1. Motivation and Contributions

- Analog-to-digital-converters (ADCs) play a vital role in modern information processing systems
- Ideally, the dynamic range (DR) of an ADC should be greater than the DR of the input signal, otherwise, it leads to loss of information
- Sampling is an important building block in an ADC. Typically, a band-limited signal is sampled at Nyquist rate, which is greater than twice the signal’s maximum frequency component
- High sampling rates are expensive and power consuming ADCs
- Desirable: High-DR-ADCs that operate at lower sampling rate
- This paper addresses the high DR issue of an ADC using the non-linear modulo operator
- We propose a fast and robust algorithm called LASSO-$B^2R^2$ for recovering bandlimited true samples from folding samples
- We show that the first-order difference of the residual samples is sparse and can be recovered from its partial Fourier measurements by formulating a sparse recovery problem
- We demonstrate that the proposed algorithm is robust to noise and computationally efficient compared to the existing methods

2. Problem Statement

- Modulo Sampling:

$$f(t) \in L^2(\mathbb{R}) \cap B_{wm}: \text{Finite-energy band-limited signal}$$

$$g(t) = f(t) \mod 2\pi$$

- Existing Literature:
  - Higher-Order Differences (HODs) Based [1]: Sensitive to noise and requires a sampling rate of 2\,\pi \times \text{Nyquist rate}
  - Prediction-Based [2]: Improved upon HODs and required a sampling rate that is greater than Nyquist rate
  - Beyond Bandwidth Residual Recovery [1]: Sensitive to noise and requires a sampling rate slightly higher than Nyquist. Computationally expensive and not fast enough

3. Properties of Finite-Energy BL Signals

- Time-Domain Separation: Riemann-Lebesgue Lemma: $$\lim_{T \to \infty} f(t) = 0$$

- Frequency-Domain Separation: $$Z(e^{j\omega}) = F(e^{j\omega}), \forall \omega < \pi$$

4. Observation

- $$f[n] = f[n] + f[n+1]$$ is sparse

- In practice, the actual $\lambda$ value is far less than its upper bound

5. LASSO-$B^2R^2$

- Time-Domain Separation: $$2[n] = \text{a finite } N \text{-length signal}$$

- Fourier-Domain Separation:

$$F_k(e^{j\omega}) = \sum_{n=-\infty}^{\infty} f[n] e^{-j\omega n}$$

$$|F_k(e^{j\omega})| < C$$

- Observation: $$2[n] = \text{is a sparse signal}$$

ISTA Algorithm:

$$\tilde{x}^{(k+1)} = S_{\lambda}(\tilde{x}^k - \mu \nabla J(\tilde{x}^k))$$

where $$S_{\lambda}(x) = \text{sign}(x) \max(|x| - \lambda, 0)$$

6. Results

- Simulation: Robustness

- Over-Sampling

- Limitation

7. Conclusions

- We proposed a fast and robust algorithm to recover the residual signal in modulo sampling

- We showed that the residual signal’s first-order difference is a sparse vector

- Then, the problem of recovering the residual signal is formulated as a sparse recovery problem

- Through simulations, we demonstrated that the proposed algorithm is fast and robust compared to existing methods