On polynomial approximations to $AC^0$

Abstract:

In this talk, we will discuss some questions related to polynomial approximations of $AC^0$. A classic result due to Tarui (1991) and Beigel, Reingold, and Spielman (1991), states that any $AC^0$ circuit of size $s$ and depth $d$ has an $\varepsilon$-error probabilistic polynomial over the reals of degree at most $(\log(s/\varepsilon))^\Theta(d)$. We will have a re-look at this construction and show how to improve the bound to $(\log s)^{\Theta(d)}\log(1/\varepsilon)$, which is much better for small values of $\varepsilon$. As an application of this result, we show that $(\log s)^{\Theta(d)}\log(1/\varepsilon)$-wise independence fools $AC^0$, improving on Tal's strengthening of Braverman's theorem that $(\log(s/\varepsilon))^\Theta(d)$-wise independence fools $AC^0$. Time permitting, we will also discuss some lower bounds on the best polynomial approximations to $AC^0$.

Joint work with Srikanth Srinivasan.