Abstract:

We investigate a class of models related to the Bak-Sneppen (BS) model, initially proposed to study evolution. The BS model is extremely simple and yet captures some forms of complex behavior such as self-organized criticality that is often observed in physical and biological systems. In this model, random fitnesses in \([0, 1]\) are associated to \(N\) agents located at the vertices of a graph \(G\), in our case a cycle. Their fitnesses are ranked from worst (0) to best (1). At every time-step the agent with the lowest fitness and its neighbors on the graph \(G\) are replaced by new agents with random fitnesses. We approximate the dynamics by making a simplifying independence assumption. We use order statistics to define a dynamical system on the set of cumulative distribution functions \(R : [0, 1] \rightarrow [0, 1]\) that mimics the evolution of the distribution of the fitnesses in these rank-driven models, among which the BS model. We then show that this dynamical system reduces to a 1-dimensional polynomial map. Using an additional conjecture we can then find the limiting distribution as a function of the initial conditions. Roughly speaking, this ansatz says that the bulk of the replacements in the Bak-Sneppen model occur in a decreasing fraction of the population as the number \(N\) of agents tends to infinity. Agreement with experimental results of the BS model is excellent.