On the subring of special cycles on orthogonal Shimura varieties

Abstract:

By old results with Millson, the generating series for the cohomology classes of special cycles on orthogonal Shimura varieties over a totally real field are Hilbert-Siegel modular forms. These forms arise via theta series. Using this result and the Siegel-Weil formula, we show that the products in the subring of cohomology generated by the special cycles are controlled by the Fourier coefficients of triple pullbacks of certain Siegel-Eisenstein series.

As a consequence, there are comparison isomorphisms between special subrings for different Shimura varieties. In the case in which the signature of the quadratic space $V$ is $(m,2)$ at an even number $d_+$ of archimedean places, the comparison gives a 'combinatorial model' for the special cycle ring in terms of the associated totally positive definite space.