Word maps and word measures: probability and geometry

Abstract:

Every word \( w(x_1,\ldots,x_r) \) in a free group, such as the commutator word \( w=xyx^(-1)y^(-1) \), induces a word map \( w:G^r\rightarrow G \) on every group \( G \). For \( g \) in \( G \), it is natural to ask whether the equation \( w(x_1,\ldots,x_r)=g \) has a solution in \( G^r \), and to estimate the "size" of this solution set, in a suitable sense. When \( G \) is finite, or more generally a compact group, this becomes a probabilistic problem of analyzing the distribution of \( w(x_1,\ldots,x_r) \), for Haar-random elements \( x_1,\ldots,x_r \) in \( G \). When \( G \) is an algebraic group, such as \( SL_n(C) \), one can study the geometry of the polynomial map \( w:SL_n(C)^r\rightarrow SL_n(C) \), using algebraic methods.

Such problems have been studied in the last few decades, in various settings such as finite simple groups, compact p-adic groups, compact Lie groups, and simple algebraic groups. Analogous problems have been studied for Lie algebra word maps as well. In this talk, I will mention some of these results, and explain the tight connections between the probabilistic and algebraic approaches.

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