Revisiting Totally Positive Differential Systems: A Tutorial and New Results

Abstract:

A matrix is called totally nonnegative (TN) if all its minors are nonnegative, and totally positive (TP) if all its minors are positive. Multiplying a vector by a TN matrix does not increase the number of sign variations in the vector. In a largely forgotten paper, Schwarz (1970) considered matrices whose exponentials are TN or TP. He also analyzed the evolution of the number of sign changes in the vector solutions of the corresponding linear system.

In a seemingly different line of research, Smillie (1984), Smith (1991), and others analyzed the stability of nonlinear tridiagonal cooperative systems by using the number of sign variations in the derivative vector as an integer-valued Lyapunov function. We provide a tutorial on these fascinating research topics and show that they are intimately related. This allows to derive generalizations of the results by Smillie (1984) and Smith (1991) while simplifying the proofs. This also opens the door to many new and interesting research directions.

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