Weyl group representations and Harish-Chandra cells

Abstract:

Suppose $g$ is a semisimple Lie algebra with Weyl group $W$. Write $L(w)$ for the irreducible highest weight module of highest weight $-w\cdot \rho - \rho$. Write $J$ (for "Joseph") for the set of primitive ideals in a semisimple enveloping algebra contained in the augmentation ideal. In a 1978 paper "$W$-module structure in the primitive spectrum..." Joseph attached to each primitive ideal $I$ in $J$ a subset $\text{Lcell}(I) = \{ w \in W | \text{Ann}(L(w)) = I \}$. He showed also how to make $\text{Lcell}(I)$ into a basis for a representation $\sigma(I)$ of $W$, in such a way that $\sum_{I \in J} \sigma(I) = \text{regular representation of } W$. These representations $\sigma(I)$ are now called "left cell representations," terminology that is apparently due to Joseph (see his 1981 paper "Goldie rank in the enveloping algebra...III," page 310). Joseph proved in a 1980 paper that each left cell representation consists of exactly one copy of Joseph's "Goldie rank representation" for the primitive ideal $I$, and some additional representations that are not Goldie rank representations. For the past forty years, understanding of these left cell representations of $W$ has been at the heart of a great deal of work on representations of reductive groups. Lusztig in his 1984 book gave a description of all left cells in terms of the geometry of nilpotent orbits. Part of Lusztig's description uses Springer's parametrization of $W$ representations by irreducible representations of the equivariant fundamental group $A(O)$ for a nilpotent orbit $O$. I will discuss the "opposite" part of Lusztig's description, involving conjugacy classes in $A(O)$.