Rational functions and piecewise-linear maps in non-archimedean geometry

Abstract:

Let $X$ be an irreducible algebraic variety over a non-archimedean field $k$. Vladimir Berkovich has attached to it an analytic space $X^{an}$ with very good topological properties (it is locally compact and locally arcwise connected), and which contains plenty of "natural" piecewise-linear spaces called skeleta. More precisely, if $S$ is a skeleton, its piecewise-linear structure is natural in the sense that it is "induced by rational functions on $X$": if $f$ is a non-zero rational function on $X$, $\log |f|$ is well-defined on $S$ (since the latter consists of Zariski-dense points), and is PL (piecewise linear); there exist finitely many non-zero rational functions $f_1, \ldots, f_m$ on $X$ such that the $\log |f_i|$ induce a PL-isomorphism between $S$ and a finite union of convex polytopes in $\mathbb{R}^m$. In this talk, I will present a joint work with Ehud Hrushovski, François Loeser and Jinhe He, in which we prove by model-theoretic methods, under the assumption that the ground field $k$ is algebraically closed, a finiteness result for the set of PL functions on a given skeleton $S$ of the form $\log |f|$ with $f$ rational and non-zero. More precisely, we show that there exist non-zero rational functions $g_1, \ldots, g_r$ on $X$ such that the functions on $S$ of the form $\log |g_j|$ are exactly those that can be obtained using only the $+,-,\min$ and $\max$ operations from the $\log |g_i|$ and the constant functions with values in $\log |k^*|$.