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When a system of real quadratic equations has a solution

Abstract:

The existence and the number of solutions of a system of polynomial equations in $n$ variables over an algebraically closed field is a classical topic in algebraic geometry. Much less is known about the existence of solutions of a system of polynomial equations over reals. Any such problem can be reduced to a system of quadratic equations by introducing auxiliary variables. Due to the generality of the problem, a computationally efficient algorithm for determining whether a real solution of a system of quadratic equations exists is believed to be impossible. We will discuss a simple and efficient sufficient condition for the existence of a solution. While the problem and the condition are of algebraic nature, the proof relies on Fourier analysis and concentration of measure. Joint work with Alexander Barvinok.