Rational functions and piecewise-linear maps in non-archimedean geometry

Abstract:

Let X be an irreducible algebraic variety over a non-archimedean field k. Vladimir Berkovich has attached to it an analytic space $X^{an}$ with very good topological properties (it is locally compact and locally arcwise connected), and which contains plenty of “natural” piecewise-linear spaces called skeleta. More precisely, if S is a skeleton, its piecewise-linear structure is natural in the sense that it is “induced by rational functions on X”: - if f is a non-zero rational function on X, $\log |f|$ is well-defined on S (since the latter consists of Zariski-dense points), and is PL (piecewise linear); - there exist finitely many non-zero rational functions $f_1, \ldots, f_m$ on X such that the log $\log |f_i|$ induce a PL-isomorphism between S and a finite union of convex polytopes in $R^m$. In this talk, I will present a joint work with Ehud Hrushovski, François Loeser and Jinhe He, in which we prove by model-theoretic methods, under the assumption that the ground field k is algebraically closed, a finiteness result for the set of PL functions on a given skeleton S of the form $\log |f|$ with f rational and non-zero. More precisely, we show that there exist non-zero rational functions $g_1, \ldots, g_r$ on X such that the functions on S of the form $\log |g_i|$ are exactly those that can be obtained using only the +,-,min and max operations from the log $\log |g_i|$ and the constant functions with values in $\log |k^*|$.