Uniformity for the Number of Rational Points on a Curve

Abstract:

By Faltings’s Theorem, formerly known as the Mordell Conjecture, a smooth projective curve of genus at least 2 that is defined over a number field $K$ has at most finitely many $K$-rational points. Vojta later gave a second proof. Many authors, including Bombieri, de Diego, Parshin, Remond, Vojta, proved upper bounds for the number of $K$-rational points. I will discuss joint work with Vesselin Dimitrov and Ziyang Gao where we prove that the number of points on the curve is bounded from above as a function of $K$, the genus, and the rank of the Mordell-Weil group of the curve’s Jacobian. We follow Vojta’s approach to the Mordell Conjecture and answer a question of Mazur. I will explain the new feature: an inequality for the Neron-Tate height in a family of abelian varieties. It allows us to bound from above the number of points when the modular height of the curve is sufficiently large. This suffices for Mazur’s Question.