Abstract:

Every $k$ entries in a permutation can have one of $k!$ different relative orders, called patterns. How
many times does each pattern occur in a large random permutation of size $n$? The distribution of
this $k!$-dimensional vector of pattern densities was studied by Janson, Nakamura, and Zeilberger
(2015). Their analysis showed that some component of this vector is asymptotically multinormal of
order $1/\sqrt{n}$, while the orthogonal component is smaller. Using representations of the
symmetric group, and the theory of U-statistics, we refine the analysis of this distribution. We show
that it decomposes into $k$ asymptotically uncorrelated components of different orders in $n$, that
correspond to representations of $S_k$. Some combinations of pattern densities that arise in this
decomposition have interpretations as practical nonparametric statistical tests.