Modulo Sampling and Recovery with Unknown and Time-Varying Folding Parameter

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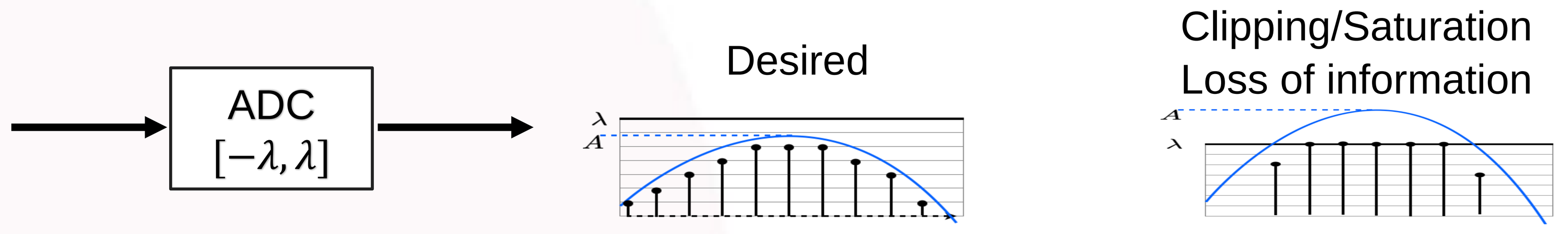
The paper

Our Lab

1. Motivation and Contributions

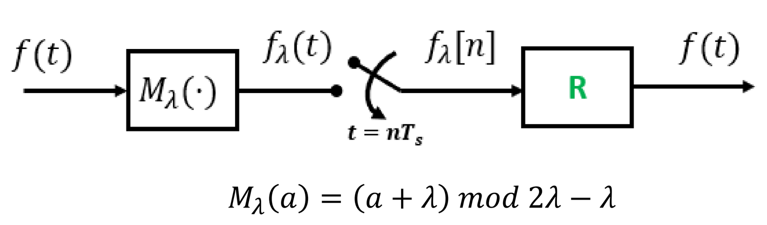
- **Addressing ADC Dynamic Range Limits:** Overcoming the challenge of capturing high dynamic range SI signals without information loss.

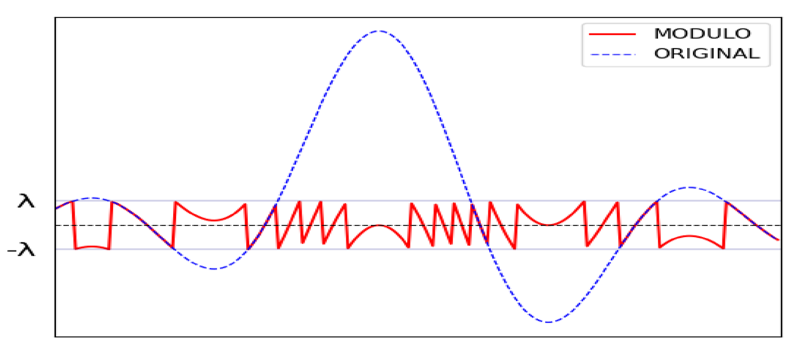


- **Modulo Sampling for unknown folding parameter:** Extending modulo sampling theoretical guarantee and recovery.
- **Unknown Time-varying folding parameter:** Extending theoretical guarantees for time-varying folding parameter.
- **Robust Performance Analysis:** Evaluating the method's effectiveness across various noise levels, oversampling rates and with or without given the folding parameter.

2. Modulo Sampling and Recovery

- **Modulo Sampling for BL:**



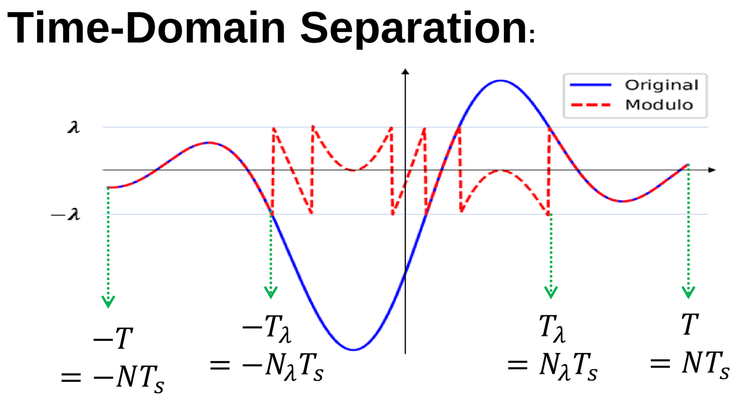


$f_\lambda[n] = f[n] + z[n],\ z[n] \in 2\lambda\mathbb{Z}$

$f(t)$ **BL signal**

$f_\lambda(t) = f(t) + z(t)$ $\xrightarrow{t = nT_s}$ $f_\lambda[n] = f[n] + z[n],\ z[n] \in 2\lambda\mathbb{Z}$

Time-Domain Separation:



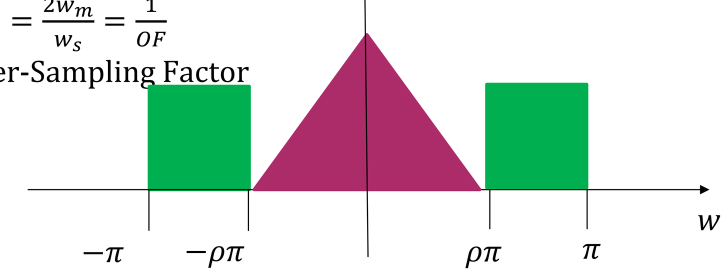
$z(t)$ Finite T_λ-duration

$z[n]$ N_λ-length signal

Frequency-Domain Separation:

where $\rho = \frac{2w_m}{w_s} = \frac{1}{OF}$

OF : Over-Sampling Factor



$$\hat{F}_\lambda\left(e^{\frac{j2\pi k}{N}}\right) = \sum_{n=0}^{N-1} \hat{z}(n) e^{\frac{-j2\pi kn}{N}},\quad \frac{2\pi k}{N} \in (\rho\pi, 2\pi - \rho\pi)$$

$$\left[\hat{\mathbf{F}}_\lambda\right]_{M\times 1} = [\mathbf{V}]_{M\times N}[\hat{\mathbf{z}}]_{N\times 1}$$

Optimization problem to recover signal

3. Theoretical Guarantees

- Theorem 3 and 4 shows identification from modulo samples with unknown folding parameter and even unknown time-varying parameter

Theorem 1
BL recovery with finite missing samples → **Theorem 3**
Modulo recovery with unknown folding parameter

↓ Extension

Theorem 2
BL recovery from tail of samples → **Theorem 4**
Modulo recovery with unknown time-varying folding

4. Recovery Algorithm

- Modulo recovery when folding parameter is unknown.

Algorithm 1 Modulo Recovery Algorithm with Unknown Folding Parameter

1: **Input:** Modulo samples $x_\lambda[nT_s]$, hyper-parameter μ
2: **Output:** Recovered signal $x[nT_s]$
3: Define $\mathcal{T}$ as DTFT followed by projection onto out-of-band frequencies and D is the discrete derivative.
4: Set $\hat{\lambda} = \max_n |x_\lambda[nT_s]|$.
5: Optimize z:

$$\hat{z} = \arg\min_{z\in\ell^2} \|\mathcal{T}x_\lambda + 2\hat{\lambda}\mathcal{T}z\|_2^2 + \mu\|Dz\|_1$$

6: Round every $\hat{z}[n]$ into the nearest integer.
7: Optimize λ again using fixed $\hat{z}$:

$$\tilde{\lambda} = \arg\min_{\lambda>0} \|\mathcal{T}x_\lambda + 2\lambda\mathcal{T}\hat{z}\|_2^2 = \frac{-\Re\langle \mathcal{T}x_\lambda, \mathcal{T}\hat{z}\rangle}{\langle \mathcal{T}\hat{z}, \mathcal{T}\hat{z}\rangle}$$

where $\Re$ is the real part.
8: Compute recovered signal $x[nT_s]$ using:

$$x[nT_s] = x_\lambda[nT_s] + 2\tilde{\lambda}\hat{z}[n]$$

- Estimate folding parameter
- Unfold signal using existing methodology and estimated folding parameter
- Estimate folding parameter again
- Reconstruction

5. Simulations

- Different SNR levels, oversampling rates, known and unknown the folding parameter

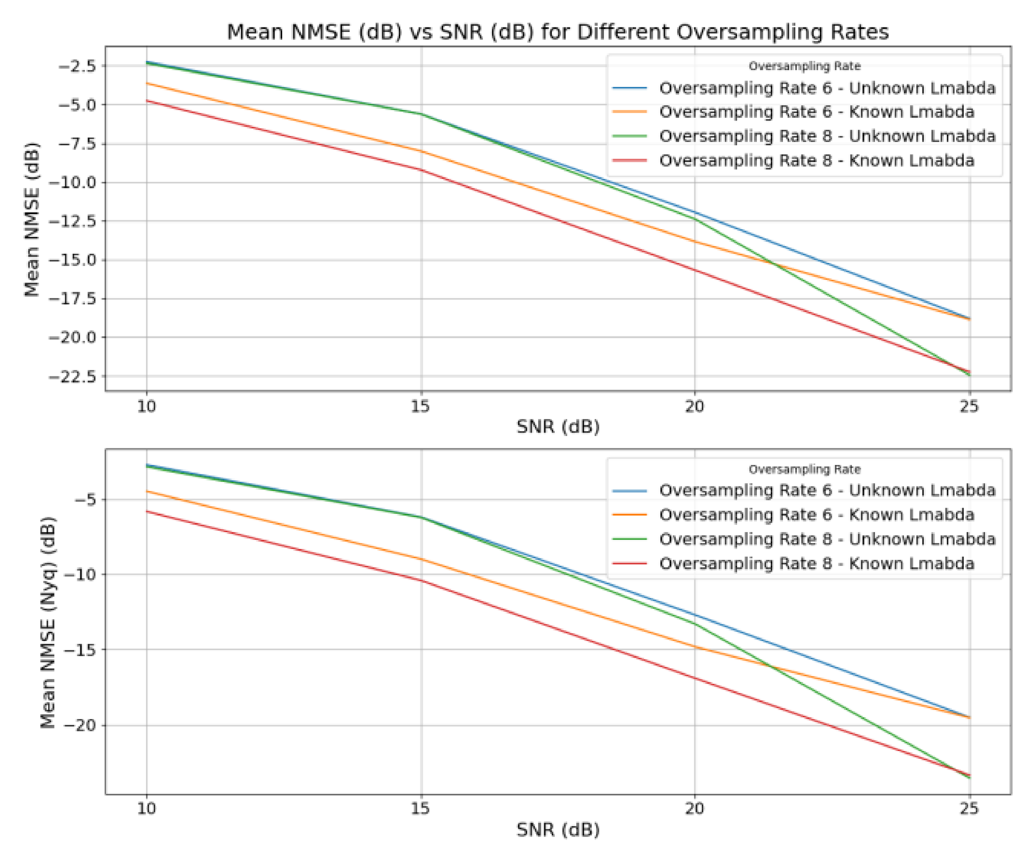


Fig. 2. Mean NMSE (dB) versus SNR (dB) for various oversampling rates, depicting performance comparisons on the sampling grid (top) and the Nyquist grid (bottom).

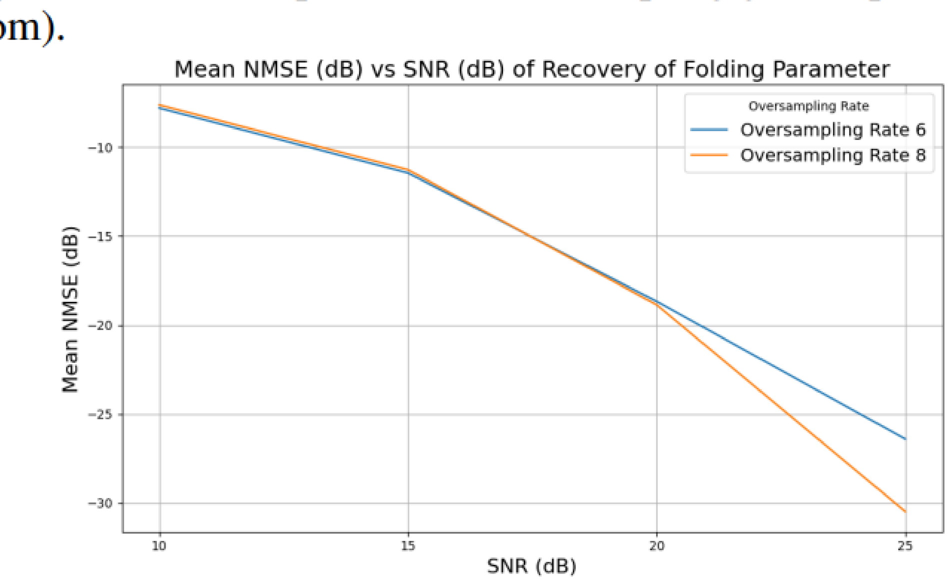


Fig. 3. Mean NMSE (dB) versus SNR (dB) for different oversampling rates, illustrating the performance of λ recovery.

6. Conclusion

- Theoretical guarantee and recovery for unknown folding parameter
- Theoretical guarantee for unknown time-varying parameter
- Performance Across Noise Levels, oversampling rate and knowing folding parameter