Tiling the integers with translates of one tile: the Coven-Meyerowitz tiling conditions

Abstract:

It is well known that if a finite set of integers A tiles the integers by translations, then the translation set must be periodic, so that the tiling is equivalent to a factorization $A+B=\mathbb{Z}_M$ of a finite cyclic group. Coven and Meyerowitz (1998) proved that when the tiling period $M$ has at most two distinct prime factors, each of the sets $A$ and $B$ can be replaced by a highly ordered "standard" tiling complement. It is not known whether this behaviour persists for all tilings with no restrictions on the number of prime factors of $M$. In joint work with Izabella Laba (UBC), we proved that this is true for all sets tiling the integers with period $M=(pqr)^2$. In my talk I will discuss this problem and introduce some ideas from the proof.