Uniformity for the Number of Rational Points on a Curve

By Faltings's Theorem, formerly known as the Mordell Conjecture, a smooth projective curve of genus at least 2 that is defined over a number field K has at most finitely many K-rational points. Votja later gave a second proof. Many authors, including Bombieri, de Diego, Parshin, Remond, Vojta, proved upper bounds for the number of K-rational points. I will discuss joint work with Vesselin Dimitrov and Ziyang Gao where we prove that the number of points on the curve is bounded from above as a function of K, the genus, and the rank of the Mordell-Weil group of the curve's Jacobian. We follow Vojta's approach to the Mordell Conjecture and answer a question of Mazur. I will explain the new feature: an inequality for the Neron-Tate height in a family of abelian varieties. It allows us to bound from above the number of points when the modular height of the curve is sufficiently large. This suffices for Mazur's Question.