Abstract:

We present a randomized algorithm that computes single-source shortest paths (SSSP) in $O(m\log^8(n)\log W)$ time when edge weights are integral, can be negative, and are $\geq -W$. This essentially resolves the classic negative-weight SSSP problem. The previous bounds are $\sim O((m+n^{1.5})\log W)$ and $m^{4/3+o(1)}\log W$. Near-linear time algorithms were known previously only for the special case of planar directed graphs. Also, independently of our work, the recent breakthrough on min-cost flow [Chen, Kyng, Liu, Peng, Probst-Gutenberg and Sachdeva] implies an algorithm for negative-weight SSSP with running time $m^{1+o(1)}$.

In contrast to all recent developments that rely on sophisticated continuous optimization methods and dynamic algorithms, our algorithm is simple: it requires only a simple graph decomposition and elementary combinatorial tools. In fact, ours is the first combinatorial algorithm for negative-weight SSSP to break through the classic $O(m\sqrt{n}\log n W)$ bound from over three decades ago [Gabow and Tarjan SICOMP'89].