A Stone-von Neumann equivalence for smooth representations and its application to degenerate Whittaker models

Abstract:

The Stone von-Neuman theorem relates the irreducible unitary representations of the Heisenberg group $H_n$ to non-trivial unitary characters of its center $Z$, and plays a crucial role in the construction of the oscillator representation for the metaplectic group. We give two extensions of this result to non-unitary and non-irreducible representations, thereby obtaining an equivalence of categories between certain representations of $Z$ and those of $H_n$. Our first result is an algebraic equivalence, which can be regarded as a generalization of Kashiwara's lemma from the theory of $\mathcal{D}$-modules. Our second result is a smooth equivalence, which involves the fundamental ideas of Ducloux on differentiable representations and smooth imprimitivity systems for Nash groups. We show how to extend the oscillator representation to the smooth setting and give an application to degenerate Whittaker models for representations of reductive groups. This is joint work with Raul Gomez and Dmitry Gourevitch.