Selfdual cuspidal representations of $\mathrm{GL}(r,D)$ and distinction by an inner involution

Abstract:

Let $n$ be a positive integer, $F$ be a non-Archimedean locally compact field of odd residue characteristic $p$ and $G$ be an inner form of $\mathrm{GL}(2n,F)$. This is a group of the form $\mathrm{GL}(r,D)$ for a positive integer $r$ and division $F$-algebra $D$ of reduced degree $d$ such that $rd=2n$. Let $K$ be a quadratic extension of $F$ in the algebra of matrices of size $r$ with coefficients in $D$, and $H$ be its centralizer in $G$. We study selfdual cuspidal representations of $G$ and their distinction by $H$, that is, the existence of a nonzero $H$-invariant linear form on such representations, from the viewpoint of type theory. When $F$ has characteristic 0, we characterize distinction by $H$ for cuspidal representations of $G$ in terms of their Langlands parameter, proving in this case a conjecture by Prasad and Takloo-Bighash.