Non-adaptive vs Adaptive Queries in the Dense Graph Testing Model by Oded Goldreich and Avi Wigderson

Abstract:
We study the relation between the query complexity of adaptive and non-adaptive testers in the dense graph model. It has been known for a couple of decades that the query complexity of non-adaptive testers is at most quadratic in the query complexity of adaptive testers. We show that this general result is essentially tight; that is, there exist graph properties for which any non-adaptive tester must have query complexity that is almost quadratic in the query complexity of the best general (i.e., adaptive) tester. More generally, for every $q(n) \leq \sqrt{n}$ and constant $c \in [1,2]$, we show a graph property that is testable in $\Theta(q(n))$ queries, but its non-adaptive query complexity is $\Theta(q(n)^c)$, omitting $\text{poly}(\log n)$ factors and ignoring the effect of the proximity parameter $\epsilon$. Furthermore, the upper bounds hold for one-sided error testers, and are at most quadratic in $1/\epsilon$. These results are obtained through the use of general reductions that transport properties of ordered structured (like bit strings) to those of unordered structures (like unlabeled graphs). The main features of these reductions are query-efficiency and preservation of distance to the properties. This method was initiated in our prior work (ECCC, TR20-149), and we significantly extend it here.