New bounds on lattice covering volumes, and nearly uniform covers

Abstract:

Let $L$ be a lattice in $\mathbb{R}^n$ and let $K$ be a convex body. The covering volume of $L$ with respect to $K$ is the minimal volume of a dilate $rK$, such that $L + rK = \mathbb{R}^n$, normalized by the covolume of $L$. Pairs $(L,K)$ with small covering volume correspond to efficient coverings of space by translates of $K$, where the translates lie in a lattice. Finding upper bounds on the covering volume as the dimension $n$ grows is a well studied problem, with connections to practical questions arising in computer science and electrical engineering. In a recent paper with Or Ordentlich (EE, Hebrew University) and Oded Regev (CS, NYU) we obtain substantial improvements to bounds of Rogers from the 1950s. In another recent paper, we obtain bounds on the minimal volume of nearly uniform covers (to be defined in the talk). The key to these results are recent breakthroughs by Dvir and others regarding the discrete Kakeya problem. I will give an overview of the questions and results. No mathematical prerequisites will be assumed (beyond the concept of Lebesgue measure and vector spaces over finite fields).