מכון ויצמן למדע

THE WEIZMANN INSTITUTE OF SCIENCE

FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

Seminar in Geometry and Topology

Room 290C ,Ziskind Building
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at 16:15

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WIS

Tempered Manifolds and Schwartz Functions on Them

Abstract:

Schwartz functions are classically defined as smooth functions such that they, and all their (partial) derivatives, decay at infinity faster than the inverse of any polynomial. This was formulated on $\mathbb{R}^n$ by Laurent Schwartz, and later on Nash manifolds (smooth semi-algebraic varieties) by Fokko du Cloux and by Rami Aizenbud and Dima Gourevitch. In a joint work with Boaz Elazar we have extended the theory of Schwartz functions to the category of (possibly singular) real algebraic varieties. The basic idea is to define Schwartz functions on a (closed) algebraic subset of $\mathbb{R}^n$ as restrictions of Schwartz functions on $\mathbb{R}^n$.

Both in the Nash and the algebraic categories there exists a very useful characterization of Schwartz functions on open subsets, in terms of Schwartz functions on the embedding space: loosely speaking, Schwartz functions on an open subset are exactly restrictions of Schwartz functions on the embedding space, which are zero "to infinite order" on the complement to this open subset. This characterization suggests a very intuitive way to attach a space of Schwartz functions to an arbitrary (not necessarily semi-algebraic) open subset of $\mathbb{R}^n$.

In this talk, I will explain this construction, and more generally the construction of the category of tempered smooth manifolds. This category is in a sense the "largest" category whose objects "look" locally like open subsets of $\mathbb{R}^n$ (for some n), and on which Schwartz functions may be defined. In the development of this theory some classical results of Whitney are used, mainly Whitney type partition of unity (this will also be explained in the talk). As time permits, I will show some properties of Schwartz functions, and describe some possible applications. This is a work in progress.

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