Smooth models of systems with complicated stochastic behaviors

Abstract:
To describe different levels of stochastic behavior Ergodic Theory introduces a hierarchy of ergodic properties. Among them ergodicity, mixing and the Bernoulli property are most used to study models in science with complicated "turbulent-like" behavior. However, "typical" systems in science are modeled by smooth (differentiable) dynamical systems acting on smooth phase spaces and the classical "smooth realization problem" asks whether any smooth phase space allows a dynamical system with prescribe collection of ergodic (stochastic) properties, in other words whether topology of the phase space may impose obstructions on a system to exhibit certain stochastic behavior. In this connection I also discuss two more important characteristics of stochastic behavior -- the rate of decay of correlations (rate of mixing) and the Central Limit Theorem -- and whether they too can be realized on any smooth phase space.