A relative de Rham theorem for Nash Submersions

Abstract:

For a Nash manifold $X$ and a Nash vector bundle $E$ on $X$, one can form the topological vector space of Schwartz sections of $E$, i.e. the smooth sections which decay fast along with all derivatives. It was shown by Aizenbud and Gourevitch, and independently by Luca Prelli, that for a Nash manifold $X$, the complex of Schwartz sections of the de Rham complex of $X$ has cohomologies isomorphic to the compactly supported cohomologies of $X$.

In my talk I will present a work in progress, joint with Avraham Aizenbud, to generalize this result to the relative case, replacing the Nash manifold $M$ with a Nash submersion $f: M \rightarrow N$. Using infinity categorical methods, I will define the notion of a Schwartz section of a Nash bundle $E$ over a complex of sheaves with constructible cohomologies, generalizing the notion of Schwartz section on an open semialgebraic set. I will then relate the Schwartz sections of the relative de Rham complex of a Nash submersion $f: M \rightarrow N$ with the Schwartz functions on $N$ over the derived push-forward with proper support of the constant sheaf on $M$. Finally, I will conclude with some applications to the relation between the Schwartz sections of the relative de Rham complex and the topology of the fibers of $f$. 