Soft extrapolation of band-limited functions

Abstract:

The problem of computational super-resolution asks to recover high-frequency features of an object from the noisy and blurred/band-limited samples of its Fourier transform, based on some a-priori information about the object class. A core theoretical question is to quantify the possible amount of bandwidth extension and the associated stability of recovering the missing frequency components, as a function of the sample perturbation level and the object prior.

In this work we assume that the object has a compact space/time extent in one dimension (but otherwise can be fairly arbitrary), while the low-pass window can have a super-exponentially decaying "soft" shape (such as a Gaussian). In contrast, previously known results considered only the ideal "hard" window (a characteristic function of the band) and objects of finite energy. The super-resolution problem in this case is equivalent to a stable analytic continuation of an entire function of finite exponential type. We show that a weighted least-squares polynomial approximation with equispaced samples and a judiciously chosen number of terms allows one to have a super-resolution factor which scales logarithmically with the noise level, while the pointwise extrapolation error exhibits a Holder-type continuity with an exponent derived from weighted potential theory. The algorithm is asymptotically minimax, in the sense that there is essentially no better algorithm yielding meaningfully lower error over the same smoothness class.

The results can be considered as a first step towards analyzing the much more realistic model of a sparse combination of compactly-supported "approximate spikes", which appears in applications such as synthetic aperture radar, seismic imaging and direction of arrival estimation, and for which only limited special cases are well-understood.

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