

The Weizmann Institute of Science
Faculty of Mathematics and Computer Science

Geometric Functional Analysis and Probability Seminar

Room 155, Ziskind Building
on Thursday, Dec 12, 2024
at 13:30

Michael Simkin
MIT

will speak on

Lower tails for triangles inside the critical window

Abstract:

How likely is $G(n; p)$ to have a less-than-typical number of triangles? This is a foundational question in non-linear large deviation theory. When $p \ll 1/\sqrt{n}$ or $p \gg 1/\sqrt{n}$ the answer is fairly well-understood, with Janson's inequality applying in the former case and regularity- or container-based methods applying in the latter. We study the regime $p = c/\sqrt{n}$, with c fixed, with the large deviation event having at most η times the expected number of triangles, for a fixed $\eta \in [0, 1)$.

We prove explicit formula for the log-asymptotics of the event in question, for a wide range of pairs (c, η) . In particular, we show that for sufficiently small η (including the triangle-free case $\eta = 0$) there is a phase transition as c increases, in the sense of a non-analytic point in the rate function. On the other hand, if $\eta > 1/2$, then there is no phase transition.

As corollaries, we obtain analogous results for the $G(n; m)$ model, when $m = Cn^{3/2}$. In contrast to the $G(n; p)$ case, we show that a phase transition occurs as C increases for η .

Finally, we show that the probability of $G(n; m)$ being triangle free, where $m = Cn^{3/2}$ for a sufficiently small constant C , conforms to a Poisson heuristic.

Joint with Matthew Jenssen, Will Perkins, and Aditya Potukuchi. Based on arXiv:2410.22951 and arXiv:2411.18563.