
THE WEIZMANN INSTITUTE OF SCIENCE
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE
Algebraic Geometry and Representation Theory Seminar

on Wednesday, Mar 10, 2021 at 16:30

<https://weizmann.zoom.us/j/98304397425>

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Weyl group representations and Harish-Chandra cells

Abstract:

Suppose \mathfrak{g} is a semisimple Lie algebra with Weyl group W . Write $L(\mathfrak{w})$ for the irreducible highest weight module of highest weight $-\mathfrak{w}\rho - \rho$. Write J (for "Joseph") for the set of primitive ideals in a semisimple enveloping algebra contained in the augmentation ideal. In a 1978 paper "W-module structure in the primitive spectrum..." Joseph attached to each primitive ideal I in J a subset $L_{\text{cell}}(I) = \{\mathfrak{w} \text{ in } W \mid \text{Ann}(L(\mathfrak{w})) = I\}$. He showed also how to make $L_{\text{cell}}(I)$ into a basis for a representation $\sigma(I)$ of W , in such a way that $\sum_{I \text{ in } J} \sigma(I) = \text{regular representation of } W$. These representations $\sigma(I)$ are now called "left cell representations," terminology that is apparently due to Joseph (see his 1981 paper "Goldie rank in the enveloping algebra...III," page 310). Joseph proved in a 1980 paper that each left cell representation consists of exactly one copy of Joseph's "Goldie rank representation" for the primitive ideal I , and some additional representations that are not Goldie rank representations. For the past forty years, understanding of these left cell representations of W has been at the heart of a great deal of work on representations of reductive groups. Lusztig in his 1984 book gave a description of all left cells in terms of the geometry of nilpotent orbits. Part of Lusztig's description uses Springer's parametrization of W representations by irreducible representations of the equivariant fundamental group $A(O)$ for a nilpotent orbit O . I will discuss the "opposite" part of Lusztig's description, involving conjugacy classes in $A(O)$.