also formulated a number of natural conjectures about these functions, which in the matrix case can be proved by appropriate integral formulas. However this technique is unavailable in the general setting and as a result these conjectures have remained open.

In recent work with G. Olafsson we have solved most of these conjectures, using ideas from the theory of Cherednik algebras and Jack polynomials. Among other results we obtain sharp estimates for the exponential kernel that allow us to establish a rigorous theory of the Fourier and Laplace transforms, and we prove an explicit formula for the Laplace transform of a Jack polynomial conjectured by Macdonald. This opens the door for several future developments in the associated harmonic analysis, some of which we also treat. This includes (1) the Paley-Wiener theorem, (2) Laplace transform identities for hypergeometric functions, and (3) the "so-called" Ramanujan master theorem.

Note the unusual room [De Picciotto Building, Room 25]