THE WEIZMANN INSTITUTE OF SCIENCE
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Algebraic Geometry and Representation Theory Seminar

Room 1 ,Ziskind Building
on Monday, Jun 15, 2015at 15:15

Note the unusual day, time and place. Note that this is the second talk from the same seminar on this date.

Gerald SchwarzBrandeis University

Oka Principles and the Linearization Problem

Abstract:

Let *Q* be a Stein space and *L* a complex Lie group. Then Grauert's Oka Principle states that the canonical map of the isomorphism classes of holomorphic principle *L*-bundles over *Q* to the isomorphism classes of topological principle *L*-bundles over *Q* is an isomorphism. In particular he showed that if *P*, *P'* are holomorphic principle *L*-bundles and $\Phi:P\to P'$ a topological isomorphism, then there is a homotopy Φ_t of topological isomorphisms with $\Phi_0=\Phi$ and $\Phi_1:P\to P'$ a holomorphic isomorphism.

Let *X* and *Y* be Stein *G*-manifolds where *G* is a reductive complex Lie group. Then there is a quotient Stein space Q_X, and a morphism $\pi_X:X\to Q_X$ such that $(\pi_X)^* \mathcal{O}(Q_X)=\mathcal{O}(X)^G$. Similarly we have $p_Y:Y\to Q_Y$.

Suppose that $\Phi:X\to Y$ is a *G*-biholomorphism. Then the induced mapping $\phi:Q_X\to Q_Y$ has the following property: for any $z\in Q_X$, $X_z:=\pi_X^{-1}(z)$ is *G*-isomorphic to $Y_{\phi(z)}$ (the fibers are actually affine *G*-varieties). We say that ϕ is admissible. Now given an admissible ϕ, assume that we have a *G*-equivariant homeomorphism $\Phi:X\to Y$ lifting ϕ. Our goal is to establish an Oka principle, saying that ϕ has a deformation Φ_t with $\Phi_0=\Phi$ and Φ_1 biholomorphic.

We establish this in two main cases. One case is where Φ is a diffeomorphism that restricts to *G*-isomorphisms on the reduced fibers of π_X and π_Y. The other case is where Φ restricts to *G*-isomorphisms on the fibers and *X* satisfies an auxiliary condition, which usually holds. Finally, we give applications to the Holomorphic Linearization Problem. Let *G* act holomorphically on $X=\backslash C^n$. When is there a change of coordinates such that the action of *G* becomes linear? We prove that this is true, for *X* satisfying the same auxiliary condition as before, if and only if the quotient Q_X is admissibly biholomorphic to the quotient of a *G*-module *V*.