Tiling the integers with translates of one tile: the Coven-Meyerowitz tiling conditions

Abstract:

It is well known that if a finite set of integers \( A \) tiles the integers by translations, then the translation set must be periodic, so that the tiling is equivalent to a factorization \( A+B=\mathbb{Z}_M \) of a finite cyclic group. Coven and Meyerowitz (1998) proved that when the tiling period \( M \) has at most two distinct prime factors, each of the sets \( A \) and \( B \) can be replaced by a highly ordered "standard" tiling complement. It is not known whether this behaviour persists for all tilings with no restrictions on the number of prime factors of \( M \). In joint work with Izabella Laba (UBC), we proved that this is true for all sets tiling the integers with period \( M=(pqr)^2 \). In my talk I will discuss this problem and introduce some ideas from the proof.