Period Relations of Standard $L$-Functions of Symplectic Type

Abstract:

A classical result of Euler says that the value of the Riemann-Zeta function at a positive even integer $2k$ is a rational multiple of $\pi^{2k}$. In the 1970s, a successive pioneering work of G. Shimura revealed the relation of different critical values of $L$-functions that are attached to modular forms of $\text{GL}_2$. This type of result, conjectured by D. Blasius for general linear groups, is called period relation of a certain automorphic $L$-function, which is closely related to a celebrated conjecture of P. Deligne. In this talk, I will discuss my work joint with Dihua Jiang and Binyong Sun on the period relation for the twisted standard $L$-function $L(s, \Pi \otimes \chi)$, where $\Pi$ is an irreducible cuspidal automorphic representation of $GL_{2n}(\mathbb{A})$ which is regular algebraic and of symplectic type. Along this talk, I will also discuss the key ingredient of this project - the existence of uniform cohomological test vector, which provides the most precise information on the archimedean local integrals.