Abstract:

We study a new direction of research in "property testing": Permutation equations. Let $A$ and $B$ be two permutations in $\text{Sym}(n)$ that "almost commute"- are they a small deformation of permutations that truly commute? More generally, if $R$ is a system of word-equations in variables $X=x_1,\ldots,x_d$ and $A_1,\ldots,A_d$ are permutations which are almost a solution; are they near true solutions? It turns out that the answer to this question depends only on the group presented by the generators $X$ and relations $R$. This leads to the notions of "stable groups" and "testable groups" and calls for some group theoretic methods for answering these questions. We will present a few results and methods which were developed in recent years to check whether a group is stable/testable (e.g., using IRS's= invariant random subgroups). We will also describe the connection of this subject with locally testable codes as well as with the long-standing problem of whether every group is sofic.