Strong Average-Case Circuit Lower Bounds from Non-trivial Derandomization

Abstract:

We prove that for all constants $a$, $\text{NQP} = \text{NTIME}[n^{\text{polylog}(n)}]$ cannot be $(1/2 + 2^{-(\log a n)})$-approximated by $2^{\log^a n}$-size $\text{ACC}^0$ of THR circuits ($\text{ACC}^0$ circuits with a bottom layer of threshold gates). Previously, it was even open whether $\text{E}^{\text{NP}}$ can be $(1/2+1/\sqrt{n})$-approximated by $\text{AC}^0[2]$ circuits.

More generally, we establish a connection showing that, for a typical circuit class $C$, non-trivial nondeterministic $\text{CAPP}$ algorithms imply strong $(1/2 + 1/n^{\omega(1)})$ average-case lower bounds for nondeterministic time classes against $C$ circuits. The existence of such (deterministic) algorithms is much weaker than the widely believed conjecture $\text{PromiseBPP} = \text{PromiseP}$.

Our new results build on a line of recent works, including [Murray and Williams, STOC 2018], [Chen and Williams, CCC 2019], and [Chen, FOCS 2019]. In particular, it strengthens the corresponding $(1/2 + 1/\text{polylog}(n))$-inapproximability average-case lower bounds in [Chen, FOCS 2019]. The two important technical ingredients are techniques from Cryptography in $\text{NC}^0$ [Applebaum et al., SICOMP 2006], and Probabilistic Checkable Proofs of Proximity with $\text{NC}^1$-computable proofs.