

A Tutorial on MIMO-OFDM ISAC: From Far-Field to Near-Field

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Abstract—Integrated sensing and communication (ISAC) is one of the key usage scenarios for future sixth-generation (6G) mobile communication networks, where communication and sensing (C&S) services are simultaneously provided through shared wireless spectrum, signal processing modules, hardware, and network infrastructure. Such an integration is strengthened by the technology trends in 6G, such as denser network nodes, larger antenna arrays, wider bandwidths, higher frequency bands, and more efficient utilization of spectrum and hardware resources, which incentivize and empower enhanced sensing capabilities. Moreover, emerging applications such as Internet-of-Everything (IoE), autonomous ground and aerial vehicles, virtual reality/augmented reality (VR/AR), and connected intelligence have intensified the demands for both high-quality C&S services, accelerating the development and implementation of ISAC in wireless networks. As the dominant waveform used in contemporary communication systems, orthogonal frequency division multiplexing (OFDM) is still expected to be a very competitive technology for 6G, rendering it necessary to thoroughly investigate the potential and challenges of OFDM ISAC. Thus, this paper aims to provide a comprehensive tutorial overview of ISAC systems enabled by large-scale multi-input multi-output (MIMO) and OFDM technologies and to discuss their fundamental principles, advantages, and enabling signal processing methods. To this end, a unified MIMO-OFDM ISAC system model is first introduced, followed by four frameworks for estimating parameters across the spatial, delay, and Doppler domains, including parallel one-domain, sequential one-domain, joint two-domain, and joint three-domain parameter estimation. Next, sensing algorithms and performance analyses are presented in detail for far-field scenarios where uniform plane wave (UPW) propagation is valid, followed by their extensions to near-field scenarios where uniform spherical wave (USW) characteristics need to be considered. Finally, this paper points out open challenges and outlines promising avenues for future research on MIMO-OFDM ISAC.

Index Terms—Integrated sensing and communication (ISAC), MIMO-OFDM ISAC, far-field, near-field, super resolution.

I. INTRODUCTION

In June 2023, the International Telecommunication Union Radiocommunication Sector (ITU-R) agreed on a draft of new recommendations regarding the framework and overall objectives for the future development of international mobile telecommunications (IMT) for 2030 and beyond, where 6G usage scenarios were addressed [1]. Among these novel scenarios, *integrated sensing and communication* (ISAC) stands out as a transformative paradigm. In fact, the 3rd Generation Partnership Project (3GPP) Service and System Aspects working group had already initiated a feasibility study on ISAC in

2022 [2]. Generally speaking, ISAC refers to a new paradigm that integrates wireless communication and sensing (C&S) capabilities into a unified system, by efficiently sharing wireless spectrum, signal processing modules, hardware, and network infrastructure, with potential joint design of waveforms and signal processing techniques [3]–[7]. The deep integration of C&S functionalities facilitates concurrent transmission of communication data and sensing of environmental information, including both static background and moving targets, thus enhancing spectrum/energy/cost efficiency and achieving appealing cooperation gains [7]–[9]. Leveraging these advantages, ISAC is expected to find a wide range of applications such as high-accuracy localization and tracking [10], target classification/imaging, environmental monitoring, augmented human senses, autonomous vehicles [11], and posture/gesture recognition [12].

ISAC originated from radar-communication coexistence (RCC) studies [13], which initially focused on interference mitigation for collocated radar and communication systems. With the evolution of wireless communication and digital radar technologies, such as multi-input multi-output (MIMO) [14], [15] and orthogonal frequency division multiplexing (OFDM) [16], the gap between C&S has been gradually bridged. This technological convergence drove the emergence of dual functional radar communication (DFRC) systems [17], [18] that achieve higher spectral efficiency (SE) and hardware integration. To enable the coexistence of radar and communication systems within the same frequency band, a portion of the sub-6 GHz bands was released for radar and communication [19], which promoted the evolution of DFRC towards ISAC. Afterwards, extensive research efforts have been devoted to optimizing ISAC performance by virtue of information theory [20]–[22], waveform design [23]–[29], signal processing techniques [30], [31], and experiments and prototyping [32], [33].

There are three typical design paradigms for ISAC, namely, communication-centric design, sensing-centric design, and co-design of C&S. In this paper, we focus on communication-centric designs that exploit communication waveforms and design frameworks to achieve sensing functions. Leveraging globally and densely deployed wireless infrastructure and networks, communication-centric ISAC has emerged as one of the most promising approaches for 6G systems, providing significant advantages over separate C&S designs as follows.

- **Efficient spectrum utilization:** The massive number of

Internet-of-Things (IoT) devices and soaring demands for high-speed data services have imposed an unprecedented strain on the limited spectrum resources available for C&S [34]. In conventional DFRC systems, C&S are performed in non-overlapping frequency bands, resulting in fragmented and poor utilization of spectrum resources. Communication-centric ISAC provides an efficient means to greatly improve spectrum efficiency, since the spectrum pool can be flexibly allocated and even completely reused by C&S tasks, as will be discussed in Section II-B.

- **Pervasive sensing via wireless communication networks:** Pervasive sensing is an important ISAC vision for 6G that cannot be achieved by traditional radar sensing networks with limited sensing coverage. This issue can be resolved by leveraging existing wireless infrastructure, including cellular base stations (BSs) and backhaul links that are globally and densely deployed. Working cooperatively, a collection of wireless networks can form a powerful and pervasive sensor network with ubiquitous coverage and connectivity.

- **Enhanced cost/energy efficiency:** Enhancing energy efficiency (EE) and cost efficiency (CE) is an important goal for 6G and ISAC. Pervasive sensing that relies on conventional standalone radars would require the deployment of a large amount of radar equipment, thus greatly degrading the EE/CE. Instead, leveraging existing communication systems to perform wireless sensing can significantly reduce the hardware cost and energy consumption for sensing, bringing great value to 6G.

Among other communication-centric designs, OFDM ISAC has received significant attention from both academia and industry due to its effective C&S performance. OFDM is the most popular waveform in contemporary communication systems, thanks to the following advantages.

- **High SE:** OFDM systems employ inverse fast Fourier transform (IFFT)/ fast Fourier transform (FFT) operations to achieve frequency domain transformations, dividing the channel bandwidth into multiple orthogonal subcarriers for parallel data transmission, thus significantly improving SE.

- **Robustness against ISI:** The use of a cyclic prefix (CP) in OFDM is a pivotal technique that can efficiently mitigate the impact of ISI caused by multipath.

- **Easy equalization:** Since OFDM equalization is performed in the frequency domain, the receiver only needs to perform one-tap equalization on each subcarrier, making it much simpler and more efficient than time-domain equalization in single-carrier systems.

- **Flexible time-frequency resource allocation:** The number and spacing of subcarriers, together with the power and bit loading to each subcarrier can be dynamically adjusted based on available channel state information (CSI), thereby maximizing network throughput and minimizing error rates while ensuring desired performance for individual users.

For target sensing, OFDM has also been considered as a promising waveform with the following advantages.

- **Array-manifold-like structure in the subcarrier and symbol domains:** After removing random information-bearing symbols, the remaining OFDM signal exhibits an array-manifold-like structure in both the subcarrier and symbol do-

mains, enabling efficient estimation of the delay and Doppler frequency for the sensing targets, as will be shown in detail in Section IV-B.

- **Efficient decoupled estimation of delay and Doppler:** The phase variations in OFDM channels induced by delay and Doppler in the subcarrier and symbol domains are independent. This allows for efficient decoupled estimation of delay and Doppler, where the data in the symbol domain can be used as snapshots for delay estimation and vice versa.

- **“Thumbtack-shaped” ambiguity function:** In both the subcarrier and symbol domains, the ambiguity function of OFDM signals with the information-bearing symbols removed exhibits a thumbtack shape, endowing OFDM waveforms with high delay and Doppler resolution, especially when the channel bandwidth is large and/or the coherent processing interval (CPI) is long.

For the above reasons, OFDM is expected to be a competitive technology for ISAC systems. Besides, its compatibility with legacy communication systems makes it a direct and standard-compliant evolutionary solution. For OFDM waveforms used in existing mobile communication systems, numerous studies have demonstrated that sensing can be achieved by using either pilot reference signals [35], [36] or the entire data frame [37] of OFDM signals, highlighting its significant potential for future 6G ISAC scenarios. However, the sensing capabilities of OFDM require further exploration. Some research has focused on optimizing MIMO-OFDM ISAC transmitter designs, such as waveforms and signal optimization [5], [38], interference management [39], power and time-frequency resource allocation [40], and beamforming design [41], but further investigation into efficient receiver signal processing techniques is still required.

To this end, three key challenges must be addressed. First, a unified mathematical model for MIMO-OFDM ISAC is required to reveal the relationship between the system configuration and sensing performance. For instance, increasing the number of subcarriers and symbols can enhance the delay and Doppler resolution of OFDM waveforms, but this also significantly increases the complexity of delay and Doppler estimation. Moreover, joint estimation of target parameters across the spatial, delay, and Doppler domains, significantly escalates signal processing complexity at the receiver, hindering real-time ISAC implementation in practice. Finally, the uniform spherical wave (USW) characteristics in near-field scenarios invalidate traditional far-field algorithms designed based on the conventional uniform plane wave (UPW) model, posing new challenges in computational complexity and estimation performance for extending far-field methods to near-field scenarios. Therefore, a comprehensive and in-depth study of mathematical models and signal processing techniques for MIMO-OFDM ISAC systems is of paramount importance.

Existing relevant overview papers [4], [6], [7], [42] have primarily focused on the fundamental limits, waveform designs, application scenarios, and challenges of generic ISAC, without focusing on MIMO-OFDM ISAC. In [5], the authors considered ISAC signal design and optimization based on OFDM, but signal processing techniques were designed only for far-field scenarios, and joint processing of three-dimensional

signals under MIMO-OFDM ISAC was not considered. Unlike prior work, this paper aims to provide a comprehensive and in-depth tutorial overview on MIMO-OFDM ISAC that focuses on signal processing techniques at the receiver side, extending from far-field to near-field scenarios. The main contributions of this work are summarized as follows.

- First, based on the six basic ISAC modes discussed by 3GPP, we present a unified mathematical ISAC model and characterize the signal transceiving relationships among different ISAC transmitters and receivers. Based on this unified model, various input-output relationships and corresponding signal processing approaches for MIMO-OFDM ISAC systems are elaborated. Moreover, we propose four frameworks for estimating parameters across the spatial, delay, and Doppler domains, including parallel one-domain, sequential one-domain, joint two-domain and joint three-domain parameter estimation. Then, a comprehensive comparison of the four frameworks is provided.
- Next, we elaborate the four parameter estimation frameworks and corresponding estimation algorithms for far-field scenarios. To this end, the application and performance of various classical radar algorithms are discussed in the context of MIMO-OFDM ISAC, including inverse discrete Fourier transform/discrete Fourier transform (IDFT/DFT)-based methods, subspace-based super-resolution methods, and compressed sensing (CS) methods. Theoretical analysis and numerical results are presented to compare the computational complexity, estimation accuracy, and resolution of these algorithms.
- Finally, we present target sensing algorithms for the more general near-field scenarios. In particular, we point out the challenges that arise when directly applying far-field based algorithms to near-field sensing problems. Subsequently, we extend the far-field algorithms to near-field sensing and study representative near-field algorithms with low complexity and super resolution. Performance comparisons and analysis of these near-field algorithms are also conducted.

The rest of this paper is organized as follows. Section II introduces the Key Performance Indicators (KPIs) and key technologies of ISAC, as well as the reasons for adopting OFDM waveforms. Section III presents a unified ISAC framework and the corresponding MIMO-OFDM ISAC signal model. Section IV and Section V detail parameter estimation methods for far-field and near-field target sensing, respectively. Section VI provides directions worthy of further investigation, and Section VII concludes this paper.

Notations: Bold face lower- and upper-case letters denote vectors and matrices, respectively. The notations $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ denote transpose, conjugate, and conjugate transpose operations, respectively. The modulus of a complex scalar is given by $|a|$, $|\mathbf{a}|$ and $|\mathbf{A}|$ denote the element-wise modulus of vector \mathbf{a} and matrix \mathbf{A} . The transformation of vector \mathbf{a} into a diagonal matrix is represented as $\text{diag}\{\mathbf{a}\}$, and $[\mathbf{a}]_m$ denotes the m th element of vector \mathbf{a} . The operator $\text{vec}(\mathbf{A})$ converts matrix \mathbf{A} into a column vector by sequentially

stacking its columns. The average of all elements of \mathbf{A} is obtained by $\text{mean}\{\mathbf{A}\}$, and $\text{angle}(a)$ represents the phase of a complex number a . The operators \mathbf{A}^\dagger and $\det(\mathbf{A})$ denote the pseudoinverse and determinant of matrix \mathbf{A} , respectively. The symbols \otimes and \odot denote the Kronecker product and Hadamard product, respectively. The rectangle pulse function is denoted as $\text{rect}(t) = 1, t \in [0, 1]$.

II. ISAC TECHNOLOGIES AND WHY OFDM ISAC

A. Key ISAC Technologies

1) *Primary ISAC Tasks and KPIs:* To evaluate the performance of ISAC systems, several KPIs have been specified for C&S services. The performance of communication tasks can be assessed from two key perspectives: efficiency and reliability. Efficiency is used to assess the amount of information successfully delivered from the transmitter to the receiver given limited resources. Reliability refers to the ability of the system to withstand adverse communication environments, such as channel fading, interference, and noise. This metric focuses on transmission quality, aiming to ensure that information is not corrupted or lost during transmission. On the other hand, sensing tasks can be generally classified into three categories, i.e., target detection, parameter estimation, and object imaging, with different KPIs. Target detection refers to determining the presence or absence of targets obscured by clutter and interference [43]. Parameter estimation refers to the problem of extracting relevant parameters from target reflections. For unbiased estimators, the Cramér-Rao bound (CRB) characterizes the theoretical lower bound on mean squared error (MSE) between the actual parameters and their estimates [22], [40]. The number of sensing degrees of freedom (SDoF) is also an important KPI for parameter estimation, characterizing the maximum number of targets whose parameters can be estimated. For multi-target sensing, resolution is an essential KPI that refers to the ability of the system to distinguish objects with similar parameter values. Object imaging is a sensing task that aims to determine the type and shape of sensed objects. Imaging deviation is used as a KPI in this case to measure the accuracy of the imaging process [44]. In addition, image entropy and image contrast can be employed to evaluate the quality of the imaging results [45].

2) *ISAC transmitter:* The design of ISAC transmission systems involves key considerations aimed at optimizing both C&S functions, such as waveform design, resource allocation, antenna configuration, and beam management. Communication is typically BS-centric with single-hop propagation, whereas sensing is governed by the classical radar distance equation [46]. A dedicated waveform that can achieve higher degrees of integration between C&S without interference is crucial. One straightforward solution is to schedule C&S on orthogonal resource blocks, so that they do not interfere with each other [47]–[49]. However, orthogonal resource allocation usually suffers from poor SE and EE. Therefore, in order to maximize the integration gain between C&S, it is more favorable to design a fully unified ISAC waveform.

Multi-antenna technology has evolved from MIMO in fourth-generation (4G) networks to massive MIMO in fifth-generation (5G) networks. For future 6G systems, several

emerging technologies may greatly improve the number of available spatial DoFs, such as extremely large-scale MIMO (XL-MIMO) [50], [51], sparse MIMO [52], [53], modular MIMO [54], [55] and fluid [56]/movable antennas [57]–[60]. As array apertures increase, the beams of future XL-MIMO and mmWave/THz ISAC systems may be extremely narrow, increasing the challenges of beam management in achieving high gain and avoiding misalignment. In the literature, transmit beamformers are often designed to minimize the discrepancy between the resulting and targeted beampattern for sensing, while minimizing multi-user interference [61]–[64]. To further address the complexity and cost incurred by fully digital MIMO, hybrid beamforming has been studied for enhancing ISAC performance in full-duplex systems [65], [66].

3) *ISAC Channel*: The mathematical models for C&S channels differ significantly. Communication channel models are typically stochastic, focusing on signal transmission quality, while sensing channels are deterministic, incorporating target scattering. The transmitters and receivers in communication systems are usually assumed to be spatially separated, while the sensing transmitters and receivers can be collocated or separated, corresponding to “monostatic sensing” or “bistatic sensing”, respectively. While multipath environments can enhance the spatial diversity of communication systems, radar sensing usually requires a line-of-sight (LoS) link for target detection. Channel fading models are also different for C&S. For instance, large- and small-scale fading are typically used to model the propagation effects in communication channels, while for radar sensing, channel fading is usually characterized in terms of target scattering characteristics, such as the radar cross section (RCS).

To integrate the acquisition of communication CSI with sensing services, several challenges need to be addressed. First, both monostatic and bistatic sensing need to be consistent with the communication system architecture. Second, the propagation characteristics exhibited by scatterers must be considered in the communication channel. In addition, precise channel models that can accurately characterize user/target mobility are needed for high-mobility applications. Recently, stochastic models and hybrid ISAC channel modeling have been proposed [67]–[69].

With the ever-increasing array apertures and the use of mmWave/terahertz frequencies, the (radiative) near-field region will significantly expand, even up to several hundreds of meters. Thus, it is crucial to consider algorithm development and performance for ISAC systems that operate in near-field scenarios. In the near-field region, the more accurate USW model needs to be considered to characterize both the phase and amplitude variations across array elements [50], [51]. The USW models for ISAC enable the joint estimation of target angle and range even with only single-anchor array processing.

Furthermore, as the operating frequency increases, ISAC signals become more susceptible to blockages, which can significantly degrade C&S performance. Fortunately, several techniques have been proposed to address this issue, such as using intelligent reflecting surfaces (IRSs) [70], [71] and fully passive metal reflectors [72] that establish virtual wireless links. Furthermore, one can also exploit prior knowledge

of the local environment to enhance C&S performance. For example, approaches based on a channel channel knowledge map (CKM) have been proposed to acquire CSI for achieving environment-aware C&S [73], [74].

4) *ISAC receiver*: Advanced signal processing techniques need to be developed for the ISAC receiver to achieve favorable C&S performance, addressing issues such as transceiver synchronization, interference management, clutter suppression and target parameter estimation. For monostatic sensing, the receiver is collocated with the ISAC transmitter, and thus it has full knowledge of the transmitted signals. As such, the sensing receiver does not need to perform a dedicated synchronization or demodulation of the communication symbols. However, self-interference occurs due to signal leakage between the transmitter and receiver, which may lead to non-linear distortion and difficulty in sensing weak targets [75], [76]. For bistatic sensing, the ISAC receiver needs to estimate and compensate for clock bias, and the communication symbols need to be demodulated before performing sensing.

Typical parameters that need to be estimated include the angle of arrival (AoA), distance, and velocity of targets. Sensing receivers usually estimate the distance and velocity parameters by analyzing the propagation delays and Doppler shifts of the echo signals, respectively. For AoA estimation, multiple antennas are needed to provide spatial resolution. These estimation problems have been studied for many decades. For example, data-based algorithms such as Capon beamforming [77] and amplitude and phase estimation (APES) [78] can be used to improve the angle estimation accuracy by optimizing the beamforming gain at the receiver. Subspace-based techniques, such as multiple signal classification (MUSIC) [79] and estimation of signal parameters via rotational invariance techniques (ESPRIT) [80], exploit the orthogonality between the signal subspace and the noise subspace to achieve super-resolution angle estimation. Compared to subspace-based algorithms, CS and sparse recovery methods exploit channel sparsity to reconstruct the transmitted signals and extract target parameters. However, sparse recovery inherently involves NP-hard ℓ_0 -norm optimization, which is typically approximated using greedy algorithms like orthogonal matching pursuit (OMP) [81]. In [82], the authors demonstrated that the ℓ_0 -norm problem can be relaxed to an ℓ_1 -norm formulation to achieve tractable solutions involving convex optimization algorithms based on the ℓ_1 -norm [83], [84] or the ℓ_1 -singular value decomposition (SVD) [85]. While both ℓ_0 - and ℓ_1 -norm minimization require discrete grids that introduce potential basis mismatch and degraded sensing accuracy, atomic norm minimization (ANM) [86], [87] avoids grid dependency but incurs higher computational complexity. ISAC techniques that use super-resolution sensing algorithms fall under the heading integrated super-resolution sensing and communication (IS²AC) [25], [88], [89], and can provide highly accurate parameter estimates. However, such algorithms are known to be sensitive to array modeling errors, and they are costly to implement for large-scale arrays. These factors must be taken into account when considering them for ISAC scenarios.

B. Potential ISAC Waveforms and OFDM ISAC

To meet the diverse C&S requirements of 6G ISAC applications, using an appropriate waveform is critical. For high communication performance, such as high data rates and low BER, waveforms should feature high SE, low peak-to-average power ratio (PAPR), robustness to Doppler effects and phase noise, resilience to nonlinear power amplifier noise, low signal processing complexity, and compatibility with legacy MIMO systems. For radar sensing, waveforms should enhance resolution, estimation accuracy, clutter rejection, and interference resistance by exhibiting low PAPR and favorable ambiguity function characteristics, such as narrow mainlobe width, low peak-to-sidelobe ratio (PSLR) or low integrated sidelobe ratio (ISLR). However, achieving all these attributes simultaneously is challenging, resulting in trade-offs between C&S performance. Based on these trade-offs, one can classify waveform design approaches as either sensing-centric, communication-centric, or joint designs.

In this paper, we focus on communication-centric ISAC systems. Communication-centric approaches leverage existing communication waveforms with various modifications to enable or enhance their sensing capability. However, the inherent randomness of communication waveforms typically degrades sensing performance due to issues like high PAPR, random autocorrelation properties, and reduced resolution [16]. Communication waveforms can be categorized as either single-carrier or multi-carrier. Single-carrier waveforms feature low PAPR and minimal nonlinear distortion [104], making them suitable for scenarios with limited coverage but high communication quality requirements. Examples include single-carrier frequency division multiple access (SC-FDMA) [90] methods, such as DFT-spread-OFDM (DFT-s-OFDM) [16], [91], widely used in the uplink of 4G and 5G systems. The authors of [23] and [92] studied single-carrier ISAC using the delay-Doppler alignment modulation (DDAM) technique, which eliminates inter-symbol interference (ISI) through path-based beamforming and delay compensation. On the other hand, multi-carrier waveforms enable increased communication rates and SE [104]. OFDM [16], [25] is a quintessential example of multi-carrier waveforms and is ubiquitously used in 4G and 5G systems, featuring a “thumbtack-shaped” ambiguity function that provides an inherent sensing capability. Additionally, emerging multi-carrier waveforms like orthogonal time frequency space (OTFS) [26], orthogonal chirp division multiplexing (OCDM) [27], and affine frequency division multiplexing (AFDM) [28] exhibit greater robustness against Doppler for high-mobility ISAC applications at the cost of higher implementation complexity.

Table I presents the main types of ISAC waveforms along with their advantages and disadvantages. In summary, sensing-centric waveforms are limited by low communication rates. For joint designs, multiplexing methods suffer from inefficient radio resource utilization, while fusion methods entail high implementation complexity and/or complex optimization problems. For communication-centric waveforms, emerging technologies like DDAM, OTFS, OCDM, and AFDM exhibit superior sensing performance in high-mobility scenarios but

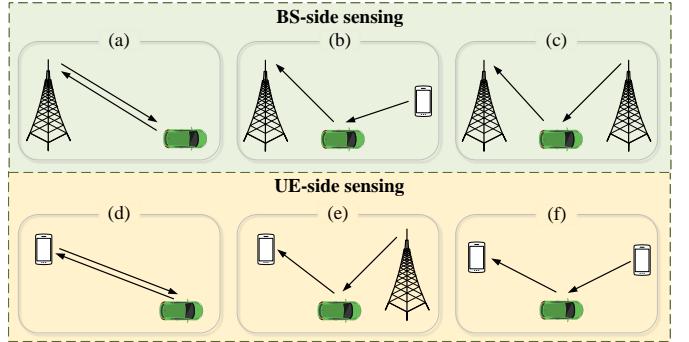


Fig. 1: Six basic ISAC modes discussed by 3GPP, with BS-side sensing modes: (a) BS monostatic, (b) UE-to-BS bistatic, (c) BS-to-BS bistatic modes, and UE-side sensing modes: (d) UE monostatic, (e) BS-to-UE bistatic, (f) UE-to-UE bistatic modes.

are often accompanied by complex signal processing and high computational demands. Consequently, OFDM remains a highly promising waveform for future ISAC wireless networks. For MIMO-OFDM ISAC systems, the use of XL-MIMO and large time-frequency bandwidths significantly enhances the sensing performance, while they simultaneously introduce substantial computational complexity for parameter estimation. This challenge becomes particularly pronounced when performing joint estimation across the spatial, delay, and Doppler domains, compounded by the additional complexity of near-field parameter estimation. Thus, in this paper, we present a tutorial overview of rigorous and unified models of MIMO-OFDM ISAC, the methods and corresponding algorithms for parameter estimation across the three domains, as well as their performance and complexity. Furthermore, the extension of far-field methods to near-field scenarios will be elaborated.

III. FUNDAMENTAL ISAC MODELS

Considering the universal applicability of ISAC frameworks to all possible waveforms, we first introduce various ISAC modes, characterizing the relationships among transmitter and receiver signals and considering the required signal processing. As shown in Fig. 1, 3GPP standardization bodies have discussed six basic ISAC modes [105], which can be divided into two categories depending on whether the sensing signal is received and processed at the BS or UE side. For BS-side sensing, as shown in Fig. 1 (a)-(c), ISAC signals could be sent by either the BS itself, a UE, or another BS, corresponding to the BS monostatic, UE-to-BS bistatic, and BS-to-BS bistatic modes, respectively. Similarly, for UE-side sensing, as shown in Fig. 1 (d)-(f), ISAC signals could be sent by either the UE itself, a BS, or another UE, corresponding to the UE monostatic, BS-to-UE bistatic, and UE-to-UE bistatic modes, respectively. In the following, we present the signal input-output relationships for C&S based on a unified ISAC model that is applicable to all the six modes above.

A. Unified ISAC Model

Fig. 2 shows a generic ISAC setup that includes the aforementioned six modes as special cases. It consists of three

TABLE I: Comparison of advantages and limitations for various ISAC waveforms.

Waveforms		Pros	Cons
Communication-centric	Single-carrier [16], [23], [90]–[92]	<ul style="list-style-type: none"> Low PAPR Low energy consumption 	<ul style="list-style-type: none"> Low SE Sensitivity to ISI Compromised sensing performance
	DDAM [23], [92]	<ul style="list-style-type: none"> Low PAPR Low guard interval overhead High SE Low complexity 	<ul style="list-style-type: none"> Reliance on channel sparsity
	OFDM [16], [25]	<ul style="list-style-type: none"> High SE Robust against ISI Flexible time-frequency resource allocation Array-manifold-like structure in subcarrier and symbol domains Efficient decoupled estimation of delay and Doppler “Thumbtack-shaped” ambiguity function Backward compatibility 	<ul style="list-style-type: none"> High PAPR Challenging self-interference cancellation Sensitivity to high Doppler spread
	OTFS [26]/OCDM [27]/AFDM [28]	<ul style="list-style-type: none"> Robust against Doppler spread High C&S performance 	<ul style="list-style-type: none"> Complex signal processing High computational complexity
Sensing-centric	Plused [93]	<ul style="list-style-type: none"> Simple for implementation Wide detection range 	<ul style="list-style-type: none"> Low communication rate Short-range sensing blind spot Long-range ambiguities
	Linear frequency modulation [29]	<ul style="list-style-type: none"> High distance resolution Low PAPR Robust against Doppler spread No short-range sensing blind spot 	<ul style="list-style-type: none"> Low communication rate
Joint design	Multiplexing [3], [94]–[96]	<ul style="list-style-type: none"> Balance C&S Simple for implementation 	<ul style="list-style-type: none"> Low radio resource utilization CDM raises the noise floor SDM limits C&S coverage
	Fusion [21], [97]–[103]	<ul style="list-style-type: none"> High C&S performance 	<ul style="list-style-type: none"> Implementation challenges Complex optimization problems

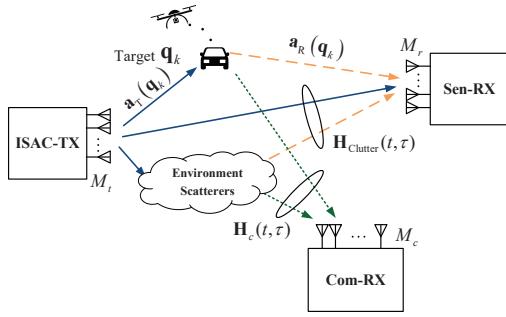


Fig. 2: A generic ISAC setup that includes all the six modes shown in Fig. 1 as special cases.

components, namely an ISAC transmitter (ISAC-TX), a communication receiver (Com-RX) and a sensing receiver (Sen-RX). The ISAC-TX transmits ISAC signals to the Com-RX to provide communication services, and to the Sen-RX for sensing K targets whose locations denoted as \mathbf{q}_k , $k = 1, 2, \dots, K$. Denote the number of antennas at the ISAC-TX, Com-RX, and Sen-RX by M_t , M_c , and M_r , respectively. For sensing, we assume that each target has an LoS path with the ISAC-TX and the Sen-RX, while such an assumption is not needed for communication. The array response vector between the ISAC-TX and target k is denoted as $\mathbf{a}_T(\mathbf{q}_k) \in \mathbb{C}^{M_t \times 1}$, which is a function of the sensing target location \mathbf{q}_k in general. Similarly, the RX array response vector is denoted as $\mathbf{a}_R(\mathbf{q}_k) \in \mathbb{C}^{M_r \times 1}$. Specific expressions for $\mathbf{a}_T(\mathbf{q}_k)$ and $\mathbf{a}_R(\mathbf{q}_k)$ in far-field and

near-field scenarios will be provided in Section III-B.

Denote the ISAC signal transmitted by the ISAC-TX as $\mathbf{x}(t) \in \mathbb{C}^{M_t \times 1}$ and the communication channel impulse response between the ISAC-TX and Com-RX as $\mathbf{H}_c(t, \tau) \in \mathbb{C}^{M_c \times M_t}$, where t and τ denote observation time and delay, respectively. Then, the communication signal $\mathbf{y}_c(t) \in \mathbb{C}^{M_c \times 1}$ received by the M_c -antenna Com-RX array is

$$\mathbf{y}_c(t) = \mathbf{H}_c(t, \tau) * \mathbf{x}(t) + \mathbf{n}(t), \quad (1)$$

where $*$ denotes convolution, and $\mathbf{n}(t) \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{M_c})$ is circularly symmetric additive white Gaussian noise (AWGN) with zero mean and variance σ^2 . Communication signal processing methods remain consistent with traditional communication systems and will not be detailed here.

For target sensing, the ISAC signal sent by the ISAC-TX is reflected/scattered by the K targets and other scatterers in the surrounding environment, and then received by the Sen-RX. An LoS link between the ISAC-TX and Sen-RX may also exist. Thus, the sensing signal $\mathbf{y}_s(t) \in \mathbb{C}^{M_r \times 1}$ received by the M_r -antenna Sen-RX array can be expressed as

$$\begin{aligned} \mathbf{y}_s(t) = & \sum_{k=1}^K \alpha_k \mathbf{a}_R(\mathbf{q}_k) \mathbf{a}_T^H(\mathbf{q}_k) \mathbf{x}(t - \tau_k - \tau_\Delta) \\ & \times e^{j2\pi(f_\Delta + v_k)t} + \mathbf{H}_{\text{Clutter}}(t, \tau) * \mathbf{x}(t) + \tilde{\mathbf{z}}(t), \end{aligned} \quad (2)$$

where α_k , τ_k , and v_k denote the complex-valued channel coefficient, the propagation delay, and Doppler frequency of the path corresponding to the k -th sensing target, respectively, τ_Δ and f_Δ are the symbol time offset (STO) and carrier

frequency offset (CFO) caused by the difference between the reference clock and local oscillators (LO) at the ISAC-TX and Sen-RX. Note that we may assume $\tau_\Delta = 0$ and $f_\Delta = 0$ for monostatic ISAC modes, while $\tau_\Delta \neq 0$ and $f_\Delta \neq 0$ in general for bistatic ISAC modes. The term $\mathbf{H}_{\text{Clutter}}(t, \tau)$ is the channel impulse response of the clutter, which consists of the signals scattered/reflected by other scatterers irrelevant to the K sensing targets and the LoS link signal between the ISAC-TX and Sen-RX. Note that for monostatic ISAC systems, the ISAC-TX and Sen-RX are collocated and operate in full-duplex mode to achieve maximum SE, making them vulnerable to self-interference (SI) due to imperfect isolation. The residual SI after the application of various cancellation techniques can also be considered as a component of the clutter. Finally, $\tilde{\mathbf{z}}(t)$ is the AWGN at the Sen-RX.

The presence of clutter signals and SI makes it more challenging to distinguish the targets from the complex environment, and the STO and CFO will cause biases in the estimation of target parameters. Thus, SI cancellation, clutter rejection, and synchronization are essential for efficient ISAC signal processing, which will be discussed in Section VI. After time and frequency synchronization, clutter rejection and SI cancellation, the resulting sensing signal $\mathbf{y}_s(t) \in \mathbb{C}^{M_r \times 1}$ in (2) reduces to

$$\mathbf{y}_s(t) = \sum_{k=1}^K \alpha_k \mathbf{a}_R(\mathbf{q}_k) \mathbf{a}_T^H(\mathbf{q}_k) \mathbf{x}(t - \tau_k) e^{j2\pi v_k t} + \mathbf{z}(t), \quad (3)$$

where the clutter rejection and SI cancellation residuals are typically considered as random noise and lumped together with the AWGN in $\mathbf{z}(t)$. The sensing signal in (3) is then exploited to estimate the parameters of the K sensing targets, including the reflection coefficient α_k , target location \mathbf{q}_k , propagation delay τ_k , and Doppler frequency shift v_k .

Note that except for some differences in the number of antennas and computational capabilities, BS-side and UE-side sensing are symmetric in terms of signal modeling and processing. Therefore, in the following, we will present detailed models and signal processing techniques for ISAC systems of the three BS-side sensing modes based on the unified ISAC model, while similar results can be applied to UE-side sensing as well. To this end, we first show that the generic ISAC setup in Fig. 2 and the resulting signal in (3) can be reduced to the three BS-side sensing modes shown in Figs. 1 (a)-(c).

- **BS monostatic ISAC mode:** For BS monostatic ISAC mode, the ISAC-TX and Sen-RX shown in Fig. 2 are collocated at the BS. In order to maximize the SE, the ISAC BS needs to employ in-band full-duplex operation for simultaneous downlink ISAC signal transmission and uplink sensing signal reception. Note that while collocated at the BS, we assume that the ISAC-TX and Sen-RX use different sets of antennas for ease of SI cancellation. The Com-RX in Fig. 2 corresponds to a UE. In this case, the generic ISAC setup in Fig. 2 reduces to the BS monostatic setup in Fig. 3(a). Furthermore, the ISAC signal $\mathbf{x}(t)$ in (1)-(3) is transmitted by the BS, and is denoted as $\mathbf{x}_{\text{BS}}(t) \in \mathbb{C}^{M_t \times 1}$.
- **UE-to-BS bistatic ISAC mode:** For UE-to-BS bistatic

ISAC mode, the ISAC-TX in Fig. 2 corresponds to a UE, while the Com-RX and Sen-RX are collocated at the BS, as shown in Fig. 3(b). Note that unlike the BS monostatic mode in Fig. 3(a), since only signal reception needs to be performed at the BS in Fig. 3(b), the Com-RX and Sen-RX may use the same set of antennas. However, synchronization is essential due to the use of different local oscillators (LO) and reference clocks by the ISAC BS and the UE. Furthermore, the ISAC signal $\mathbf{x}(t)$ in (1)-(3) is transmitted by the UE, and is denoted as $\mathbf{x}_{\text{UE}}(t) \in \mathbb{C}^{M_t \times 1}$.

- **BS-to-BS bistatic ISAC mode:** For BS-to-BS bistatic ISAC mode, the ISAC-TX in Fig. 2 corresponds to a BS, while the Com-RX and Sen-RX occur at a UE and another BS, respectively, as shown in Fig. 3(c). Note that unlike UE-to-BS bistatic mode in Fig. 3(b), although BS bistatic mode also requires synchronization, it can be achieved through a wired connection between the two ISAC BSs. Furthermore, the ISAC signal $\mathbf{x}(t)$ in (1)-(3) is broadcast by the transmit BS, and is denoted as $\mathbf{x}_{\text{BS}}(t) \in \mathbb{C}^{M_t \times 1}$.

The corresponding ISAC-TX, Com-RX, and Sen-RX of the unified model in Fig. 2, as well as the required processing for the six ISAC modes, are summarized in Table II.

In the following, we focus on ISAC systems implemented with OFDM waveforms, which is the dominant waveform in 4G and 5G communication systems, and is likely to continue to play a critical role for 6G. In OFDM-ISAC systems, target sensing can be performed based on pilots only or on the fully reconstructed transmitted OFDM data frame. The former approach can be easily implemented, but its sensing performance is limited by the availability of time-frequency resources allocated to pilots. The latter can fully exploit all of the time-frequency resources for target sensing using known (for monostatic sensing) or demodulated communication symbols (for bistatic sensing) for parameter estimation, at the expense of increased complexity. In this paper, we employ the OFDM frame-based approach for sensing tasks using all time-frequency resources allocated to both pilots and communication data.

B. MIMO-OFDM ISAC Signal Processing and Modeling

For MIMO-OFDM ISAC, the ISAC-TX transmits OFDM signals over N subcarriers and P OFDM symbols. The subcarrier spacing and the OFDM symbol duration without the CP are denoted by Δf and T , respectively, where $T = \frac{1}{\Delta f}$. The system bandwidth is thus $B = N\Delta f$. Let T_{cp} denote the CP duration, so the OFDM symbol duration including CP is $T_s = T + T_{\text{cp}}$. To avoid ISI for communication, the CP duration should be no smaller than the multi-path delay spread τ_{com} of the communication channel $\mathbf{H}_c(t, \tau)$, since the Com-RX only begins receiving communication signals upon receiving its first path. However, for target sensing, the CP duration should be larger than the propagation delay τ_{sen} corresponding to the ISI-free sensing distance [106], which is different from the case of communications since the Sen-RX begins to receive signals as soon as the ISAC-TX transmits, so that no nearby

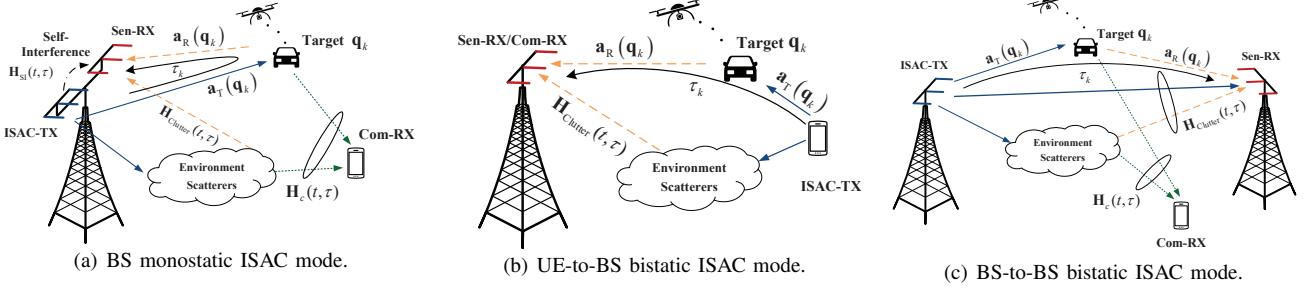


Fig. 3: An illustration of the three BS-side sensing ISAC modes.

TABLE II: The unified ISAC model and its special cases corresponding to the six basic modes.

ISAC modes		Transmitter and receiver			Required processing		
Unified model		ISAC-TX	Com-RX	Sen-RX	SI cancellation	Synchronization	Clutter rejection
BS-side sensing	BS monostatic	BS	UE	BS	✓		✓
	UE-to-BS bistatic	UE	BS	BS		✓	✓
	BS-to-BS bistatic	BS-1	UE	BS-2		✓	✓
UE-side sensing	UE monostatic	UE	BS	UE	✓		✓
	BS-to-UE bistatic	BS	UE	UE		✓	✓
	UE-to-UE bistatic	UE-1	UE-2	UE-2		✓	✓

targets are missed. Thus, in ISAC systems, the CP should be set as $T_{\text{cp}} \geq \tau_{\text{max}}$, where $\tau_{\text{max}} = \max\{\tau_{\text{com}}, \tau_{\text{sen}}\}$. Thus, the OFDM signal $\mathbf{x}(t) \in \mathbb{C}^{M_t \times 1}$ transmitted by the M_t -antenna ISAC-TX array is given by

$$\mathbf{x}(t) = \sum_{p=0}^{P-1} \sum_{n=0}^{N-1} \mathbf{w}_{n,p} b_{n,p} e^{j2\pi n \Delta f (t - pT_s - T_{\text{cp}})} \text{rect}\left(\frac{t - pT_s}{T_s}\right), \quad (4)$$

where $\mathbf{w}_{n,p} \in \mathbb{C}^{M_t \times 1}$ and $b_{n,p}$ are the transmit beamforming vector and the transmitted symbol on the n -th subcarrier of the p -th OFDM symbol, respectively. Contrary to traditional OFDM radar, where all subcarriers and symbols can be exploited to sense targets, a user typically cannot be allocated all of the time-frequency resources for communications. Thus, $\mathbf{w}_{n,p}$ depends on the users' resource allocation and beam management. In addition, if a user occupies the (n, p) -th resource element (RE), $b_{n,p} \neq 0$ is the information-bearing symbol; otherwise, $b_{n,p} = 0$. Later, in Section VI-C, we will further discuss this issue.

Substituting (4) into (3), the resulting sensing signal $\mathbf{y}_s(t) \in \mathbb{C}^{M_r \times 1}$ for radar processing at the Sen-RX is

$$\mathbf{y}_s(t) = \sum_{k=1}^K \sum_{p=0}^{P-1} \sum_{n=0}^{N-1} \alpha_k^{n,p} \mathbf{a}_R(\mathbf{q}_k) b_{n,p} e^{j2\pi n \Delta f (t - \tau_k - pT_s - T_{\text{cp}})} \times e^{j2\pi v_k t} \text{rect}\left(\frac{t - \tau_k - pT_s}{T_s}\right) + \mathbf{z}(t), \quad (5)$$

where $\alpha_k^{n,p} = \alpha_k \mathbf{a}_T^H(\mathbf{q}_k) \mathbf{w}_{n,p}$ is the equivalent channel coefficient including the impact of transmit beamforming. Then, the signal in (5) is partitioned into P blocks, each of duration T_s . After discarding the CP, the p -th ($p = 0, \dots, P-1$) block is given by

$$\mathbf{y}_s^p(t) = \mathbf{y}_s(t + pT_s + T_{\text{cp}}) \text{rect}\left(\frac{t}{T_s}\right), \quad t \in [0, T). \quad (6)$$

The Sen-RX samples the sensing signal with sampling rate $B = N\Delta f$. To avoid inter-carrier interference (ICI) caused by large Doppler frequency shifts, the subcarrier spacing is typically assumed to be much larger than the Doppler frequency shift, i.e., $\Delta f \gg |v_k|, \forall k = 1, 2, \dots, K$. Under this assumption, the q -th ($q = 0, 1, \dots, N-1$) sample of the p -th block can be expressed as

$$\begin{aligned} \mathbf{y}_s^p[q] &= \mathbf{y}_s^p(q/B) \\ &= \sum_{k=1}^K \sum_{n=0}^{N-1} \alpha_k^{n,p} \mathbf{a}_R(\mathbf{q}_k) b_{n,p} e^{j2\pi n \Delta f (q/B - \tau_k)} \\ &\quad \times e^{j2\pi v_k (q/B + pT_s + T_{\text{cp}})} + \mathbf{z}^p[q] \quad (7) \\ &\approx \sum_{k=1}^K \sum_{n=0}^{N-1} \bar{\alpha}_k^{n,p} \mathbf{a}_R(\mathbf{q}_k) b_{n,p} e^{j2\pi n q/N} e^{-j2\pi n \Delta f \tau_k} \\ &\quad \times e^{j2\pi p T_s v_k} + \mathbf{z}^p[q], \end{aligned}$$

where $\bar{\alpha}_k^{n,p} = \alpha_k^{n,p} e^{j2\pi v_k T_{\text{cp}}}$, $\mathbf{z}^p[q]$ is the resulting noise at the p -th block. Note that in (7), we have used the fact that $B = N\Delta f$ and $|v_k q/B| < |v_k N/B| = |v_k|/\Delta f$, so that $e^{j2\pi v_k q/B} \approx 1$ for $|v_k|/\Delta f \ll 1$.

After performing an N -point DFT for each of the P blocks in (7), we obtain the following frequency-domain signal for the n -th ($n = 0, 1, \dots, N-1$) subcarrier of the p -th ($p = 0, 1, \dots, P-1$) OFDM block $\bar{\mathbf{y}}_s^p[n] \in \mathbb{C}^{M_r \times 1}$ as

$$\begin{aligned} \bar{\mathbf{y}}_s^p[n] &= \frac{1}{N} \sum_{q=0}^{N-1} \mathbf{y}_s^p[q] e^{-j2\pi n q/N} \\ &= b_{n,p} \sum_{k=1}^K \bar{\alpha}_k^{n,p} \mathbf{a}_R(\mathbf{q}_k) e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k} + \bar{\mathbf{z}}^p[n], \end{aligned} \quad (8)$$

where $\bar{\mathbf{z}}^p[n]$ is the resulting AWGN in the frequency domain. As a result, we obtain a total of $M_r \times N \times P$ signal values across the spatial, subcarrier, and symbol domains, which can

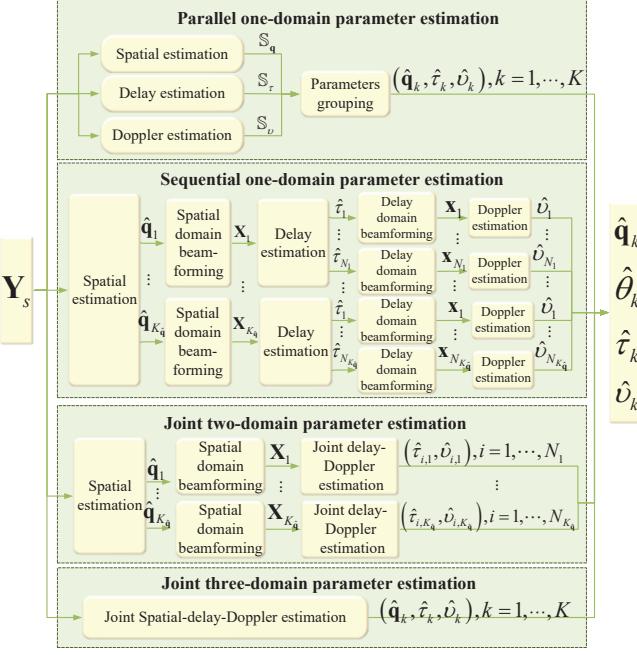


Fig. 4: The four parameter estimation frameworks in MIMO-OFDM ISAC.

be organized as a tensor $\mathbf{Y}_s \in \mathbb{C}^{M_r \times N \times P}$ by setting

$$\mathbf{Y}_s(:, n, p) = \bar{\mathbf{y}}_s^p[n]. \quad (9)$$

Parameter estimation is one of the most important sensing tasks in ISAC systems, as it underpins functions like target localization, tracking, and imaging. Therefore, in the following, we will focus on the estimation of the location \mathbf{q}_k (or the angle θ_k in the far-field case), the propagation delay τ_k , and the Doppler frequency shift v_k of the sensing targets. Generally speaking, the three parameters of each target can be estimated from the spatial, delay and Doppler domains. Inspired by traditional joint parameter estimation methods [107], [108], we propose four frameworks to estimate target parameters across the three domains. As shown in Fig. 4, depending on whether the parameters are estimated jointly or separately in each domain, the four frameworks are parallel one-domain, sequential one-domain, joint two-domain and joint three-domain parameter estimation. The parallel one-domain approach separately estimates the sets of target locations $\mathbb{S}_q = \{\mathbf{q}_1, \dots, \mathbf{q}_K\}$ or angles $\mathbb{S}_\theta = \{\theta_1, \dots, \theta_K\}$, delays $\mathbb{S}_\tau = \{\tau_1, \dots, \tau_K\}$ and Doppler shifts $\mathbb{S}_v = \{v_1, \dots, v_K\}$, and then associates the individual parameter estimated into groups $(\mathbf{q}_k/\theta_k, \tau_k, v_k)$ for each of the K sensing targets. This approach has a relatively low computational complexity, but its sensing performance is limited by potential errors in the parameter grouping.

On the other hand, the sequential one-domain approach performs spatial, delay and Doppler domain estimation sequentially. For example, spatial domain estimation can be applied first to determine the K_q distinguishable locations/angles. When the spatial resolution is limited, it may not be possible to distinguish all K targets in the spatial domain alone. Thus, in general, $K_q \leq K$. After location/angle estimation, beamforming can be applied to separate the K_q signals in the spatial domain, denoted as $\mathbf{X}_{k_q}, k_q = 1, \dots, K_q$.

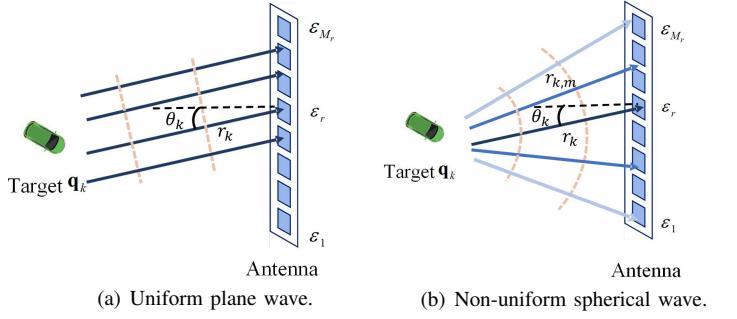


Fig. 5: Illustration of far-field UPW versus near-field USW.

Subsequently, delay and Doppler domain processing can be applied. Note that the order in which the different types of parameters are estimated depends on the resolution and target separation in the three domains. The sequential one-domain estimation method does not require parameter grouping, but its complexity is greater than parallel one-domain processing.

Similar to the sequential one-domain method, joint two-domain parameter estimation first determines the parameter in one domain, and then exploits beamforming methods to separate the signals of different targets, but it jointly estimates the parameters in the remaining two domains. Fig. 4 illustrates the special case of joint processing of the delay-Doppler domains after location/angle estimation has been performed, while other combinations are also possible. This method can achieve higher estimation accuracy, but its complexity is much higher than that of the decoupled estimation methods. Last, joint three-domain parameter estimation refers to joint estimation of all parameters. This approach can achieve the highest estimation accuracy but also incurs the highest computational complexity.

The details of each method will be discussed in Sections IV and V for the far-field and near-field cases, respectively. Note that the TX and RX steering vectors $\mathbf{a}_T(\mathbf{q}_k)$ and $\mathbf{a}_R(\mathbf{q}_k)$ are functions of target location \mathbf{q}_k , which in general depend on both the angle of departure (AoD)/AoA θ_k and distance r_k between the target and the reference antenna. In the following, we present expressions for the RX steering vectors for the far-field and near-field scenarios. Similar results can be obtained for the TX array response.

C. Array Response Vector: From Far-Field to Near-Field

When the BS array aperture is small, targets are typically located in the far-field region, and the conventional UPW model accurately characterizes the variations of signal phases across the array, as shown in Fig. 5(a). However, for large scale arrays, targets are more likely to be located in the near-field region, and the USW assumption should be used [50], [51], as shown in Fig. 5(b). Let the location of target k be represented as $\mathbf{q}_k = [r_k \cos \theta_k, r_k \sin \theta_k]^T, k = 1, \dots, K$, where r_k is the distance between target k and a given reference antenna, and $\theta_k \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is the angle of target k with respect to the normal vector of the array. Then, the receive steering vector $\mathbf{a}_R(\mathbf{q}_k)$ can be expressed as

$$[\mathbf{a}_R(\mathbf{q}_k)]_m = e^{j \frac{2\pi}{\lambda} (r_{k,m} - r_k)}, m = 1, \dots, M_r, \quad (10)$$

where $r_{k,m}$ denotes the distance between the k -th target and the m -th receive antenna. For narrowband signals, half-wavelength inter-antenna spacing is typically assumed, i.e., $d = \frac{\lambda}{2}$. If we assume the r -th antenna is the reference located at the origin of the array's coordinate system, the positions of the M_r antennas can be written as $[0, \varepsilon_m d], m = 1, \dots, M_r$, where $|\varepsilon_m d|$ is the distance between the m -th antenna and the reference, and $\varepsilon_r = 0$. Thus, $r_{k,m}$ can be written as

$$r_{k,m} = \sqrt{r_k^2 - 2\varepsilon_m d r_k \sin \theta_k + \varepsilon_m^2 d^2}. \quad (11)$$

The classical criterion for distinguishing the near- and far-field regions is the Rayleigh distance, given by $r_{\text{Rayleigh}} = \frac{2D^2}{\lambda}$, where D is the aperture of the antenna array [51], [109], [110]. When $r_k > r_{\text{Rayleigh}}$, the target is assumed to be located in the far-field, so that the UPW model is appropriate and $r_{k,m}$ can be simplified by performing a first-order Taylor series approximation of (11), as shown in Fig. 5(a). Thus, $r_{k,m}$ can be simplified as

$$r_{k,m} \approx r_k - \varepsilon_m d \sin \theta_k, \quad (12)$$

and the corresponding steering vector in (10) becomes

$$\mathbf{a}_R(\theta_k) = \left[e^{-j\frac{2\pi}{\lambda}\varepsilon_m d \sin \theta_k} \right], m = 1, \dots, M_r. \quad (13)$$

For near-field sensing with $r_k < r_{\text{Rayleigh}}$ shown in Fig. 5(b), substituting the more general expression in (11) into (10) leads to

$$\mathbf{a}_R(r_k, \theta_k) = \left[e^{j\frac{2\pi}{\lambda}(\sqrt{r_k^2 - 2\varepsilon_m d r_k \sin \theta_k + \varepsilon_m^2 d^2} - r_k)} \right]. \quad (14)$$

Comparing (13) and (14), we see that in the far-field, only the angle can be estimated from one snapshot signal on different antenna elements, while both target angle and distance can be estimated in the near-field. The general expression in (14) can be simplified using a second-order Taylor approximation of (11) [51], leading to

$$r_{k,m} \approx r_k - \varepsilon_m d \sin \theta_k + \frac{\varepsilon_m^2 d^2}{2r_k} \cos^2 \theta_k, \quad (15)$$

with corresponding array response vector

$$\mathbf{a}_R(r_k, \theta_k) = \left[e^{j(\omega_k \varepsilon_m + \psi_k \varepsilon_m^2)} \right], m = 1, \dots, M_r, \quad (16)$$

where $\omega_k = -\frac{2\pi d \sin \theta_k}{\lambda}$ and $\psi_k = \frac{\pi d^2 \cos^2 \theta_k}{\lambda r_k}$. Note that in this model ω_k only depends on θ_k , while ψ_k depends on both θ_k and r_k . The quadratic phase model of (16) is referred to as the Fresnel approximation for near-field sources.

In Sections IV and V, we will present typical parameter estimation algorithms based on the four methods presented in Fig. 4, first for far-field and then for near-field scenarios.

IV. FAR-FIELD MIMO-OFDM ISAC

In this section, we elaborate on how to sense far-field targets based on the signal processing methods illustrated in Fig. 4, demonstrate the application variants of various standard radar algorithms under MIMO-OFDM ISAC and analyze their complexity and performance.

A. Far-field Sensing Signal Model

For ease of exposition, we assume that the Sen-RX is equipped with a ULA with half-wavelength inter-antenna

spacing. Setting the first element as the reference element, the position of the m -th element can be obtained as

$$\varepsilon_m d = (m-1)d, m = 1, \dots, M_r. \quad (17)$$

Substituting (17) into (13), the steering vector $\mathbf{a}_R(\mathbf{q}_k) = \mathbf{a}_R(\theta_k) \in \mathbb{C}^{M_r \times 1}$ of the k -th target can be written as

$$\mathbf{a}_R(\theta_k) = \left[1, e^{j2\pi \frac{d}{\lambda} \sin \theta_k}, \dots, e^{j2\pi(M_r-1) \frac{d}{\lambda} \sin \theta_k} \right]^H, \quad (18)$$

and the steering matrix $\mathbf{A}_\theta \in \mathbb{C}^{M_r \times K}$ is defined as

$$\mathbf{A}_\theta = [\mathbf{a}_R(\theta_1), \mathbf{a}_R(\theta_2), \dots, \mathbf{a}_R(\theta_K)]. \quad (19)$$

Substituting (18) into (8) and (9), the resulting sensing data tensor \mathbf{Y}_s for far-field MIMO-OFDM ISAC can be expressed as

$$\begin{aligned} \mathbf{Y}_s(:, n, p) = & b_{n,p} \sum_{k=1}^K \bar{\alpha}_k^{n,p} \mathbf{a}_R(\theta_k) e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k} \\ & + \bar{\mathbf{z}}^p[n]. \end{aligned} \quad (20)$$

For far-field MIMO-OFDM ISAC, the parameters to be estimated for each sensing target are the AoA θ_k , propagation delay τ_k , and Doppler frequency v_k . In the following, we will discuss how various classic radar algorithms for parameter estimation can be applied to far-field MIMO-OFDM ISAC systems, such as the IDFT/DFT and subspace-based methods. IDFT/DFT-based methods like the periodogram [111] can be easily implemented, but the resolution is limited by the array aperture, signal bandwidth and CPI, respectively. On the other hand, subspace-based methods such as MUSIC [79] and ESPRIT [80], can achieve super-resolution parameter estimation, but they usually require high computational complexity. Additionally, CS and sparse recovery methods, such as OMP [81], are also commonly used for parameter estimation. Compared to IDFT/DFT and subspace-based methods, CS achieves higher accuracy under low SNR and sparse channel conditions but incurs higher complexity, especially with large signal dimensions, making it difficult to implement in real time. Moreover, in scenarios with dense targets or clutter, CS methods are prone to locally optimal or suboptimal solutions, hence degrading estimation performance.

The signals to/from different users can be separated using orthogonal time-frequency resources and then processed individually in OFDM ISAC systems. In the following, for ease of exposition, we consider the case of a single user with collocated time-frequency allocation, namely, the transmit beamforming vectors $\mathbf{w}_{n,p}$ in (4) are identical for all subcarriers and symbols $\bar{\alpha}_k^{n,p} = \bar{\alpha}_k = \alpha_k \mathbf{a}_T^H(\mathbf{q}_k) \mathbf{w} e^{j2\pi v_k T_{cp}}, \forall n = 0, 1, \dots, N-1, p = 0, 1, \dots, P-1$. As a result, the signal in (20) reduces to

$$\begin{aligned} \mathbf{Y}_s(:, n, p) = & b_{n,p} \sum_{k=1}^K \bar{\alpha}_k \mathbf{a}_R(\theta_k) e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k} \\ & + \bar{\mathbf{z}}^p[n]. \end{aligned} \quad (21)$$

Based on the signal model in (21), we below detail the various far-field MIMO-OFDM ISAC signal processing methods illustrated in Fig. 4.

B. Decoupled Parameter Estimation

Since the phase variations of the sensing signal in (21) in the angle, delay and Doppler domains are completely decoupled,

the three parameters (θ_k, τ_k, v_k) can be estimated separately or sequentially using the parallel and sequential one-domain parameter estimation methods in Fig. 4.

1) Parallel One-domain Parameter Estimation:

AoA estimation problem: In far-field scenarios, the spatial domain processing can only provide AoA information for each target. In this case, the resulting OFDM sensing data in the subcarrier and symbol domains can be treated as snapshots. As such, the $M_r \times N \times P$ dimensional signal \mathbf{Y}_s in (21) can be re-organized as an $M_r \times Q_{N,P}$ dimensional matrix \mathbf{X}_θ , with $Q_{N,P} = NP$, where $\mathbf{X}_\theta(:, q_{n,p}) = \mathbf{Y}_s(:, n, p)$, $q_{n,p} = n + pN$, $n \in [0, N-1]$, $p \in [0, P-1]$. Thus, \mathbf{X}_θ can be equivalently expressed in matrix form as

$$\mathbf{X}_\theta = \mathbf{A}_\theta \mathbf{S}_\theta + \mathbf{Z}_\theta, \quad (22)$$

where $\mathbf{A}_\theta \in \mathbb{C}^{M_r \times K}$ is the steering matrix given in (19), \mathbf{Z}_θ is the resulting noise, and $\mathbf{S}_\theta \in \mathbb{C}^{K \times Q_{N,P}}$ is given by

$$\mathbf{S}_\theta(k, q_{n,p}) = b_{n,p} \bar{\alpha}_k e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k}. \quad (23)$$

The AoA estimation problem corresponds to estimating the angles $\mathbb{S}_\theta = \{\theta_1, \dots, \theta_K\}$ of the K sensing targets embedded in steering matrix \mathbf{A}_θ , given data matrix \mathbf{X}_θ . Specific algorithms to address this problem will be presented in Section IV-B3.

Delay estimation problem: The random information-bearing symbols $b_{n,p}$ in (21) depend only on subcarrier index n and symbol index p , not on antenna index m . They affect the manifold structure in the subcarrier and symbol domains, but not in the spatial domain. Thus, unlike AoA estimation in (22), to ensure the manifold structure in the subcarrier and symbol domains, the information-bearing symbols $b_{n,p}$ in (21) need to be removed. The transmitted data is known a priori at the Sen-RX in the monostatic ISAC mode, or it can be demodulated and detected in the bi-static ISAC mode. Thus, the communication symbols can be removed by performing element-wise division on the received signal in (21), i.e.,

$$\begin{aligned} \bar{\mathbf{Y}}_s(:, n, p) &= \frac{\mathbf{Y}_s(:, n, p)}{b_{n,p}} \\ &= \sum_{k=1}^K \bar{\alpha}_k \mathbf{a}_R(\theta_k) e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k} + \bar{\mathbf{Z}}(:, n, p), \end{aligned} \quad (24)$$

where $\bar{\mathbf{Z}}(:, n, p) = \frac{\bar{\mathbf{z}}^p[n]}{b_{n,p}}$ is the resulting noise.

For delay estimation, the data in the antenna and symbol domains can be treated as snapshots. As such, the $M_r \times N \times P$ dimensional data matrix after element-wise symbol division in (24) can be restructured into an $N \times Q_{M_r,P}$ dimensional matrix \mathbf{X}_τ , with $Q_{M_r,P} = M_r P$, where $\mathbf{X}_\tau(:, q_{m,p}) = \bar{\mathbf{Y}}_s(m, :, p)$, $q_{m,p} = m + pM_r$, $m \in [0, M_r-1]$, $p \in [0, P-1]$. Then, matrix \mathbf{X}_τ can be expressed as

$$\mathbf{X}_\tau = \mathbf{A}_\tau \mathbf{S}_\tau + \mathbf{Z}_\tau, \quad (25)$$

where $\mathbf{A}_\tau \in \mathbb{C}^{N \times K}$ is the steering matrix in the subcarrier domain given by

$$\mathbf{A}_\tau = [\mathbf{a}_\tau(\tau_1), \dots, \mathbf{a}_\tau(\tau_K)], \quad (26)$$

and $\mathbf{a}_\tau(\tau_k) \in \mathbb{C}^{N \times 1}$ is the steering vector in the delay domain given by

$$\mathbf{a}_\tau(\tau_k) = [1, e^{j2\pi \Delta f \tau_k}, \dots, e^{j2\pi (N-1) \Delta f \tau_k}]^H. \quad (27)$$

Besides, the $(k, q_{m,p})$ -th element of $\mathbf{S}_\tau \in \mathbb{C}^{K \times Q_{M_r,P}}$ is

$$\mathbf{S}_\tau(k, q_{m,p}) = \bar{\alpha}_k e^{-j2\pi m \frac{d}{\lambda} \sin(\theta_k)} e^{j2\pi p T_s v_k}. \quad (28)$$

The delay estimation problem is to estimate the propagation delays $\mathbb{S}_\tau = \{\tau_1, \dots, \tau_K\}$ of the K sensing targets embedded in steering matrix \mathbf{A}_τ , given data matrix \mathbf{X}_τ in (25).

Doppler estimation problem: For Doppler estimation, the data in the antenna and subcarrier domains can be treated as snapshots. The signal after element-wise symbol division in (24) can be restructured into a $P \times Q_{M_r,N}$ dimensional matrix \mathbf{X}_v , with $Q_{M_r,N} = M_r N$, where $\mathbf{X}_v(:, q_{m,n}) = \bar{\mathbf{Y}}_s(m, n, :)$, $q_{m,n} = m + nM_r$. Then, matrix \mathbf{X}_v can be written as

$$\mathbf{X}_v = \mathbf{A}_v \mathbf{S}_v + \mathbf{Z}_v, \quad (29)$$

where \mathbf{Z}_v is corresponding noise, and $\mathbf{A}_v \in \mathbb{C}^{P \times K}$ is the steering matrix in the Doppler domain, given by

$$\mathbf{A}_v = [\mathbf{a}_v(v_1), \dots, \mathbf{a}_v(v_K)], \quad (30)$$

and $\mathbf{a}_v(v_k) \in \mathbb{C}^{P \times 1}$ is the steering vector in the symbol domain given by

$$\mathbf{a}_v(v_k) = [1, e^{-j2\pi T_s v_k}, \dots, e^{-j2\pi (P-1) T_s v_k}]^H. \quad (31)$$

The $(k, q_{m,n})$ -th element of $\mathbf{S}_v \in \mathbb{C}^{K \times Q_{M_r,N}}$ is given by

$$\mathbf{S}_v(k, q_{m,n}) = \bar{\alpha}_k e^{-j2\pi m \frac{d}{\lambda} \sin(\theta_k)} e^{-j2\pi n \Delta f \tau_k}. \quad (32)$$

The Doppler estimation problem is to estimate the Doppler frequencies $\mathbb{S}_v = \{v_1, \dots, v_K\}$ of the K sensing targets based on steering matrix \mathbf{A}_v , given data matrix \mathbf{X}_v in (29).

After estimating the AoAs $\mathbb{S}_\theta = \{\hat{\theta}_1, \dots, \hat{\theta}_K\}$, delays $\mathbb{S}_\tau = \{\hat{\tau}_1, \dots, \hat{\tau}_K\}$, and Doppler frequencies $\mathbb{S}_v = \{\hat{v}_1, \dots, \hat{v}_K\}$ separately via parallel one-domain parameter estimation, the individual estimation results have to be associated with the corresponding targets. In general, $K_{\hat{\theta}}, K_{\hat{\tau}}, K_{\hat{v}} \leq K$, since the targets are not necessarily distinguishable in all three domains. Two common parameter grouping methods are based on signal correlation and power detection, which first group the parameters estimated from the three domains one by one.

There are a total of $K_{\hat{\theta}} K_{\hat{\tau}} K_{\hat{v}}$ possible parameter groups, obtained as $(\hat{\theta}_{k_{\hat{\theta}}}, \hat{\tau}_{k_{\hat{\tau}}}, \hat{v}_{k_{\hat{v}}})$, $\forall k_{\hat{\theta}} = 1, \dots, K_{\hat{\theta}}, k_{\hat{\tau}} = 1, \dots, K_{\hat{\tau}}, k_{\hat{v}} = 1, \dots, K_{\hat{v}}$. In the signal correlation method, the reconstructed signal $\hat{\mathbf{Y}}_{k_{\hat{\theta}}, k_{\hat{\tau}}, k_{\hat{v}}} \in \mathbb{C}^{M_r \times N \times P}$ for parameter group $(\hat{\theta}_{k_{\hat{\theta}}}, \hat{\tau}_{k_{\hat{\tau}}}, \hat{v}_{k_{\hat{v}}})$ is expressed as

$$\hat{\mathbf{Y}}(m, n, p) = e^{-j2\pi m \frac{d}{\lambda} \sin(\hat{\theta}_{k_{\hat{\theta}}})} e^{-j2\pi n \Delta f \hat{\tau}_{k_{\hat{\tau}}}} e^{j2\pi p T_s \hat{v}_{k_{\hat{v}}}}, \quad (33)$$

where $m \in [0, M_r-1]$, $n \in [0, N-1]$, $p \in [0, P-1]$. Then, the correlation coefficient \hat{C} between $\hat{\mathbf{Y}}$ and $\bar{\mathbf{Y}}_s$ in (24) is calculated as

$$\hat{C} = \left| \text{mean} \left\{ \hat{\mathbf{Y}}^* \odot \bar{\mathbf{Y}}_s \right\} \right|. \quad (34)$$

The correct parameter groups will exhibit a strong correlation between the reconstructed signal and signal $\bar{\mathbf{Y}}_s$. Thus, the K parameter pairs $(\hat{\theta}_{k_{\hat{\theta}}}, \hat{\tau}_{k_{\hat{\tau}}}, \hat{v}_{k_{\hat{v}}})$ with the highest correlation coefficient \hat{C} correspond to the K sensing targets.

For power based detection, the power of $\bar{\mathbf{Y}}_s$ in (24) in the angle-delay-Doppler domain at the location of each parameter group $(\hat{\theta}_{k_{\hat{\theta}}}, \hat{\tau}_{k_{\hat{\tau}}}, \hat{v}_{k_{\hat{v}}})$ can be obtained as

$$\hat{P} = \frac{1}{M_r N P} \left| \mathbf{a}_\tau^H(\hat{\tau}_{k_{\hat{\tau}}}) \left(\mathbf{a}_R^H(\hat{\theta}_{k_{\hat{\theta}}}) \bar{\mathbf{Y}}_s \right) \mathbf{a}_v^*(\hat{v}_{k_{\hat{v}}}) \right|^2, \quad (35)$$

where $\mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}})$, $\mathbf{a}_\tau(\hat{\tau}_{k_{\hat{\tau}}})$, and $\mathbf{a}_v(\hat{v}_{k_{\hat{v}}})$ are the steering vectors in the spatial, delay, and Doppler domains in (18), (27), and

(31), respectively. The K parameter groups $(\hat{\theta}_{k_{\hat{\theta}}}, \hat{\tau}_{k_{\hat{\theta}}}, \hat{v}_{k_{\hat{\theta}}})$ with the highest power \hat{P} are taken to be the parameters of the K sensing targets.

2) Sequential One-domain Parameter Estimation:

The sequential one-domain parameter estimation method, as shown in Fig. 4, estimates the target parameters in sequence over the three domains. It typically begins in the domain with the highest resolution so that more targets can be distinguished. For example, consider a case in which the array has many elements, while the bandwidth is narrow and the CPI is short. In this case, the angular resolution is highest, followed by the delay and Doppler resolutions, so it is best to first estimate the AoA of the sensing targets using \mathbf{X}_θ in (22), and then use beamforming to extract the signal of targets at different angles. Then, the delays are estimated for targets at each angle. Since there may be multiple targets at the same angle, beamforming (matched filtering) can be applied in the delay domain to further distinguish targets with different delays. Finally, the corresponding Doppler frequency shift is estimated for targets at each delay. Thus, the target parameters in the three domains are directly grouped in this sequential method.

Specifically, as shown in Fig. 4, assume that a total of $K_{\hat{\theta}}$ AoAs are estimated as $\hat{\theta}_{k_{\hat{\theta}}}, k_{\hat{\theta}} = 1, \dots, K_{\hat{\theta}}$, based on (22), where $K_{\hat{\theta}} \leq K$ since not all targets are distinguishable in the spatial domain. Let $N_{k_{\hat{\theta}}} \geq 1$ denote the number of targets at each estimated angle and $K = \sum_{k_{\hat{\theta}}=1}^{K_{\hat{\theta}}} N_{k_{\hat{\theta}}}$. Before performing delay domain estimation, we first apply spatial domain beamforming based on the estimated AoAs in order to exploit the beamforming gain and to decouple the signals in the spatial domain. After performing beamforming on (24), the resulting signal $\mathbf{X}_{k_{\hat{\theta}}} \in \mathbb{C}^{N \times P}, k_{\hat{\theta}} = 1, \dots, K_{\hat{\theta}}$, is

$$\begin{aligned} \mathbf{X}_{k_{\hat{\theta}}}(n, p) &= \mathbf{r}^H(\hat{\theta}_{k_{\hat{\theta}}}) \bar{\mathbf{Y}}_s(:, n, p) \\ &= \sum_{k \in \Omega_{k_{\hat{\theta}}}} \bar{\alpha}_k \mathbf{r}^H(\hat{\theta}_{k_{\hat{\theta}}}) \mathbf{a}_R(\theta_k) e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k} \\ &\quad + \sum_{k \notin \Omega_{k_{\hat{\theta}}}} \bar{\alpha}_k \mathbf{r}^H(\hat{\theta}_{k_{\hat{\theta}}}) \mathbf{a}_R(\theta_k) e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k} + \bar{z}(q_{n,p}) \\ &\approx \sum_{i=1}^{N_{k_{\hat{\theta}}}} \tilde{\alpha}_{i,k_{\hat{\theta}}} e^{-j2\pi n \Delta f \tau_{i,k_{\hat{\theta}}}} e^{j2\pi p T_s v_{i,k_{\hat{\theta}}}} + \bar{z}(q_{n,p}), \end{aligned} \quad (36)$$

where $\Omega_{k_{\hat{\theta}}} = \{k | \theta_k \approx \hat{\theta}_{k_{\hat{\theta}}}\}$, $\tilde{\alpha}_{i,k_{\hat{\theta}}} = \bar{\alpha}_{i,k_{\hat{\theta}}} \mathbf{r}^H(\hat{\theta}_{k_{\hat{\theta}}}) \mathbf{a}_R(\theta_{i,k_{\hat{\theta}}})$ is the resulting complex channel gain, $\tau_{i,k_{\hat{\theta}}}$ and $v_{i,k_{\hat{\theta}}}$ are the delay and Doppler of the i -th target at the estimated angle $\hat{\theta}_{k_{\hat{\theta}}}$, and $\mathbf{r}(\hat{\theta}_{k_{\hat{\theta}}})$ is the beamforming vector used in the spatial domain for the estimated angle $\hat{\theta}_{k_{\hat{\theta}}}$.

There are three classic beamforming methods that are used to achieve a balance between complexity and performance. These methods include maximum-ratio combining (MRC), zero-forcing (ZF), and MMSE. The expressions for the corresponding beamforming vectors are given by

$$\begin{aligned} \mathbf{r}_{\text{MRC}}(\hat{\theta}_{k_{\hat{\theta}}}) &\triangleq \mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}}) / \|\mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}})\|, \\ \mathbf{r}_{\text{ZF}}(\hat{\theta}_{k_{\hat{\theta}}}) &\triangleq \hat{\mathbf{A}}_{k_{\hat{\theta}}} \mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}}) / \|\hat{\mathbf{A}}_{k_{\hat{\theta}}} \mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}})\|, \\ \mathbf{r}_{\text{MMSE}}(\hat{\theta}_{k_{\hat{\theta}}}) &\triangleq \mathbf{C}_{k_{\hat{\theta}}}^{-1} \mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}}) / \|\mathbf{C}_{k_{\hat{\theta}}}^{-1} \mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}})\|, \end{aligned} \quad (37)$$

where $\hat{\mathbf{A}}_{k_{\hat{\theta}}} \triangleq \mathbf{I}_{M_r} - \mathbf{A}_{k_{\hat{\theta}}} \left(\mathbf{A}_{k_{\hat{\theta}}}^H \mathbf{A}_{k_{\hat{\theta}}} \right)^{-1} \mathbf{A}_{k_{\hat{\theta}}}^H$, $\mathbf{A}_{k_{\hat{\theta}}} = [\mathbf{a}_R(\hat{\theta}_1), \dots, \mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}-1}), \mathbf{a}_R(\hat{\theta}_{k_{\hat{\theta}}+1}), \dots, \mathbf{a}_R(\hat{\theta}_{K_{\hat{\theta}}})]$, and $\mathbf{C}_{k_{\hat{\theta}}} = \sum_{k \neq k_{\hat{\theta}}}^{K_{\hat{\theta}}} \frac{|\bar{\alpha}_k|^2}{\sigma^2} \mathbf{a}_R(\hat{\theta}_k) \mathbf{a}_R(\hat{\theta}_k)^H + \mathbf{I}_{M_r}$. In the delay and Doppler domains, similar beamformers can be employed.

The approximation in (36) holds if different AoA clusters are sufficiently separated such that the residual signals from other angles can be considered as noise. Then, $\mathbf{X}_{k_{\hat{\theta}}}$ in (36) can be expressed in matrix form as

$$\mathbf{X}_{k_{\hat{\theta}}} = \mathbf{A}_{\tau, k_{\hat{\theta}}} \text{diag}(\tilde{\alpha}) \mathbf{A}_{v, k_{\hat{\theta}}}^T + \mathbf{Z}_{k_{\hat{\theta}}}, \quad (38)$$

where $\tilde{\alpha} = [\tilde{\alpha}_1, \dots, \tilde{\alpha}_{N_{k_{\hat{\theta}}}}]^T$ is the vector of resulting complex-valued gains, $\mathbf{A}_{\tau, k_{\hat{\theta}}} = [\mathbf{a}_\tau(\tau_1), \dots, \mathbf{a}_\tau(\tau_{N_{k_{\hat{\theta}}}})]$ and $\mathbf{A}_{v, k_{\hat{\theta}}} = [\mathbf{a}_v(v_1), \dots, \mathbf{a}_v(v_{N_{k_{\hat{\theta}}}})]$ are the steering matrices in the delay and Doppler domains for the targets located at angle $\hat{\theta}_{k_{\hat{\theta}}}$. Then, the matrices $\mathbf{X}_{k_{\hat{\theta}}}, k_{\hat{\theta}} = 1, \dots, K_{\hat{\theta}}$, can be exploited to estimate the delays $\mathbb{S}_{\tau, k_{\hat{\theta}}} = \{\tau_{i,k_{\hat{\theta}}} | i = 1, \dots, N_{k_{\hat{\theta}}}\}$ of the targets at different angles. Furthermore, for targets with different delays at each angle, further target discrimination can be achieved through beamforming in the delay domain. In particular, the signal for a target whose AoA and delay are estimated as $(\hat{\theta}_{k_{\hat{\theta}}}, \hat{\tau}_{i,k_{\hat{\theta}}})$ can be reduced to $\mathbf{x}_{i,k_{\hat{\theta}}}^T = \mathbf{r}^H(\hat{\tau}_{i,k_{\hat{\theta}}}) \mathbf{X}_{k_{\hat{\theta}}}, i = 1, \dots, N_{k_{\hat{\theta}}}$. Then, the Doppler $\hat{v}_{i,k_{\hat{\theta}}}$ can be estimated from $\mathbf{x}_{k_{\hat{\theta}}} \in \mathbb{C}^{P \times 1}$ for the target.

3) One-dimensional Parameter Estimation Algorithms:

In the previous subsections, we have modeled the one-dimensional (1D) parameter estimation problems in (22), (25), (29), (38) for AoA, delay, and Doppler estimation. It is not difficult to see that all these problems share a similar structure, and thus the problems can be solved using essentially the same algorithms. In this section, we provide examples of 1D AoA estimation algorithms, while they can be easily modified for delay and Doppler estimation.

Periodogram: For any snapshot $q_{n,p}$ of signal $\mathbf{X}_\theta \in \mathbb{C}^{M_r \times Q_{N,P}}$ in (22), the noise-free power spectrum $f(\theta)$ in the angular domain can be obtained as

$$\begin{aligned} f(\theta) &= \frac{1}{M_r} \left| \mathbf{a}_R^H(\theta) \mathbf{X}_\theta(:, q_{n,p}) \right|^2 \\ &= \frac{1}{M_r} \left| b_{n,p} \sum_{k=1}^K \bar{\alpha}_k e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k} \right. \\ &\quad \left. \times \sum_{m=0}^{M_r-1} e^{j2\pi m \frac{d}{\lambda} (\sin \theta - \sin \theta_k)} \right|^2 \\ &= \frac{1}{M_r} \left| b_{n,p} \sum_{k=1}^K \bar{\alpha}_k e^{-j2\pi n \Delta f \tau_k} e^{j2\pi p T_s v_k} \right. \\ &\quad \left. \times \frac{\sin(\pi \frac{d}{\lambda} M_r (\sin \theta - \sin \theta_k))}{\sin(\pi \frac{d}{\lambda} (\sin \theta - \sin \theta_k))} \right|^2 \\ &\stackrel{(a)}{\approx} \frac{|b_{n,p}|^2}{M_r} \sum_{k=1}^K |\bar{\alpha}_k|^2 \left| \frac{\sin(\pi \frac{d}{\lambda} M_r (\sin \theta - \sin \theta_k))}{\sin(\pi \frac{d}{\lambda} (\sin \theta - \sin \theta_k))} \right|^2 \end{aligned} \quad (39)$$

where $\mathbf{a}_R(\theta)$ is the steering vector in the spatial domain in (18), $\theta \in [-90^\circ, 90^\circ]$ is the observation angle, and θ_k is the k -th target angle. The approximation in (a) holds if the angular difference between the targets is larger than the

angular resolution of the array. The angular resolution of an array is based on the Rayleigh limit [112], which corresponds to half the width between the null points of a focused beam of the array. Form (39), the null points for a beam pointed at θ_k can be obtained by $\pi \frac{d}{\lambda} M_r (\sin \theta - \sin \theta_k) = \pm \pi$, so the beamwidth is given by

$$\text{BW}_\theta = \sin \theta - \sin \theta_k = \frac{2\lambda}{M_r d} \approx \frac{2\lambda}{D}, \quad (40)$$

where $D = M_r d$ denotes the array aperture. The maximum resolution is achieved when $\theta_k \rightarrow 0^\circ$, and then

$$\Delta\theta = \arcsin \left(\frac{\lambda}{M_r d} \right) \approx \frac{\lambda}{D}. \quad (41)$$

Thus, the angular resolution of a ULA is inversely proportional to the effective array aperture. This means that as the effective array aperture increases, the beamwidth becomes narrower, thus enhancing the ability of the array to resolve different targets.

Similarly, for delay and Doppler estimation, the power spectrum in the delay and Doppler domains can be derived from signals \mathbf{X}_τ and \mathbf{X}_v in (25) and (29), respectively. By calculating the corresponding null-to-null bandwidth, the minimum resolvable difference in the target delay and Doppler frequency are given by

$$\Delta\tau = \frac{1}{N\Delta f} = \frac{1}{B}, \quad \Delta v = \frac{1}{PT_s} = \frac{1}{\text{CPI}}. \quad (42)$$

Thus, the delay and Doppler resolution are inversely proportional to system bandwidth B and CPI, respectively. In contrast to the angular resolution, which depends on the target angle, the delay and Doppler resolution are independent of the target parameters.

In practice, calculating the power spectrum in the angular domain (see (39)) involves an exhaustive search over $\theta \in [-90^\circ, 90^\circ]$, resulting in high computational complexity. To address this issue, the periodogram algorithm discretizes the phase $\varphi_\theta = \frac{d}{\lambda} \sin \theta \in [-\frac{d}{\lambda}, \frac{d}{\lambda}]$ in the angular domain into N_θ^{IFFT} points and employs the IFFT to accelerate the computation. When $N_\theta^{\text{IFFT}} > M_r$, zero-padding is utilized to convert $\mathbf{X}_\theta \in \mathbb{C}^{M_r \times Q_{N,P}}$ in (22) to $\mathbf{X}_\theta^{\text{ZP}} \in \mathbb{C}^{N_\theta^{\text{IFFT}} \times Q_{N,P}}$. In particular, the periodogram $\mathbf{f}_\theta \in \mathbb{C}^{N_\theta^{\text{IFFT}} \times 1}$ can be obtained by performing an IFFT on each column of matrix $\mathbf{X}_\theta^{\text{ZP}}$ and then applying column-wise summation, which is given by

$$\begin{aligned} \mathbf{f}_\theta &= \frac{1}{Q_{N,P}} \sum_{q=0}^{Q_{N,P}-1} \left| \text{IFFT}_{N_\theta^{\text{IFFT}}} \{ \mathbf{X}_\theta^{\text{ZP}}(:, q) \} \right|^2 \\ &= \frac{1}{Q_{N,P}} \sum_{q=0}^{Q_{N,P}-1} \frac{1}{M_r} \left| \mathbf{W}_{N_\theta^{\text{IFFT}}} \mathbf{X}_\theta^{\text{ZP}}(:, q) \right|^2, \end{aligned} \quad (43)$$

where $|\cdot|$ denotes element-wise modulus, and \mathbf{W}_N is the N -point IDFT matrix, expressed as

$$\mathbf{W}_N = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & W^{-1} & \dots & W^{-(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & W^{-(N-1)} & \dots & W^{-(N-1)(N-1)} \end{bmatrix}, \quad (44)$$

with $W = e^{-j\frac{2\pi}{N}}$. Additionally, since the default phase of the IFFT is $[0, \pi]$, a period transformation for \mathbf{f}_θ is required to shift

TABLE III: Simulation setup for monostatic MIMO-OFDM ISAC.

Parameters	Values
Number of antennas at the Sen-RX	$M_r = 16$
Number of subcarriers	$N = 128$
Number of OFDM symbols	$P = 64$
Subcarrier spacing	$\Delta f = 120 \text{ kHz}$
CP duration	$T_{\text{cp}} = T/4 = 8.33 \mu\text{s}$
Carrier frequency	$f_c = 28 \text{ GHz}$
SNR of the radar processing signal at the Sen-RX in (5)	SNR = 10 dB
Number of targets	$K = 3$
AoA	$(-20, 10, 45)^\circ$
Distance	$(20, 80, 50) \text{ m}$
Delay	$(0.13, 0.53, 0.33) \mu\text{s}$
Radial velocity	$(8, 12, 20) \text{ m/s}$
Doppler frequency	$(1.49, 2.24, 3.73) \text{ kHz}$

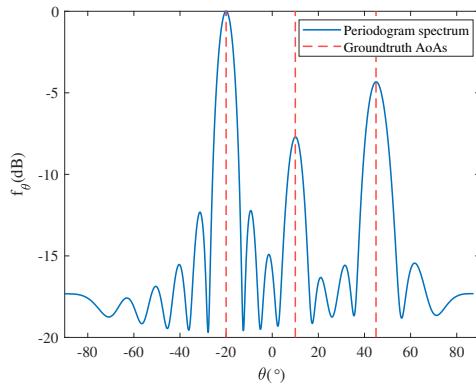


Fig. 6: Periodogram spectrum for AoA estimation in MIMO-OFDM ISAC.

the period to $[-\pi/2, \pi/2]$. Then, the peak index \hat{n}_θ^k of the periodogram $\mathbf{f}_\theta \in \mathbb{C}^{N_\theta^{\text{IFFT}} \times 1}$ indicates the phase $\frac{d}{\lambda} \sin \hat{\theta}_k = \frac{\hat{n}_\theta^k}{N_\theta^{\text{IFFT}}}, \hat{n}_\theta^k \in \left[-\frac{N_\theta^{\text{IFFT}}}{2}, \frac{N_\theta^{\text{IFFT}}}{2} \right]$, in the angular domain. As such, the AoAs of the sensing targets can be estimated as

$$\hat{\theta}_k = \arcsin \left(\frac{\hat{n}_\theta^k \lambda}{N_\theta^{\text{IFFT}} d} \right), \forall k = 1, \dots, K. \quad (45)$$

Consider a monostatic ISAC with the simulation setup given in Table III. The periodogram spectrum for AoA estimation after normalization and conversion to dB is shown in Fig. 6. It is observed that higher spectral peaks occur at groundtruth AoAs, while lower sidelobes are present at other angles. The height of the spectral peaks correspond to the strength of the target echos, and targets with weaker echo signals may be obscured by the sidelobes of stronger targets. When the target angle approaches 0° , the width of the spectral peak becomes narrower, which indicates that it can be more easily distinguished by the periodogram method.

MUSIC: The MUSIC algorithm exploits the orthogonality between the noise and signal subspaces to estimate AoAs/delays/Doppler frequencies. These subspaces can be obtained via an eigenvalue decomposition (EVD) of the covariance matrix of \mathbf{X}_θ in (22). This can be expressed mathematically as

$$\begin{aligned} \mathbf{R}_\mathbf{X} &= \frac{1}{NP} \mathbf{X}_\theta \mathbf{X}_\theta^H = \mathbf{A}_\theta \mathbf{R}_s \mathbf{A}_\theta^H + \sigma^2 \mathbf{I}_{M_r} \\ &= \mathbf{E}_s \mathbf{\Sigma}_s \mathbf{E}_s^H + \mathbf{E}_n \mathbf{\Sigma}_n \mathbf{E}_n^H, \end{aligned} \quad (46)$$

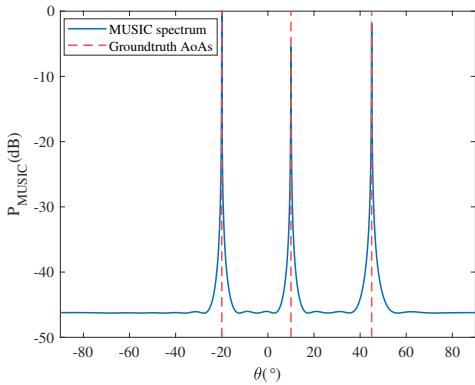


Fig. 7: MUSIC spectrum for AoA estimation in MIMO-OFDM ISAC.

where $\mathbf{R}_s = \frac{1}{NP} \mathbf{S}_\theta \mathbf{S}_\theta^H$, σ^2 is the noise power, $\mathbf{\Sigma}_s$ and $\mathbf{\Sigma}_n$ are diagonal matrices composed of the K largest eigenvalues and the remaining $M_r - K$ eigenvalues, respectively, \mathbf{E}_s and \mathbf{E}_n denote the eigenvectors that span the signal and noise subspaces. As such, the MUSIC spectrum can be written as

$$P_{\text{MUSIC}}(\theta) = \frac{1}{\mathbf{a}_R^H(\theta) \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}_R(\theta)}, \quad (47)$$

where $\mathbf{a}_R(\theta)$ is the steering vector in the spatial domain in (18), and the peaks of the MUSIC spectrum correspond to the AoAs of sensing targets.

For the simulation setup in Table III, the AoA spectrum of the MUSIC algorithm after normalization is shown in Fig. 7. It is observed that both the MUSIC and periodogram algorithms can accurately estimate the AoAs of all three targets. However, the spectrum of MUSIC and periodogram differ primarily in two aspects. First, the spectral peaks of MUSIC are sharper, leading to higher angular resolution. Second, the spectrum of MUSIC is smoother than the periodogram, which can prevent weaker targets from being obscured by the sidelobes of stronger targets, hence reducing estimation errors. It is also important to note that subspace methods such as MUSIC and ESPRIT (discussed next) rely on knowledge of the number of AoAs in order to work properly. This is a drawback compared with the non-parametric periodogram approach.

ESPRIT: The ESPRIT algorithm exploits the rotational invariance between two subarrays in an array to estimate AoAs. For the signal model in (22), by taking the first $M_r - 1$ antennas of the ULA as subarray 1, and the last $M_r - 1$ antennas as subarray 2, the received signals of the two subarrays can be expressed as

$$\mathbf{X}_1 = \mathbf{X}_\theta(1 : M_r - 1, :) = \mathbf{A}_1 \mathbf{S}_\theta + \mathbf{Z}_1, \quad (48)$$

$$\mathbf{X}_2 = \mathbf{X}_\theta(2 : M_r, :) = \mathbf{A}_2 \mathbf{S}_\theta + \mathbf{Z}_2 = \mathbf{A}_1 \mathbf{\Phi} \mathbf{S}_\theta + \mathbf{Z}_2,$$

where $\mathbf{A}_1 = \mathbf{A}_\theta(1 : M_r - 1, :)$ and $\mathbf{A}_2 = \mathbf{A}_\theta(2 : M_r, :)$ denote the steering matrices of the two subarrays, respectively, \mathbf{Z}_1 and \mathbf{Z}_2 are corresponding noises. The two subarray steering matrices are related by $\mathbf{A}_2 = \mathbf{A}_1 \mathbf{\Phi}$, where $\mathbf{\Phi}$ denotes the rotation operator determined by the target angles, and is expressed as

$$\mathbf{\Phi} = \text{diag} \left(e^{-j \frac{2\pi d \sin \theta_1}{\lambda}}, \dots, e^{-j \frac{2\pi d \sin \theta_K}{\lambda}} \right). \quad (49)$$

Thus, the target AoAs can be estimated by solving the rotation

operator.

For ease of calculation, the data from the two subarrays are concatenated into a new data matrix, i.e.,

$$\bar{\mathbf{X}} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_1 \mathbf{\Phi} \end{bmatrix} \mathbf{S}_\theta + \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix}. \quad (50)$$

Then, the signal subspace \mathbf{E}_s and noise subspace \mathbf{E}_n can be obtained by performing an EVD on the covariance matrix of the data matrix $\bar{\mathbf{X}}$, given by

$$\mathbf{R}_{\bar{\mathbf{X}}} = \frac{1}{NP} \bar{\mathbf{X}} \bar{\mathbf{X}}^H = \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^H + \mathbf{E}_n \mathbf{\Lambda}_n \mathbf{E}_n^H. \quad (51)$$

Due to the rotational invariance between the two subarrays, the signal subspace can be decomposed as

$$\mathbf{E}_s = \begin{bmatrix} \mathbf{E}_{s1} \\ \mathbf{E}_{s2} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \mathbf{T} \\ \mathbf{A}_1 \mathbf{\Phi} \mathbf{T} \end{bmatrix}, \quad (52)$$

where $\mathbf{E}_{s1} = \mathbf{E}_s(1 : M_r - 1, :)$, $\mathbf{E}_{s2} = \mathbf{E}_s(2 : M_r, :)$, and \mathbf{T} denotes a nonsingular matrix. Therefore, we have

$$\mathbf{E}_{s2} = \mathbf{A}_1 \mathbf{\Phi} \mathbf{T} = \mathbf{E}_{s1} \mathbf{T}^{-1} \mathbf{\Phi} \mathbf{T}. \quad (53)$$

Let $\mathbf{\Psi} = \mathbf{T}^{-1} \mathbf{\Phi} \mathbf{T}$, and note that the eigenvalues of $\mathbf{\Psi}$ are given by the diagonal elements of $\mathbf{\Phi}$. A least squares estimate of $\mathbf{\Phi}$ can be obtained by

$$\hat{\mathbf{\Phi}} = \mathbf{E}_{s1}^\dagger \mathbf{E}_{s2}. \quad (54)$$

The diagonal elements of $\mathbf{\Psi}$ can be obtained by computing the eigenvalues $\hat{\lambda}_k$ of the estimated $\hat{\mathbf{\Phi}}$, so the AoAs of the targets can be estimated as

$$\theta_k = -\arcsin(\lambda \text{angle}(\hat{\lambda}_k)/2\pi d), k = 1, 2, \dots, K. \quad (55)$$

OMP: The OMP algorithm is an efficient sparse recovery method based on iteratively selecting the basis vectors most correlated with the received signal to estimate target parameters, thereby avoiding complex optimization procedures. For AoA estimation, by discretizing observation angle $\theta \in [-90^\circ, 90^\circ]$ into N^s points, the steering dictionary matrix is defined as

$$\mathbf{A}_{\text{dic}} = [\mathbf{a}_R(\theta_1), \dots, \mathbf{a}_R(\theta_{N^s})] \in \mathbb{C}^{M_r \times N^s}, \quad (56)$$

where each spatial steering vector $\mathbf{a}_R(\theta)$ serves as an atom. Then, the matrix \mathbf{X}_θ in (22) is reduced to

$$\mathbf{X}_\theta = \mathbf{A}_\theta \mathbf{S}_\theta + \mathbf{Z}_\theta = \mathbf{A}_{\text{dic}} \mathbf{\Lambda} \mathbf{S}_\theta + \mathbf{Z}_\theta, \quad (57)$$

where $\mathbf{\Lambda}$ is the sparse representation basis. For any snapshot q of \mathbf{S}_θ , the sparse recovery problem can be formulated as

$$\begin{aligned} & \min_{\mathbf{S}_\theta(:,q)} \|\mathbf{S}_\theta(:,q)\|_0 \\ & \text{s.t. } \mathbf{X}_\theta(:,q) = \mathbf{A}_{\text{dic}} \mathbf{\Lambda} \mathbf{S}_\theta(:,q) + \mathbf{Z}_\theta(:,q), \end{aligned} \quad (58)$$

where $\|\cdot\|_0$ denotes the ℓ_0 norm. Since ℓ_0 -norm minimization is NP-hard, greedy optimization algorithms are typically employed to approximate the original signal via a sparse representation. The OMP algorithm iteratively selects the most correlated atoms from the discretized dictionary, updates the residual via least squares minimization, and ultimately estimates the angles of K targets.

Specifically, OMP first initializes the residual as the signal \mathbf{X}_θ in (22) and the support set as the empty set \emptyset , i.e.,

$$\mathbf{R}_0 = \mathbf{X}_\theta, \mathbb{S}_{\hat{\theta}} = \emptyset. \quad (59)$$

The algorithm then performs K iterations, each selecting the atom most correlated with the current residual, which can be expressed by

$$i_k = \arg \max_i \|\mathbf{A}_{\text{dic}}(:,i)^H \mathbf{R}_{k-1}\|_2^2, i = 1, 2, \dots, N^s, \quad (60)$$

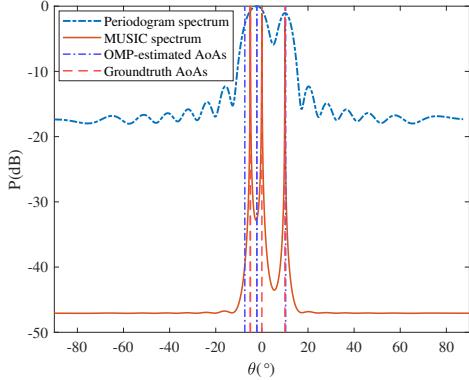


Fig. 8: Comparison of periodogram, MUSIC, OMP for AoA estimation in MIMO-OFDM ISAC with closely spaced targets.

and then the support set is updated as $\mathbb{S}_{\hat{\theta}} = \mathbb{S}_{\hat{\theta}} \cup \{i_k\}$. Here, $\|\cdot\|_2$ is the Frobenius norm and \mathbf{R}_k the residual after the k -th iteration, where the sparse recovery signal is estimated via least squares as

$$\begin{aligned} \hat{\mathbf{S}}_{\theta,k} &= \arg \max_{\mathbf{S}} \|\mathbf{X}_{\theta} - \mathbf{A}_{\text{dic}}(:, \mathbb{S}_{\hat{\theta}}) \mathbf{S}\|_2 \\ &= \mathbf{A}_{\text{dic}}(:, \mathbb{S}_{\hat{\theta}})^\dagger \mathbf{X}_{\theta}, \end{aligned} \quad (61)$$

and the residual is obtained by

$$\mathbf{R}_k = \mathbf{X}_{\theta} - \mathbf{A}_{\text{dic}}(:, \mathbb{S}_{\hat{\theta}}) \hat{\mathbf{S}}_{\theta,k}. \quad (62)$$

Finally, the AoAs are estimated as $\hat{\theta} = \theta_{\text{grid}}(\mathbb{S}_{\hat{\theta}})$, where θ_{grid} denotes the discretized angular grid.

From (41), the angular resolution of the 16-antenna ULA is $\Delta\theta = 7.1^\circ$. As a further illustration, we consider another scenario with more closely spaced targets located at $(-5, 0, 10)^\circ$, while the other parameters in Table III remain unchanged. We show a comparison in Fig. 8 between the periodogram, MUSIC, and OMP. It is observed that when the angle difference between targets is smaller than the resolution limit of the array, their spectral peaks blend together in the periodogram, making them indistinguishable. However, the OMP and MUSIC algorithms are still able to resolve all three targets. Unlike MUSIC, although the OMP algorithm can also achieve super-resolution, its estimation accuracy significantly deteriorates due to mutual interference between the closely spaced targets. In general, the OMP algorithm exhibits degraded estimation performance in scenarios where targets are densely distributed. For the OMP algorithm, the parameters of each target are estimated separately, preventing weak targets from being masked by strong ones. The sequential one-domain estimation approach in Fig. 4 is fundamentally based on the path-separation idea.

In Section IV-B, we introduced the two decoupled parameter estimation methods and the corresponding 1D estimation algorithms. In Figs. 6, 7 and 8, results for the periodogram, MUSIC, and OMP are presented for AoA estimation in parallel one-domain estimation. In the following, we give an example of using MUSIC for sequential one-domain estimation. For the simulation setup in Table III, we consider $K = 5$ targets with AoA, distance, and radial velocity parameters being $(-20^\circ, 20 \text{ m}, 8 \text{ m/s})$, $(10^\circ, 45 \text{ m}, 14 \text{ m/s})$, $(10^\circ, 80 \text{ m}, 20 \text{ m/s})$, $(45^\circ, 60 \text{ m}, 6 \text{ m/s})$, $(45^\circ, 45 \text{ m}, 12 \text{ m/s})$. The corresponding delays and Doppler frequency shifts of the five

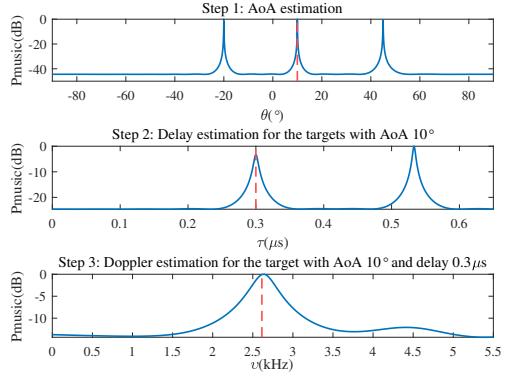


Fig. 9: Sequential one-domain parameter estimation based on MUSIC for estimating the target with parameters $(10^\circ, 0.3 \mu\text{s}, 2.61 \text{kHz})$ in MIMO-OFDM ISAC.

sensing targets are $(0.13 \mu\text{s}, 1.49 \text{ kHz})$, $(0.3 \mu\text{s}, 2.61 \text{ kHz})$, $(0.53 \mu\text{s}, 3.73 \text{ kHz})$, $(0.4 \mu\text{s}, 1.12 \text{ kHz})$, $(0.3 \mu\text{s}, 2.24 \text{ kHz})$, respectively. Under the above setting, the estimation results of sequential one-domain estimation based on MUSIC in the three domains for the target with parameters $(10^\circ, 0.3 \mu\text{s}, 2.61 \text{kHz})$ are shown in Fig. 9, where ZF-based beamforming is exploited to separate the target signals with different AoAs and delays. It is observed that for step 1, only three AoAs can be estimated for the five sensing targets. After spatial domain beamforming towards 10° , two delays are estimated in step 2. Finally, after delay domain beamforming for the target with delay $0.3 \mu\text{s}$, the Doppler frequency 2.61 kHz can be estimated in step 3.

The sequential domain method first performs AoA estimation once, followed by performing spatial domain beamforming and delay estimation $K_{\hat{\theta}}$ times, and finally delay domain beamforming and Doppler estimation K times. Compared to sequential one-domain estimation, parallel one-domain estimation requires performing AoA, delay, and Doppler estimation only once, thus reducing complexity. However, the parallel method requires the extra step of parameter grouping, whereas the sequential method associates the parameters automatically. If the sensing targets cannot be distinguished in a single domain, the parallel method may result in poor estimation performance caused by a failure in parameter grouping. In contrast, the sequential method can use beamforming to extract the signal of each target as long as the targets can be distinguished based on one parameter, thus enabling accurate estimation of the other two target parameters. The sequential method can also effectively prevent weaker targets from being obscured by the sidelobes of stronger targets by first separating out the signals of the stronger targets [113]. Therefore, the parallel method is preferred if the AoA, delay, and Doppler resolutions are all high; otherwise, the sequential method is preferable. Moreover, for sequential one-domain parameter estimation, since the failure to distinguish different targets before performing beamforming will degrade estimation performance, the target parameter with the highest resolution should be estimated first, followed by that with the second highest one and finally that with the lowest one.

C. Joint Two-domain Parameter Estimation

1) Joint Two-domain Parameter Estimation Model:

In this subsection, we present the joint two-domain parameter estimation methods shown in Fig. 4. Compared to the decoupled parameter estimation discussed in Section IV-B, joint two-domain parameter estimation can achieve higher accuracy at the cost of higher computational complexity. The two-dimensional (2D) joint target parameter estimation can be divided into three cases, corresponding to joint delay-Doppler estimation, joint AoA-delay estimation, and joint AoA-Doppler estimation. Since the three cases have mathematically similar forms, we focus on the joint delay-Doppler estimation, while all signal processing techniques and algorithms can be similarly applied to joint AoA-delay estimation and joint AoA-Doppler estimation.

As illustrated in Fig. 4, before performing joint delay-Doppler estimation, we first estimate the AoAs of the sensing targets based on (22), and then perform beamforming on (24) to decouple signals from different angles. However, joint two-domain parameter estimation aims to jointly estimate the delay and Doppler for the targets at each angle, rather than sequentially estimating these two parameters. Assume that $K_{\hat{\theta}}$ AoAs are estimated, with $N_{k_{\hat{\theta}}}$ targets at angle $\hat{\theta}_{k_{\hat{\theta}}}$. After spatial domain beamforming, the extracted signals $\mathbf{X}_{k_{\hat{\theta}}}$ in (38) for each estimated angle are exploited to simultaneously estimate delay τ_k and Doppler frequency shift v_k of the sensing targets.

2) 2D Joint Parameter Estimation Algorithms:

2D-Periodogram: For the joint delay-Doppler estimation problem, based on the signal in (38), the 2D power spectrum $f(\tau, v)$ in the delay-Doppler domain with noise ignored can be obtained in (63) as shown on top of the next page, where $\mathbf{a}_{\tau}(\tau)$ and $\mathbf{a}_v(v)$ are the steering vectors in the delay and Doppler domains in (27) and (31), $\tau \in \left[0, \frac{1}{\Delta f}\right]$ and $v \in \left[-\frac{1}{2T_s}, \frac{1}{2T_s}\right]$ are the observation delay and Doppler frequency, respectively. The approximation (a) in (63) holds if the delay or Doppler difference between sensing targets exceeds the corresponding resolution. (63) reveals that the power spectrum in the delay-Doppler domain is the product of the powers in the two domains, and the two components are mutually independent. Therefore, as long as the targets can be distinguished in either domain, the power spectrum can be used to distinguish the targets.

Similar to the 1D-periodogram, to reduce the computational complexity of searching $f(\tau, v)$ over observation delay τ and Doppler v , an N_{τ}^{IFFT} -point IFFT and N_v^{FFT} -point FFT can be performed in the subcarrier and symbol domains of $\mathbf{X}_{k_{\hat{\theta}}}$ in (38) for fast computation. When $N_{\tau}^{\text{IFFT}} > N$ or $N_v^{\text{FFT}} > P$, zero-padding is utilized to convert $\mathbf{X}_{k_{\hat{\theta}}} \in \mathbb{C}^{N \times P}$ to $\mathbf{X}_{k_{\hat{\theta}}}^{\text{ZP}} \in \mathbb{C}^{N_{\tau}^{\text{IFFT}} \times N_v^{\text{FFT}}}$. Then, the resulting 2D-periodogram $\mathbf{F} \in \mathbb{C}^{N_{\tau}^{\text{IFFT}} \times N_v^{\text{FFT}}}$ can be expressed as

$$\begin{aligned} \mathbf{F} &= \left| \text{FFT}_{N_v^{\text{FFT}}}^2 \left\{ \text{IFFT}_{N_{\tau}^{\text{IFFT}}}^1 \left\{ \mathbf{X}_{k_{\hat{\theta}}}^{\text{ZP}} \right\} \right\} \right|^2 \\ &= \frac{1}{NP} \left| \mathbf{W}_{N_{\tau}^{\text{IFFT}}} \mathbf{X}_{k_{\hat{\theta}}}^{\text{ZP}} \mathbf{W}_{N_v^{\text{FFT}}}^* \right|^2, \end{aligned} \quad (64)$$

where \mathbf{W}^* is the conjugate of IDFT matrix \mathbf{W} . Negative Doppler frequency estimation necessitates a spectral shift in

the Doppler domain of \mathbf{F} to center the zero frequency component within the spectrum. Then, the peak index $(\hat{n}_{\tau}^{i,k_{\hat{\theta}}}, \hat{n}_v^{i,k_{\hat{\theta}}})$ of \mathbf{F} indicates the phases $\Delta f \hat{\tau}_{i,k_{\hat{\theta}}} = \frac{\hat{n}_{\tau}^{i,k_{\hat{\theta}}}}{N_{\tau}^{\text{IFFT}}}$ and $T_s \hat{v}_{i,k_{\hat{\theta}}} = \frac{\hat{n}_v^{i,k_{\hat{\theta}}}}{N_v^{\text{FFT}}}$ in the delay and Doppler domains, respectively. Thus, the delay $\hat{\tau}_{i,k_{\hat{\theta}}}$ and Doppler frequency $\hat{v}_{i,k_{\hat{\theta}}}$, $\forall i = 1, \dots, N_{k_{\hat{\theta}}}$, of the $N_{k_{\hat{\theta}}}$ sensing targets at angle $\hat{\theta}_{k_{\hat{\theta}}}$ can be estimated as

$$\hat{\tau}_{i,k_{\hat{\theta}}} = \frac{\hat{n}_{\tau}^{i,k_{\hat{\theta}}}}{\Delta f N_{\tau}^{\text{IFFT}}}, \hat{v}_{i,k_{\hat{\theta}}} = \frac{\hat{n}_v^{i,k_{\hat{\theta}}}}{T_s N_v^{\text{FFT}}}. \quad (65)$$

2D-MUSIC: In order to apply super-resolution algorithms to the joint delay-Doppler estimation problem, the data matrix $\mathbf{X}_{k_{\hat{\theta}}} \in \mathbb{C}^{N \times P}$ in (38) is vectorized as $\mathbf{x}_{k_{\hat{\theta}}} \in \mathbb{C}^{NP \times 1}$, leading to

$$\begin{aligned} \mathbf{x}_{k_{\hat{\theta}}} &= \text{vec}(\mathbf{X}_{k_{\hat{\theta}}}) \\ &= \sum_{i=1}^{N_{k_{\hat{\theta}}}} \tilde{\alpha}_{i,k_{\hat{\theta}}} \mathbf{a}_v(v_{i,k_{\hat{\theta}}}) \otimes \mathbf{a}_{\tau}(\tau_{i,k_{\hat{\theta}}}) + \text{vec}(\mathbf{Z}_{k_{\hat{\theta}}}). \end{aligned} \quad (66)$$

Based on this observation vector, the delay and Doppler can be jointly estimated based on the corresponding steering vector $\mathbf{a}(\tau, v) = \mathbf{a}_v(v) \otimes \mathbf{a}_{\tau}(\tau) \in \mathbb{C}^{NP \times 1}$. However, since there is only one snapshot in (66) for joint delay-Doppler estimation, it is not possible to estimate the covariance matrix for $\mathbf{x}_{k_{\hat{\theta}}}$. Thus, modified “spatial smoothing” preprocessing (MSSP) [114] is performed to achieve decoherence. By performing MSSP in the subcarrier and symbol domains of $\mathbf{X}_{k_{\hat{\theta}}} \in \mathbb{C}^{N \times P}$ in (38) with a $N_{\text{sub}} \times P_{\text{sub}}$ window, and reformulating each submatrix into a vector, $N_{\text{snap}} = (N - N_{\text{sub}} + 1) \times (P - P_{\text{sub}} + 1)$ snapshots can thus be obtained. Mathematically, the submatrix of each smoothing window is

$$\begin{aligned} \mathbf{x}_{k_{\hat{\theta}}}^{n_s, p_s} &= \mathbf{X}_{k_{\hat{\theta}}} (n_s : n_s + N_{\text{sub}} - 1, p_s : p_s + P_{\text{sub}} - 1), \\ n_s &= 1, \dots, N - N_{\text{sub}} + 1, p_s = 1, \dots, P - P_{\text{sub}} + 1. \end{aligned} \quad (67)$$

By reconstructing each submatrix into a column vector with $\mathbf{x}_{q_{n_s, p_s}}^S = \text{vec}(\mathbf{X}_{k_{\hat{\theta}}}^{n_s, p_s}) \in \mathbb{C}^{N_{\text{sub}} P_{\text{sub}} \times 1}$, $q_{n_s, p_s} = n_s + p_s (N - N_{\text{sub}} + 1)$, N_{snap} vectors can be obtained. Then, a new observation matrix can be created by combining these vectors as columns of the matrix

$$\mathbf{X}_{k_{\hat{\theta}}}^S = \left[\mathbf{x}_1^S, \mathbf{x}_2^S, \dots, \mathbf{x}_{N_{\text{snap}}}^S \right] \in \mathbb{C}^{N_{\text{sub}} P_{\text{sub}} \times N_{\text{snap}}}. \quad (68)$$

Similar to (46) and (47), the 2D-MUSIC spectrum for joint delay and Doppler estimation can be obtained by

$$P_{\text{MUSIC}}(\tau, v) = \frac{1}{\mathbf{a}_S^H(\tau, v) \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}_S(\tau, v)}, \quad (69)$$

where \mathbf{E}_n denotes the noise subspace, $\mathbf{a}_S(\tau, v) = \mathbf{a}_v(v, P_{\text{sub}}) \otimes \mathbf{a}_{\tau}(\tau, N_{\text{sub}}) \in \mathbb{C}^{N_{\text{sub}} P_{\text{sub}} \times 1}$ is the corresponding steering vector, $\mathbf{a}_{\tau}(\tau, N_{\text{sub}})$ and $\mathbf{a}_v(v, P_{\text{sub}})$ are the steering vectors in the delay and Doppler domains in (27) and (31). The peaks of $P_{\text{MUSIC}}(\tau, v)$ correspond to the delays and Doppler frequency shifts of the sensing targets.

For the simulation setup in Table III, consider the challenging scenario where the AoAs of the three targets are all 10° , while the other parameters remain unchanged. A comparison of the 2D-periodogram and 2D-MUSIC for joint delay-Doppler estimation in this case is shown in Figs. 10(a) and 10(b), where MSSP is performed for the 2D-MUSIC algorithm with a window size of $N_{\text{sub}} \times P_{\text{sub}} = \frac{N}{2} \times \frac{P}{2} = 64 \times 32$. Similar to the 1D-periodogram and 1D-MUSIC, the spectrum of 2D-

$$\begin{aligned}
f(\tau, v) &= \frac{1}{NP} \left| \mathbf{a}_\tau^H(\tau) \mathbf{X}_{k_\theta} \mathbf{a}_v^*(v) \right|^2 \\
&= \frac{1}{NP} \left| \sum_{i=1}^{N_{k_\theta}} \tilde{\alpha}_{i,k_\theta} \sum_{n=0}^{N-1} e^{j2\pi n \Delta f (\tau - \tau_{i,k_\theta})} \sum_{p=0}^{P-1} e^{j2\pi p T_s (v_{i,k_\theta} - v)} \right|^2 \\
&\stackrel{(a)}{\approx} \sum_{i=1}^{N_{k_\theta}} \frac{|\tilde{\alpha}_{i,k_\theta}|^2}{NP} \left| \frac{\sin(\pi N \Delta f (\tau - \tau_{i,k_\theta}))}{\sin(\pi \Delta f (\tau - \tau_{i,k_\theta}))} \right|^2 \left| \frac{\sin(\pi P T_s (v - v_{i,k_\theta}))}{\sin(\pi T_s (v - v_{i,k_\theta}))} \right|^2.
\end{aligned} \tag{63}$$

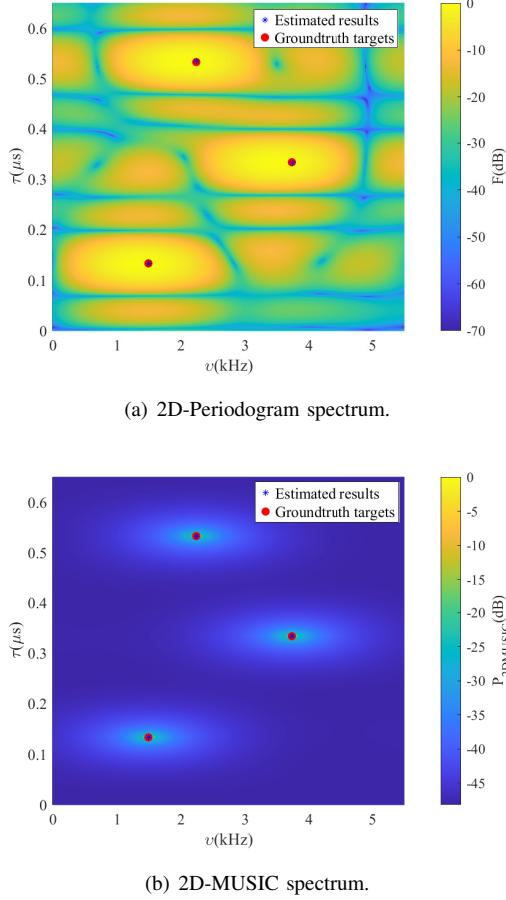


Fig. 10: A comparison of 2D-periodogram and 2D-MUSIC for joint delay-Doppler estimation in MIMO-OFDM ISAC.

MUSIC is smoother and exhibits sharper peaks compared to that of the 2D-periodogram. Consequently, in principle 2D-MUSIC offers higher delay and Doppler resolution and can prevent weak targets from being obscured by the sidelobes of strong targets, thereby reducing inter-target interference and achieving better sensing accuracy. However, for 2D-MUSIC, one must correctly choose the dimension of the signal subspace in order to include the impact of the weak targets, and this is a difficult task in general.

D. Joint Three-domain Parameter Estimation

In this subsection, we present the joint three-domain parameter estimation method shown in Fig. 4. The joint AoA-delay-Doppler estimation simultaneously determines the angles, delays, and Doppler frequency shifts of the sensing targets using the signal $\bar{\mathbf{Y}}_s$ after element-wise symbol division in (24).

3D-Periodogram: To reduce the computational complexity of calculating $f(\theta, \tau, v)$ over the observation angle θ , delay τ , and Doppler v , an N_θ^{IFFT} -point IFFT, N_τ^{IFFT} -point IFFT, and N_v^{FFT} -point FFT are performed in the antenna, subcarrier, and symbol domains of $\bar{\mathbf{Y}}_s$ in (24). When $N_\theta^{\text{IFFT}} > M_r$, $N_\tau^{\text{IFFT}} > N$ or $N_v^{\text{FFT}} > P$, zero-padding is utilized to convert $\bar{\mathbf{Y}}_s \in \mathbb{C}^{M_r \times N \times P}$ into $\bar{\mathbf{Y}}_s^{\text{ZP}} \in \mathbb{C}^{N_\theta^{\text{IFFT}} \times N_\tau^{\text{IFFT}} \times N_v^{\text{FFT}}}$. Then, the resulting 3D-periodogram $\mathbf{F}_{3D} \in \mathbb{C}^{N_\theta^{\text{IFFT}} \times N_\tau^{\text{IFFT}} \times N_v^{\text{FFT}}}$ can be expressed as

$$\mathbf{F}_{3D} = \left| \text{FFT}_{N_v^{\text{FFT}}}^3 \left\{ \text{IFFT}_{N_\tau^{\text{IFFT}}}^2 \left\{ \text{IFFT}_{N_\theta^{\text{IFFT}}}^1 \left\{ \bar{\mathbf{Y}}_s^{\text{ZP}} \right\} \right\} \right\} \right|^2. \tag{70}$$

Similar to 1D and 2D cases, a spectral shift in the angle and Doppler domains of \mathbf{F}_{3D} to center the zero frequency component within the spectrum is required. Then, from the peak indices $(\hat{n}_\theta^k, \hat{n}_\tau^k, \hat{n}_v^k)$ of \mathbf{F}_{3D} , the angle $\hat{\theta}_k$, delay $\hat{\tau}_k$, and Doppler frequency \hat{v}_k , $\forall k = 1, \dots, K$, of the K sensing targets can be estimated as

$$\hat{\theta}_k = \arcsin \left(\frac{\hat{n}_\theta^k \lambda}{N_\theta^{\text{IFFT}} d} \right), \hat{\tau}_k = \frac{\hat{n}_\tau^k}{\Delta f N_\tau^{\text{IFFT}}}, \hat{v}_k = \frac{\hat{n}_v^k}{T_s N_v^{\text{FFT}}}. \tag{71}$$

3D-MUSIC: Similar to 2D-MUSIC, MSSP is essential for decoherence. With a smoothing window size of $M_{\text{sub}} \times N_{\text{sub}} \times P_{\text{sub}}$ over $\bar{\mathbf{Y}}_s$ in (24), $N_{\text{snap}} = (M - M_{\text{sub}} + 1) \times (N - N_{\text{sub}} + 1) \times (P - P_{\text{sub}} + 1)$ snapshots can be obtained. Then, the resulting observation matrix $\tilde{\mathbf{Y}}^S \in \mathbb{C}^{M_{\text{sub}} N_{\text{sub}} P_{\text{sub}} \times N_{\text{snap}}}$ after MSSP is

$$\tilde{\mathbf{Y}}^S = \left[\bar{\mathbf{y}}_1^S, \bar{\mathbf{y}}_2^S, \dots, \bar{\mathbf{y}}_{N_{\text{snap}}}^S \right]. \tag{72}$$

Thus, the 3D-MUSIC spectrum for joint AoA-delay-Doppler estimation can be obtained as

$$P_{\text{MUSIC}}(\theta, \tau, v) = \frac{1}{\mathbf{a}_S^H(\theta, \tau, v) \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}_S(\theta, \tau, v)}, \tag{73}$$

where $\mathbf{a}_S(\theta, \tau, v) = \mathbf{a}_v(v, P_{\text{sub}}) \otimes \mathbf{a}_\tau(\tau, N_{\text{sub}}) \otimes \mathbf{a}_R(\theta, M_{\text{sub}}) \in \mathbb{C}^{M_{\text{sub}} N_{\text{sub}} P_{\text{sub}} \times 1}$ is the corresponding steering vector. The peaks of $P_{\text{MUSIC}}(\theta, \tau, v)$ correspond to the AoA, delay, and Doppler frequency shift of the sensing targets, provided the signal subspace dimension is correctly chosen.

The ESPRIT and OMP algorithms can also be applied to the 2D and 3D parameter estimation problems. Similar to the MUSIC algorithm, spatial smoothing is also required for ESPRIT to achieve decorrelation. The conventional 2D and 3D ESPRIT algorithms require parameter grouping [115] to match the estimation results of the different domains, whereas some modified ESPRIT algorithms [116], [117] can achieve automatic grouping. Furthermore, extensive efforts have been devoted to reducing the complexity for super-resolution methods. For example, the ROOT-MUSIC [118], [119] algorithm replaces the spectrum search in MUSIC with

TABLE IV: Complexity of 1D parameter estimation algorithms for AoA estimation in MIMO-OFDM ISAC.

Algorithm	Theoretical complexity
Periodogram [111]	$QN_\theta^{\text{IFFT}} \log_2 N_\theta^{\text{IFFT}}$
MUSIC [79]	$QM_r^2 + M_r^3 + (2M_r(M_r - K) + M_r)N_s^s$
PM-MUSIC [120], [121]	$QM_r^2 + M_r K(M_r - K) + (2M_r(M_r - K) + M_r)N_\theta^s$
FFT-MUSIC [122]	$QM_r^2 + M_r^3 + (M_r - K)N_\theta^{\text{IFFT}} \log_2 N_\theta^{\text{IFFT}}$
LS-ESPRIT [80]	$QM_r^2 + M_r^3 + K^2(M_r - 1) + K^3$
TLS-ESPRIT [80]	$QM_r^2 + M_r^3 + (2K)^2(M_r - 1) + 10K^3$
PM-ESPRIT [120], [121]	$QM_r^2 + M_r K(M_r - K) + Q^2(M_r - 1) + K^3$
OMP [81], [123], [124]	$KMN_\theta^s Q + MK^3 + MK^2 Q$

polynomial rooting, the propagator method (PM) [120], [121] uses an alternative to achieve the eigenvalue decomposition, and the FFT-MUSIC [122] algorithm accelerates the MUSIC spectrum peak search using FFT operations. In the following, we analyze the complexity and estimation accuracy of the aforementioned algorithms.

E. Complexity and Performance Analysis

1) Complexity Analysis:

We examine the theoretical computational complexity of the aforementioned algorithms in terms of their required number of complex-valued multiplications. As such, the complexity of the multiplication of $M \times Q$ and $Q \times N$ matrices is $\mathcal{O}(QMN)$, and that of the EVD of an $M \times M$ matrix is $\mathcal{O}(M^3)$. The complexity of various 1D parameter estimation algorithms for AoA estimation is shown in Table IV, while these results also apply to delay and Doppler estimation. In these expressions, Q denotes the number of snapshots, $N^{\text{FFT}}/N^{\text{IFFT}}$ and N^s denote the number of FFT/IFFT points and spectrum search points respectively, which are determined by the estimation interval η , and $N^{\text{FFT}} = N^{\text{IFFT}} = N^s = \frac{\Delta_{\text{max}}}{\eta}$ with Δ_{max} representing the maximum estimation range. The estimation interval affects estimation accuracy, and it should be adjustable in order to achieve specific accuracy requirements. To our best knowledge, it is difficult to express the theoretical complexity of ROOT-MUSIC due to the difficulty of quantifying the complexity of the polynomial rooting process. For a typical example, we set the number of OFDM subcarriers and symbols as $N = 128$ and $P = 64$, respectively, the number of targets is $K = 3$, the estimation interval is $\eta = 0.1^\circ$, and the estimation range is $[-90^\circ, 90^\circ]$. Then, the theoretical complexity of the various 1D estimation algorithms for different numbers of Sen-RX antennas M_r is shown in Table V.

As shown in Table V, for MIMO-OFDM ISAC systems, when the numbers of antennas is small, while the number of subcarriers and OFDM symbols are large, numerous snapshots are provided for AoA estimation, and the periodogram algorithm even exhibits higher complexity than MUSIC and ESPRIT. However, the situation changes as the number of antennas increases, the periodogram becomes more computationally efficient due to the fast computation of the FFT,

TABLE V: The complexity of 1D algorithms for different numbers of antennas for AoA estimation in MIMO-OFDM ISAC.

Algorithm	Basic setup and complexity			
	$N = 128, P = 64, Q = NP = 8192$, $K = 3, N^s = N_\theta^{\text{IFFT}} = 1800$			
	$M_r = 16$	$M_r = 64$	$M_r = 256$	$M_r = 512$
Periodogram	1.596e8	1.596e8	1.596e8	1.596e8
MUSIC	2.879e6	4.799e7	7.874e8	3.221e9
PM-MUSIC	2.876e6	4.774e7	7.708e8	3.088e9
FFT-MUSIC	2.354e6	3.500e7	5.586e8	2.292e9
LS-ESPRIT	2.101e6	3.382e7	5.537e8	2.282e9
TLS-ESPRIT	2.102e6	3.382e7	5.537e8	2.282e9
PM-ESPRIT	2.098e6	3.357e7	5.371e8	2.148e9
OMP	7.094e8	2.837e9	1.135e10	2.270e10

followed by ESPRIT and MUSIC, while OMP has the highest complexity. The periodogram and OMP algorithms can estimate the desired parameters using only partial snapshots or even a single snapshot to reduce complexity, at the cost of increased error. Among MUSIC's variants, FFT-MUSIC has the lowest complexity, followed by ROOT-MUSIC, with PM-MUSIC offering the least reduction in complexity. Total LS (TLS)-ESPRIT [80] is a variation of the LS-ESPRIT algorithm that offers slightly improved performance, but with a corresponding slight increase in computation. Unlike MUSIC, the PM algorithm has the potential to significantly reduce the complexity of ESPRIT. Overall, the periodogram, FFT-MUSIC, and PM-ESPRIT algorithms offer the lowest complexity.

The complexity analysis for the 2D/3D algorithms follows a similar approach as 1D algorithms and will not be elaborated further here. Note that 2D/3D algorithms not only have a high computational complexity but also involve calculations with large-dimensional matrices, imposing significant demands on computer hardware. To achieve high-accuracy parameter estimation while mitigating these challenges, coarse-fine grid search [125] and signal decimation [81], [126] techniques can significantly reduce both computational complexity and hardware requirements.

2) Estimation Accuracy and Resolution Analysis:

To study the relative performance of the 1D parameter estimation algorithms discussed above, we consider a scenario similar to that outlined in Table III, but with the spacing between the parameters increased to guarantee that all the algorithms resolve the targets. We conducted 300 Monte Carlo experiments to estimate parameters in three domains via these 1D parameter estimation algorithms for the SNR in (8) varying from -40 dB to 20 dB. The root mean-squared error (RMSE) for each algorithm in the three domains is shown in Fig. 11.

We observe a threshold effect at approximately -20 dB where the algorithm performance improves dramatically for the above setup. Among them, the periodogram shows the best performance, followed by MUSIC, with ESPRIT being the least estimation accuracy. The performance of PM and ESPRIT in the spatial domain surpasses that in the delay and Doppler domains since ICI affects the latter, while signals in spatial domain remain approximately orthogonal. For delay

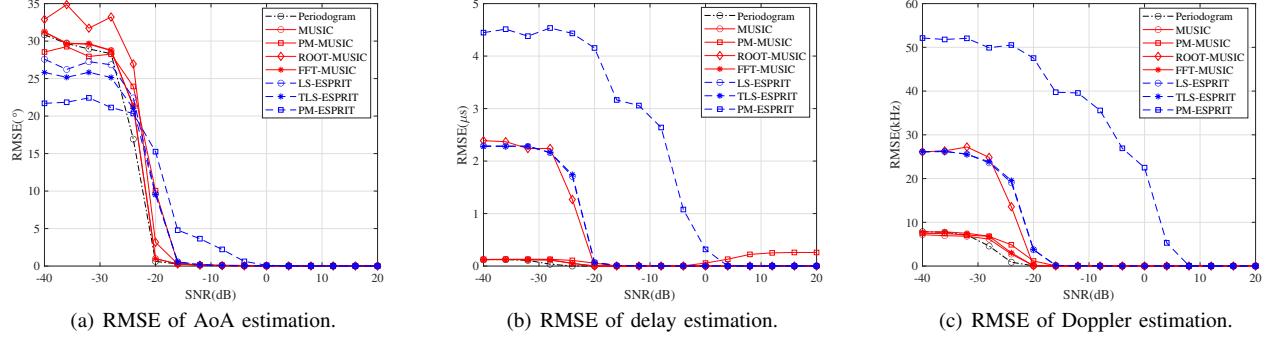


Fig. 11: Estimation accuracy of the considered 1D algorithms in the three individual domains for MIMO-OFDM ISAC.

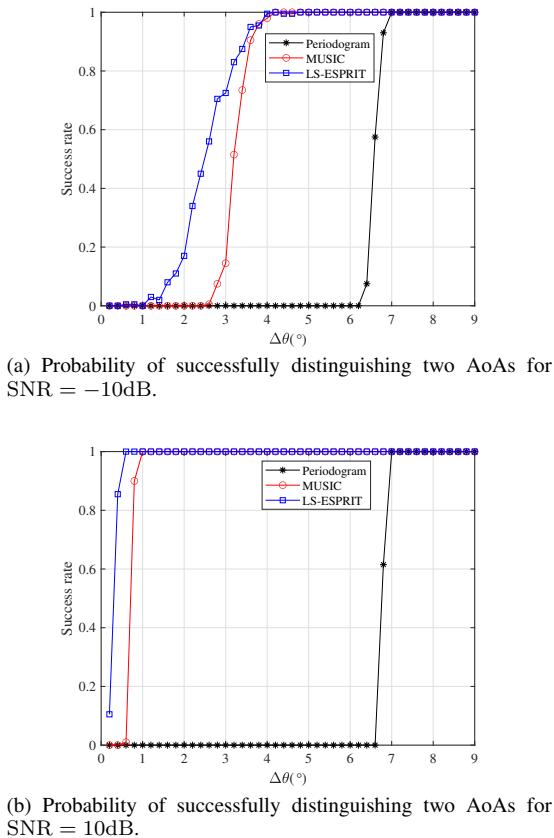


Fig. 12: Probability of successfully resolving two targets using the periodogram, MUSIC, and ESPRIT algorithms for SNR = -10dB and SNR = 10dB with the angle difference varying from 0.2° to 9° in MIMO-OFDM ISAC.

estimation, PM-MUSIC fails due to signal coherence at high SNR, although this can be addressed by spatial smoothing. FFT-MUSIC and MUSIC have similar estimation accuracy, while ROOT-MUSIC performs poorly at low SNR but approaches MUSIC at high SNR. Finally, TLS-ESPRIT has slightly higher accuracy than LS-ESPRIT at low SNR. PM-ESPRIT exhibits superior performance exclusively at high SNR, particularly for delay and Doppler estimation.

Regarding resolution, we derived the theoretical resolution of the periodogram algorithm for AoA, delay, and Doppler estimation in (41) and (42). However, deriving the theoretical

resolution for super-resolution algorithms such as MUSIC and ESPRIT is challenging. Therefore, we design a simulation experiment to study this issue. Using the estimation error of two closely positioned targets as a constraint, we statistically determine the probability of successfully resolving the two targets for each of the considered algorithm to analyze their resolution. Taking the angular resolution as an example, the Sen-RX is set as a ULA equipped with $M_r = 16$ antennas with half-wavelength element-spacing. The ISAC-TX transmits an OFDM signal with $N = 128$, the CPI $P = 64$ symbols, sub-carrier spacing $\Delta f = 120$ kHz, and CP duration $T_{\text{cp}} = \frac{1}{4}T$. Since the theoretical resolution of the periodogram is 7.1° under these conditions, we establish two targets, for which the AoA of the first target is 0° , and that of the second one varies from 0.2° to 9° . An algorithm is considered to have resolved the two targets successfully if the RMSE is within 0.5° for SNR = -10 dB, and within 0.3° for SNR = 10 dB. As the angle of the second target changes, the probabilities of successfully resolving the two targets using the periodogram, MUSIC, and ESPRIT algorithms for SNR = -10dB and SNR = 10dB are shown in Figs. 12(a) and 12(b), respectively, based on 200 Monte Carlo experiments.

The simulation results in Figs. 12(a) and 12(b) show the angular resolution of the periodogram algorithm is 7° for both SNR = -10dB and SNR = 10dB, consistent with the theoretical resolution in (41). The resolution of the MUSIC and ESPRIT algorithms depends on the SNR. For lower SNR, MUSIC and ESPRIT exhibit similar resolution, approximately 4° . For higher SNR, ESPRIT achieves superior resolution around 0.6° , whereas MUSIC provides resolution around 1° . Overall, compared to the periodogram, super-resolution algorithms significantly enhance parameter estimation resolution, particularly at high SNR levels.

V. NEAR-FIELD MIMO-OFDM ISAC

In this section, we extend the results in Section IV to the case where the targets are located in the near-field region of the Sen-RX. While the main signal processing strategies outlined in Fig. 4 are still valid in the near-field, the spatial domain estimation must be extended from AoA estimation only in far-field to joint angle-range estimation in the near-field by exploiting the characteristics of spherical wavefronts. Therefore, this section focuses mainly on spatial domain estimation.

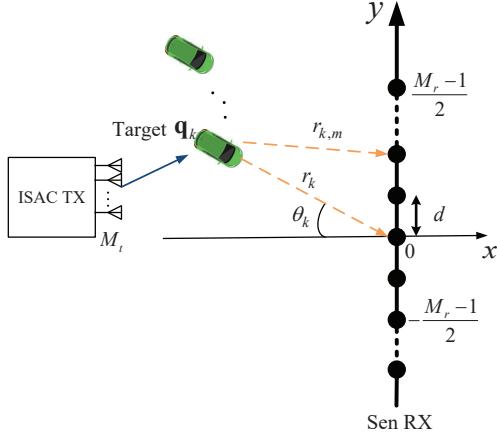


Fig. 13: Near-field sensing for MIMO-OFDM ISAC.

A. Near-field Sensing Signal Model

As shown in Fig. 13, we assume that the Sen-RX is equipped with an extremely large-scale ULA, which is placed along the y -axis with $\varepsilon_r = 0$, and $\varepsilon_m = m - (M_r + 1)/2$ in (14). The corresponding steering matrix is denoted as $\mathbf{A}_R(r, \theta) = [\mathbf{a}_R(r_1, \theta_1), \dots, \mathbf{a}_R(r_K, \theta_K)]$. Based on the model in (22), the $M_r \times Q_{N,P}$ received signal is given by

$$\mathbf{X} = \mathbf{A}_R(r, \theta) \mathbf{S} + \mathbf{Z}, \quad (74)$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_K]^T \in \mathbb{C}^{K \times Q_{N,P}}$ denotes the equivalent reflected target signals described in (23). Note that compared with the far-field model in (22), the array response vector for near-field sensing is a function of both the target angle and distance, hence allowing simultaneous estimation of both parameters independent of delay-based distance estimation. This will not only improve the distance estimation accuracy (especially if the distance estimation is performed jointly in the AoA and delay domains), but it can also alleviate the synchronization errors that typically arise in traditional delay-based distance estimation techniques. In order to estimate the angle and distance parameters based on spatial-domain processing, several methods are introduced below, which can be generally categorized into directly applying far-field algorithms, extending far-field algorithms to three types, namely, near-field, and low-complexity near-field algorithms.

B. Directly Applying Far-field Algorithms to Near-field Models

In this section, we illustrate the performance of directly applying the far-field algorithms discussed in Section IV-B while ignoring near-field effects. The result for the standard Periodogram is shown in Fig. 14, where $K = 2$ targets are considered. We see that only the target with $r = 150m$ has an accurate AoA estimate based on (45). For the closer target at $r = 30m$, multiple peaks exist around the ground-truth AoA, which is due to the mismatch between the near-field channel and the assumed far-field model. This is known as the so-called energy spread effect, which illustrates the necessity of developing dedicated near-field target localization algorithms.

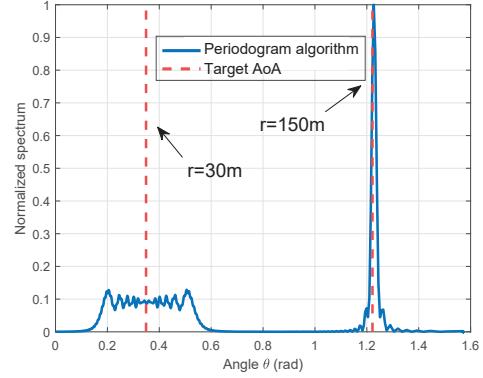


Fig. 14: Directly applying far-field algorithms in near-field scenario for AoA estimation.

C. Extending Far-field Algorithms to the Near-field

Below we discuss how to extend algorithms developed for far-field parameter estimation to the near-field case.

1) *Near-field Beam Focusing*: For near-field models, the distance and angle parameters are closely coupled in the near-field array response vector. In addition, the non-linear phase variation from antenna to antenna in near-field models also means that the IFFT is no longer directly applicable. Therefore, we introduce 2D beamforming in both the angle and distance domains in this section. Based on (74), the spatial domain sample covariance matrix of \mathbf{X} is given by

$$\mathbf{R}_X = \frac{1}{Q_{NP}} \mathbf{X} \mathbf{X}^H, \quad (75)$$

and the 2D periodogram spectrum can be written as

$$P_{2D-BF}(r, \theta) = \mathbf{a}_R^H(r, \theta) \mathbf{R}_X \mathbf{a}_R(r, \theta). \quad (76)$$

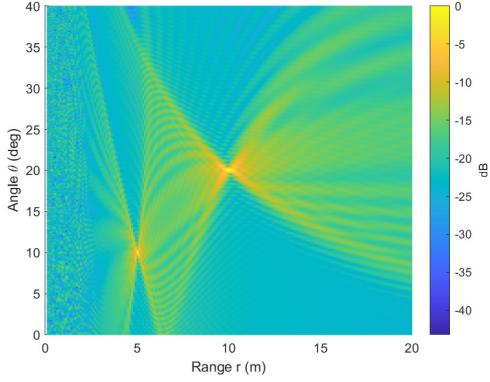
The peaks of P_{2D-BF} in the 2-D angle-distance can be used to estimate the target angle and distance parameters.

2) *Near-field MUSIC*: Similar to (47), by performing EVD to the covariance matrix \mathbf{R}_X and obtaining the noise subspace \mathbf{E}_n , the 2-D MUSIC spectrum can be expressed as

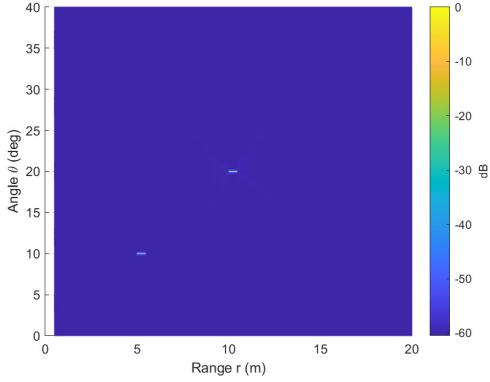
$$P_{2D-MUSIC}(r, \theta) = \frac{1}{\mathbf{a}_R^H(r, \theta) \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}_R(r, \theta)}. \quad (77)$$

3) *Near-field ESPRIT and Root-MUSIC*: As shown in Section IV-B, the far-field ESPRIT algorithm exploits the rotational invariance between identical subarrays, requiring a linear phase variation with the parameters between the subarrays. Therefore, conventional ESPRIT is not directly applicable in near-field sensing. However, for symmetric antenna arrays, a generalized version of ESPRIT can be performed, which will be introduced in Section V-D4. Root-MUSIC also relies on the assumption of steering vectors with a linear phase shift between adjacent elements, and thus is also not directly applicable in the near-field case.

4) *Simulations and discussions*: Figs. 15(a) and 15(b) show the 2D spectra for near-field beam focusing and MUSIC respectively. The number of antennas is $M_r = 256$, and the locations of the two targets are $(r_1, \theta_1) = (5m, 10^\circ)$ and $(r_2, \theta_2) = (10m, 20^\circ)$, respectively. It is observed that both beamforming and MUSIC can localize the closer target with



(a) Near-field beam focusing spectrum.



(b) Near-field 2D-MUSIC spectrum.

Fig. 15: Near-field angle-distance estimation by extending far-field algorithms to near-field.

higher accuracy than the distant target, primarily due to the limited accuracy with respect to distance.

We see that several issues arise that complicate parameter estimation in near-field scenarios. Besides the fact that some algorithms can no longer be directly applied, we see that 2D rather than 1D searches are necessary to jointly estimate the AoAs and ranges, which adds complexity. The accuracy of the range estimate is considerably less reliable than the AoA estimate, especially as the range increases to the point where the far-field model holds [127]. This is due to the decreasing sensitivity of the model in such scenarios, but is a factor that must be taken into account when jointly estimating the range by including the delay domain.

To address these challenges, one feasible approach is to decouple estimation of the angle and distance parameters to reduce the complexity of implementing the parameter optimization. Based on this idea, the following low-complexity near-field sensing algorithms are presented.

D. Low-complexity Near-Field Estimation Algorithms

Recently, several efficient algorithms have been proposed to improve the accuracy and resolution of near-field sensing with low computational complexity by efficiently decoupling the distance and angle parameters. To that end, most existing works have considered the approximate Fresnel array response

model in (16) based on second-order Taylor approximation, and the parameters to be estimated are transformed from θ_k and r_k to $\omega_k = -\frac{2\pi d \sin \theta_k}{\lambda}$ and $\psi_k = \frac{\pi d^2 \cos^2 \theta_k}{\lambda r_k}$, respectively.

1) *Second-order cumulant (SoC)*: Note that in (16), ω_k only relates to θ_k , while ψ_k depends on both θ_k and r_k . Therefore, dedicated second-order cumulants can be constructed to transform the multi-dimensional search into two single-parameter estimation problems. Based on (74) and (16), the noise-free correlation sequence between the $(am+b)$ th and m th antenna elements can be written as [128]

$$\begin{aligned} r(am+b, m) &= \frac{1}{Q_{N,P}} \mathbf{X}(am+b+J+1, :) \mathbf{X}^H(m+J+1, :) \\ &= \frac{1}{Q_{N,P}} \mathbf{A}_{R,am+b+J+1} \mathbf{S} \mathbf{S}^H \mathbf{A}_{R,m+J+1}^H \\ &\stackrel{(a)}{\approx} \sum_{k=1}^K p_k e^{j[(a-1)m+b]\omega_k + j[(a^2-1)m^2+2abm+b^2]\psi_k}, \end{aligned} \quad (78)$$

where $\mathbf{A}_{R,m}$ is the m th row of \mathbf{A}_R . Approximation (a) holds by assuming $\mathbf{s}_p^H \mathbf{s}_q \approx 0, p \neq q$. $J = \frac{M_r-1}{2}$ and $p_k = \frac{1}{Q_{N,P}} \mathbf{S}(k, :) \mathbf{S}^H(k, :)$ denotes the equivalent transmitted signal. Furthermore, by properly choosing a and b , ψ_k can be eliminated from the resulting expression, and estimation of the distance and angle parameters can be decoupled. For instance, substituting $(a, b) = (-1, 0)$ and $(a, b) = (1, 1)$ into (78), we obtain the following correlation sequences

$$r_1(-m, m) = \sum_{k=1}^K p_k e^{-2j\omega_k m}, m = 0, 1, \dots, J, \quad (79)$$

$$r_2(m+1, m) = \sum_{k=1}^K p_k e^{j(\omega_k + \psi_k)} e^{2j\psi_k m}, m = 0, 1, \dots, J-1.$$

Stacking the r_1 and r_2 terms into vectors, we obtain the signal model

$$\mathbf{r}_1 = \mathbf{A}_1 \mathbf{p}_1 \in \mathbb{C}^{(J+1) \times 1}, \quad \mathbf{r}_2 = \mathbf{A}_2 \mathbf{p}_2 \in \mathbb{C}^{J \times 1}, \quad (80)$$

where the k -th column of the $(J+1) \times K$ matrix \mathbf{A}_1 is $\mathbf{a}_{1k} = [1, e^{-2j\omega_k}, \dots, e^{-j(M_r-1)\omega_k}]^T$, and $\mathbf{p}_1 = [p_1, \dots, p_K]^T$. Similarly, the k th column of \mathbf{A}_2 and \mathbf{p}_2 can be expressed as $\mathbf{a}_{2k} = [1, e^{2j\psi_k}, \dots, e^{-j(M_r-3)\psi_k}]^T$ and $\mathbf{p}_2 = [p_1 e^{j(\omega_1 + \psi_1)}, \dots, p_K e^{j(\omega_K + \psi_K)}]^T$, respectively.

It can be observed that \mathbf{r}_1 and \mathbf{r}_2 have a form similar to the far-field model. Thus, ω_k and ψ_k can be obtained using conventional AoA estimation algorithms. In the following, we refer to this approach as “SoC-AoA”. However, the spatial resolution of this approach is reduced to $BW = \frac{4\lambda}{D}$ since the equivalent antenna aperture is half of the physical aperture. Based on SoC, a weighted linear prediction algorithm has been proposed for near-field localization [128] and higher-order cumulants (HoC) have been proposed for higher accuracy and wider application [129], [130].

2) *Reduced dimension (RD) and reduced rank (RR) algorithms*: Another method to decouple ω_k and ψ_k is by

decomposing $\mathbf{a}_R(r, \theta)$ in (16) as [131]

$$\mathbf{a}_R(\omega, \psi) = \underbrace{\begin{bmatrix} e^{j(-J)\omega} & & & \\ & e^{j(-J+1)\omega} & & \\ & & \ddots & \\ & & & 1 \\ & & & & \ddots \\ & & & & & e^{j(J-1)\omega} \\ & & & & & & e^{jJ\omega} \end{bmatrix}}_{\Gamma(\omega)} \cdot \underbrace{\begin{bmatrix} e^{j(-J)^2\psi} \\ e^{j(-J+1)^2\psi} \\ \vdots \\ e^{j(-1)^2\psi} \\ 1 \end{bmatrix}}_{\xi(\psi)}, \quad (81)$$

where $J = \frac{M_r-1}{2}$, $\Gamma(\omega) \in \mathbb{C}^{(2J+1) \times (J+1)}$ only depends on the angle, while $\xi(\psi) \in \mathbb{C}^{(J+1) \times 1}$ is related to both angle and distance. Note that (81) allows the 2D search for 2D-MUSIC (77) to be converted into two 1D searches, thus reducing the complexity. Specifically, the MUSIC spectrum can be constructed as

$$\begin{aligned} P_{\text{2D-MUSIC}}(\omega, \psi) &= \frac{1}{\xi^H(\psi)\Gamma^H(\omega)\mathbf{E}_n\mathbf{E}_n^H\Gamma(\omega)\xi(\psi)} \\ &= \frac{1}{\xi^H(\psi)\mathbf{Q}(\omega)\xi(\psi)}. \end{aligned} \quad (82)$$

In the following, we present angle and distance estimation algorithms for the RD and RR cases, respectively.

Reduced-rank algorithm: Since the rank of $\mathbf{E}_n \in \mathbb{C}^{M_r \times (M_r-K)}$ will be no smaller than $M_r - K$, $\mathbf{Q}(\omega) = \Gamma^H(\omega)\mathbf{E}_n\mathbf{E}_n^H\Gamma(\omega) \in \mathbb{C}^{(J+1) \times (J+1)}$ can be obtained by performing an EVD on \mathbf{R}_X , and $\mathbf{Q}(\omega)$ generally has full rank if $M_r - K \geq (J+1)$, i.e., $K \leq J$. On the other hand, when $\omega \in \Omega = \{\omega_1, \dots, \omega_K\}$, we have $\xi^H(\psi)\mathbf{Q}(\omega_k)\xi(\psi) \rightarrow 0$ based on the MUSIC algorithm, which is true only if $\mathbf{Q}(\omega)$ is rank deficient since generally $\xi(\psi) \neq 0$ [132]. Based on these observations, the value of θ_k can be obtained from the K highest peaks of the following 1D spectral function

$$\hat{\omega}_k = \arg \max_{\omega} \frac{1}{\det(\mathbf{Q}(\omega))}, \quad (83)$$

and the corresponding r_k is obtained by searching for peaks in the distance domain using

$$\hat{r}_k = \arg \max_r P_{\text{2D-MUSIC}}(\hat{\theta}_k, r). \quad (84)$$

Reduced-dimension algorithm: The spectrum search in (82) can also be viewed as the following optimization problem

$$\min_{\psi, \omega} \xi^H(\psi)\mathbf{Q}(\omega)\xi(\psi) \quad \text{s.t. } \mathbf{e}_1^H\xi(\psi) = 1, \quad (85)$$

where $\mathbf{e}_1 = [0, 0, \dots, 1]^T \in \mathbb{C}^{(J+1) \times 1}$. Solving (85), $\xi(\psi)$ can be expressed as $\xi(\psi) = \frac{\mathbf{Q}^{\dagger}(\omega)\mathbf{e}_1}{\mathbf{e}_1^H\mathbf{Q}^{\dagger}(\omega)\mathbf{e}_1}$. Therefore, (82) can be transformed into a 1D search in terms of the angle parameter [131]

$$\hat{\omega}_k = \arg \min_{\omega} \frac{1}{\mathbf{e}_1^H\mathbf{Q}^{\dagger}(\omega)\mathbf{e}_1}. \quad (86)$$

Furthermore, ψ_k can be obtained by constructing the LS problem

$$\min_{\mathbf{c}_k} \|\mathbf{P}\mathbf{c}_k - \hat{\mathbf{g}}_k\|^2, \quad (87)$$

where $\mathbf{c}_k = [c_{k,0}, \psi_k]^T$, $\hat{\mathbf{g}}_k = \text{angle}\left(\frac{\mathbf{Q}^{\dagger}(\hat{\omega})\mathbf{e}_1}{\mathbf{e}_1^H\mathbf{Q}^{\dagger}(\hat{\omega})\mathbf{e}_1}\right)$, and $\mathbf{P} = \begin{pmatrix} 1, \dots, 1 \\ J^2, \dots, 0 \end{pmatrix}^T$. After finding ψ_k using (87), the angle and distance parameters can be estimated as

$$\hat{\theta}_k = -\arcsin\left(\frac{\hat{\omega}_k \lambda}{2\pi d}\right), \quad \hat{r}_k = \frac{\pi d^2 \cos^2 \hat{\theta}_k}{\lambda \hat{\psi}_k}. \quad (88)$$

3) FFT-enhanced low-complexity and super-resolution algorithm: In order to further reduce the complexity, the authors in [133] proposed an efficient algorithm to narrow the search region for 2D angle-distance estimation in (77). The main idea of this algorithm is to first eliminate those regions that do not contain any targets. Afterwards, 2D MUSIC is performed within a region of reduced size in order to estimate the angle and distance, hence effectively balancing computational complexity and super-resolution. This algorithm involves three steps:

FFT-enhanced angle cluster determination: Applying an S -point FFT and IFFT to the columns and rows of sample covariance matrix \mathbf{R}_X in (75), respectively, the output can be used to eliminate angle regions where no targets exist. The resulting spectrum can be expressed as $\mathbf{p} = \text{diag}(\mathbf{W}\mathbf{R}_X\mathbf{W}^{-1})$, where \mathbf{W} is the DFT matrix, and $\text{diag}(\mathbf{A})$ means extracting the diagonal elements of \mathbf{A} to form a vector. The n th angle cluster is represented by the minimum and maximum angle value within this cluster, denoted as $\alpha_n = [\underline{\alpha}_n, \bar{\alpha}_n]$, $n = 1, \dots, L$. L is the number of clusters.

Distance cluster determination: For each angle cluster $\alpha_n = [\underline{\alpha}_n, \bar{\alpha}_n]$, 1D beamforming is performed over the distance domain for $\underline{\alpha}_n$ and $\bar{\alpha}_n$, respectively.

$$\begin{aligned} P_{\text{low}}(r) &= \mathbf{a}_R^H(r, \underline{\alpha}_n)\mathbf{R}_X\mathbf{a}_R(r, \underline{\alpha}_n), \\ P_{\text{up}}(r) &= \mathbf{a}_R^H(r, \bar{\alpha}_n)\mathbf{R}_X\mathbf{a}_R(r, \bar{\alpha}_n). \end{aligned} \quad (89)$$

By setting a threshold Γ , the distance cluster β_n can be expressed as $P_{\text{low}}(r) < \Gamma$, and $\bar{\beta}_n$ can be obtained similarly. Therefore, the overall distance cluster is expressed as $\beta_n = \underline{\beta}_n \cup \bar{\beta}_n$.

Super-resolution 2D sensing: Finally, for each cluster n , 2D MUSIC is performed for $\theta \in \alpha_n$ and $r \in \beta_n$. By clustering the angles and distance, the number of grids for the 2D search decreases significantly, thus reducing the overall complexity.

4) Symmetry-based near-field estimation algorithms: Another strategy to reduce the computational complexity of the 2D peak search is by exploiting the symmetry of array structure. Here, we present two approaches to decouple the problems of angle and distance estimation by leveraging the symmetric structure of the antenna array for multiple targets.

Modified MUSIC [134]: After computing the covariance matrix of the received signal \mathbf{R}_X in (75), its anti-diagonal elements without noise are given by

$$\begin{aligned} \mathbf{R}_X[n, M_r + 1 - n] &= \mathbb{E}\{\mathbf{X}\mathbf{X}^H\} \\ &= \mathbb{E}\left\{\left(\sum_{k=1}^K e^{j(n\omega_k + n^2\psi_k)} \mathbf{s}_k^T\right) \left(\sum_{k=1}^K e^{-j(-n\omega_k + n^2\psi_k)} \mathbf{s}_k^*\right)\right\} \\ &= \sum_{k=1}^K p_k e^{j2n\omega_k}, \quad n = 1, 2, \dots, M, \end{aligned} \quad (90)$$

where p_k denotes the equivalent transmitted signal power of target k , i.e., $\mathbb{E}\{\mathbf{s}_k^H \mathbf{s}_k\} = p_k$. From the anti-diagonal samples of the covariance samples we can reconstruct a new vector that depends only on the angle parameters, given by

$$[\tilde{\mathbf{y}}]_m = \sum_{k=1}^K p_k e^{j2m\omega_k}. \quad (91)$$

To perform 1D MUSIC, a full-rank set of observations is required, so spatial smoothing techniques are employed by splitting $\tilde{\mathbf{y}}$ into L subvectors, each comprising $(M+1-L)$ entries. The ℓ -th split signal can be expressed as

$$\tilde{\mathbf{y}}_\ell = [[\tilde{\mathbf{y}}]_\ell, [\tilde{\mathbf{y}}]_{\ell+1}, \dots, [\tilde{\mathbf{y}}]_{\ell+M-L}]^H. \quad (92)$$

Then, $\tilde{\mathbf{y}}_\ell$ can be rewritten as $\tilde{\mathbf{y}}_\ell = \mathbf{A}\mathbf{p}_\ell$, where $\mathbf{A} = [\mathbf{a}(\alpha_1), \mathbf{a}(\alpha_2), \dots, \mathbf{a}(\alpha_K)]$ with $[\mathbf{a}(\alpha_K)]_i = e^{j2i\omega_k}$, $i = 1, 2, \dots, M+1-L$. Moreover, the k -th entry of \mathbf{p}_ℓ is $[\mathbf{p}_\ell]_k = p_k e^{j2(\ell-1)\omega_k}$. The matrix \mathbf{A} has the form of a far-field steering matrix with an array of dimension $(M+1-L)$. Thus, classical 1D MUSIC can be applied to search for the target angles. The covariance matrix of $\tilde{\mathbf{y}}_\ell$ is given by

$$\tilde{\mathbf{R}} = \frac{1}{L} \sum_{\ell=1}^L \tilde{\mathbf{y}}_\ell \tilde{\mathbf{y}}_\ell^H = \frac{1}{L} \mathbf{A} \mathbf{R}_p \mathbf{A}^H, \quad (93)$$

where $\mathbf{R}_p = \sum_{\ell=1}^L \mathbf{p}_\ell \mathbf{p}_\ell^H$. To ensure that \mathbf{R}_p is full-rank, we impose the constraint $K < L$. After obtaining the angles of targets, we can perform a 1D search in the distance domain.

Generalized ESPRIT [138]: For generalized ESPRIT, the antenna array is partitioned into two subarrays, with the first subarray comprising the first J antennas arranged in an ascending order, and the second subarray consisting of the last J antennas arranged in a descending order. The steering vectors of the two subarrays can be expressed as

$$\begin{aligned} \mathbf{A}_1 &= [\mathbf{a}_1(r_1, \theta_1), \dots, \mathbf{a}_1(r_K, \theta_K)] \in \mathbb{C}^{J \times K}, \\ \mathbf{A}_2 &= [\mathbf{a}_2(r_1, \theta_1), \dots, \mathbf{a}_2(r_K, \theta_K)] \in \mathbb{C}^{J \times K}. \end{aligned} \quad (94)$$

For the k -th target, we have

$$\begin{aligned} \mathbf{a}_1 &= \left[e^{j(-N\omega_k + N^2\psi_k)}, \dots, e^{j((-N+J-1)\omega_k + (N-J+1)^2\psi_k)} \right]^T, \\ \mathbf{a}_2 &= \left[e^{j(N\omega_k + N^2\psi_k)}, \dots, e^{j((N-J+1)\omega_k + (N-J+1)^2\psi_k)} \right]^T. \end{aligned} \quad (95)$$

The array response matrix \mathbf{A} can thus be divided as

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \text{Last } (M-J) \text{ rows} \end{bmatrix} = \begin{bmatrix} \text{First } (M-J) \text{ rows} \\ \mathbf{J} \mathbf{A}_2 \end{bmatrix}, \quad (96)$$

where \mathbf{J} is used to reverse the order of the matrix rows, and $\mathbf{J}^2 = \mathbf{I}$. The array response vector of the second subarray can be represented as

$$\mathbf{A}_2 = [\mathbf{D}(\theta_1)\mathbf{a}_1(r_1, \theta_1), \dots, \mathbf{D}(\theta_K)\mathbf{a}_1(r_K, \theta_K)], \quad (97)$$

where

$$\mathbf{D}(\theta_k) = \text{diag} \left[e^{-j2N\omega_k}, \dots, e^{-j2(N-L+1)\omega_k} \right]. \quad (98)$$

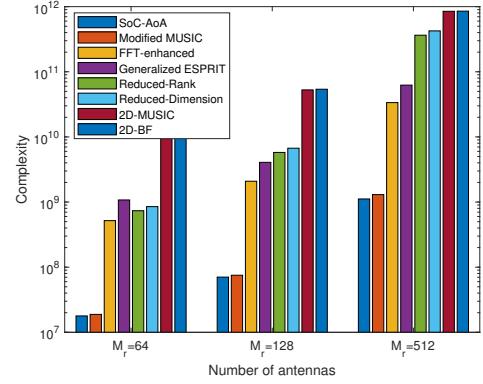


Fig. 16: Theoretical complexity of algorithms for near-field MIMO-OFDM ISAC. ($K = 4, N = 256, P = 10$)

Similar to (52), the rotation invariance between the two subarrays is manifest in terms of the signal subspace eigenvectors as follows:

$$\mathbf{E}_{s1} = \mathbf{A}_1 \mathbf{T}, \quad \mathbf{E}_{s2} = \mathbf{J} \mathbf{A}_2 \mathbf{T}. \quad (99)$$

Then, we define the new steering matrix

$$\begin{aligned} \mathbf{A}_3(\theta) &= [\mathbf{D}(\theta)\mathbf{a}_1(r_1, \theta_1), \dots, \mathbf{D}(\theta)\mathbf{a}_1(r_K, \theta_K)] \\ &= \mathbf{D}(\theta)\mathbf{A}_1, \end{aligned} \quad (100)$$

when θ aligns with the i -th target, and the i -th column of \mathbf{A}_3 is identical to the i -th column of \mathbf{A}_1 . Matrix $\mathbf{F}(\theta) = \mathbf{J} \mathbf{E}_{s2} - \mathbf{D}(\theta) \mathbf{E}_{s1}$ is rank deficient when θ corresponds to one of the target angles, which means that for an arbitrary full column rank matrix $\mathbf{W} \in \mathbb{C}^{J \times K}$, matrix $\mathbf{W}^H \mathbf{F}(\theta_k)$ is singular and its determinant is zero. The generalized ESPRIT exploits this observation by defining the following spectrum function whose peaks will correspond to the angles of targets

$$f(\theta) = \frac{1}{\det(\mathbf{W}^H \mathbf{F}(\theta))}. \quad (101)$$

Once the targets' angles are estimated, a 1D search can be conducted to estimate the distance parameters.

E. Performance Analysis and Comparison

Table VI shows the properties of the near-field sensing algorithms discussed above, including their complexity and resolution capability. Fig. 16 gives a numerical representation of the theoretical complexity of each algorithm based on the values in Table VI for receive array sizes of $M_r = 64, 128$ and 512 . The search grids for the angle and distance parameters are chosen to have $n_g = \frac{180}{0.05} = 3600$ and $n_l = \frac{90}{0.1} = 900$ cells, respectively. It is observed that for all M_r , SoC and modified MUSIC have the lowest complexity, while 2D MUSIC and 2D BF have the highest. However, as M_r increases, the complexity gap between 2D MUSIC/2D BF and other algorithms becomes smaller, because the complexity of the EVD becomes more dominant for large M_r .

Figs. 17 and 18 show the RMSE of the angle and distance parameters versus the SNR, respectively. The number of antenna elements is $M_r = 64$ with $d = \frac{\lambda}{4}$. The target is located at $(10.2^\circ, 5.64 \text{ m})$. As the SNR increases, the estimation

TABLE VI: Performance comparison for near-field sensing algorithms

Algorithm	Antenna spacing	Estimation resolution	Parameter pairing required?	Complexity ($M_r = 2J + 1$)
2D Beamforming [135]	$\leq \frac{\lambda}{2}$	Low	No	$M_r^2 NP + n_g n_l M_r^2$
2D MUSIC [136]	$\leq \frac{\lambda}{2}$	Highest	No	$M_r^3 + M_r^2 NP + n_g n_l (M_r - K)(M_r + 1)$
Second order cumulant [137]	$\leq \frac{\lambda}{4}$	High	Yes	$J^2 NP + Slog_2 S + K n_l M^2$
Reduced rank algorithm [132]	$\leq \frac{\lambda}{4}$	High	No	$M_r^3 + M_r^2 NP + (M_r - K)[n_g(J + 1)(M_r + J + 1) + n_l K(M_r + 1)]$
Reduced dimension algorithm [131]	$\leq \frac{\lambda}{4}$	High	Yes	$M_r^3 + M_r^2 NP + n_g[(M_r - K)(J + 1)(M + J + 1) + (J + 1)^3]$
FFT-enhanced near-field sensing [133]	$\leq \frac{\lambda}{2}$	High	No	$M_r^3 + M_r^2 NP + 2M_r Slog_2(S) + 2Ln_l M_r^2 + Ln'_g n'_l (M_r - K)(M_r + 1)$
Modified MUSIC [134]	$\leq \frac{\lambda}{4}$	High	No	$M_r^3 + M_r^2 NP + (J + 1)^3 + n_g(J + 1)^2 + n_l M_r^2 K$
Generalized ESPRIT [138]	$\leq \frac{\lambda}{4}$	High	No	$M_r^3 + M_r^2 NP + n_g K^4 (KJ + J^2) + n_l M_r^2 K$

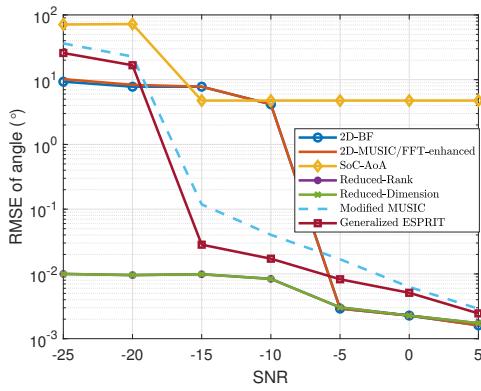


Fig. 17: RMSE of angle parameter for near-field MIMO-OFDM ISAC.

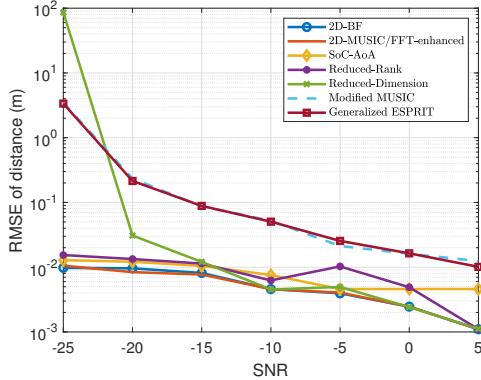


Fig. 18: RMSE of distance parameter for near-field MIMO-OFDM ISAC.

accuracy of all algorithms improves. For angle estimation, RD and RR perform best, thanks to their high-sensitivity angle search procedures. On the other hand, SoC based algorithm performs worst since the exploited structure of the covariance only holds asymptotically. We further see that 2D BF and 2D MUSIC are sensitive to SNR, and can only achieve good estimation performance when the SNR exceeds -5 dB.

In practice, several challenges still exist when implementing the existing algorithms as elaborated below.

• **Element spacing requirement:** To avoid phase ambiguity after calculating the second-order correlation for the SoC-based algorithm, the inter-antenna spacing should satisfy $d \leq \frac{\lambda}{4}$, which may lead to high mutual coupling between antenna

and is not compatible with existing communication systems that typically have half-wavelength spacing.

• **Reduced resolution:** For the SoC algorithm, the dimension of the data vector in (79) is only $dM_r/2$ due to the process of constructing cumulants. Therefore, the size of the effective aperture is reduced, and the spatial resolution decreases from $\frac{2\lambda}{D}$ to $\frac{4\lambda}{D}$.

• **Prior knowledge of the number of targets:** Almost all the algorithms mentioned above require prior knowledge of the number of targets, so the ability to accurately obtain this information will greatly impact the estimation performance. For MUSIC, this can be achieved by calculating the number of significantly large eigenvalues. For RR and RD algorithms, the number of targets is crucial to properly choose the number of peaks in the angle spectrum.

• **Error propagation:** By decoupling the angle and distance parameters, the complexity by parameter search algorithms can be significantly reduced. However, the accuracy of distance parameter estimate is highly dependent on the estimation results for the angle parameter. For the extreme case when several targets are located in the same direction, SoC and RD algorithms are no longer applicable, which may lead to severe errors in the subsequent distance estimation.

• **Parameter pairing:** Parameter pairing is an additional step after the estimation of K separate angles and distance values. For algorithms involving a 2D search, parameters are naturally paired. In addition, for the SoC-AoA and RR algorithms, the distances are obtained by performing a 1D search for each estimated angle, thus avoiding the problem of parameter pairing as well. For other algorithms, the problem and complexity associated with parameter pairing should be considered.

VI. EXTENDED DISCUSSION FOR MIMO-OFDM ISAC

In this section, we point out open problems and outline promising avenues that warrant further investigation for MIMO-OFDM ISAC to inspire future work.

A. Wideband XL-MIMO OFDM ISAC

To achieve higher angular and delay resolution, larger scale MIMO and wider bandwidths are needed for MIMO-OFDM ISAC systems. However, wideband XL-MIMO OFDM ISAC introduces spatial wideband effects [139], [140], also known

as the beam squint effects [141], where the propagation delay across the array aperture can not be ignored. The spatial wideband effect means that the array steering vector is dependent not only on the location or angle of the sensing target but also on frequency. As a result, the phase shift components in the spatial and subcarrier domains are no longer independent but become coupled for XL-MIMO OFDM ISAC. In addition, the use of XL-MIMO causes a variable Doppler frequency across the array aperture for mobile targets in the near-field, resulting in the coupling of the phase shift components in the spatial and Doppler domains. These challenging issues will fundamentally affect target sensing in XL-MIMO OFDM ISAC systems. Thus, the sensing methods and results established in this paper for conventional MIMO OFDM ISAC systems will require reformulation and modification.

B. Signal Pre-processing for MIMO-OFDM ISAC

Clutter rejection and clustering: Clutter from environmental scatterers makes it difficult to distinguish sensing targets from other reflections. One efficient approach for clutter rejection is through the use of adaptive filtering methods, such as MMSE and Kalman filtering [142]. Additionally, techniques such as spatial filtering and Doppler processing can enhance the ability of the radar to distinguish between targets and clutter, especially in dynamic environments [143]. Clustering methods also play a significant role in improving the SNR and facilitating more accurate detection in complex sensing environments. For instance, K-means clustering and Gaussian mixture models (GMM) can be employed to effectively separate target echoes from clutter [144]. Emerging techniques based on machine learning are increasingly explored for clutter rejection in ISAC systems. For instance, convolutional neural networks (CNNs) have shown promise in automatically learning the features that distinguish clutter from desired signals, leading to improved classification and interference rejection [145]. Filtering clutter based on pre-stored channel information in CKM is also an emerging and promising approach for clutter rejection. As an example, the authors of [74] proposed a new CKM approach referred to as CLAM, which can effectively remove environmental clutter through pre-acquired clutter angle information.

SI cancellation: In BS monostatic ISAC systems, the ISAC BS needs to operate in full-duplex mode, making it vulnerable to SI due to imperfect isolation. SI significantly increases the noise level at the Sen-RX, thereby degrading sensing performance. Various methods have been proposed to address this issue, such as a combination of analog and digital cancellers [3], where the analog canceller is implemented through an additional RF chain, while the digital canceller operates in the baseband of the sampled digital sensing signal.

Synchronization: For bistatic and multi-static ISAC systems, the ISAC-TX and RX are spatially separated and not synchronized since they have separate LO and sampling clock references. Therefore, the received signal experiences an additional STO τ_Δ , CFO f_Δ , and a sampling frequency offset (SFO) δ_Δ caused by the deviation in the sampling frequency between the digital-to-analog converters (DAC) at the ISAC-TX and the analog-to-digital converters (ADC) at the RX

[37]. Typically, synchronization is performed based on the LoS channel. Therefore, the RX needs to estimate and compensate for the time offset (TO) $\tau_{off} = \tau_0 + \tau_\Delta$, the frequency offset (FO) $f_{off} = v_0 + f_\Delta$, and δ_Δ . The TO must be compensated for first to determine the starting point for the OFDM signal. A coarse TO and FO can be estimated based on the C&S algorithm [146]. Furthermore, cross-correlation between the received signal and the first preamble symbol is performed for fine TO estimation [147]. After finding the starting point of the received signal, the SFO can be estimated based on the remaining preambles via a weighted LS algorithm [148]. For more accurate SFO estimation, the entire received signal can be interpolated by a multi-rate finite impulse response (FIR) filter [149], [150] and fed into a sample rate converter based on a polynomial filter [151]. Finally, based on the estimate of the coarse FO, the residual FO can be obtained by constructing a delay Doppler profile (DDP) based on pilots [152].

C. Resource Allocation and Beam Management

In practice, an ISAC system cannot continuously occupy all time-frequency resources to execute a single communication or sensing task. For example, in 5G NR TDD mode, the slots for uplink and downlink communications are alternating and non-uniform [153], [154], presenting a new challenge for sensing tasks in ISAC systems. For practical multi-user ISAC scenarios, users obtain C&S services through orthogonal frequency division multiple access (OFDMA). Thus, different users can only be allocated part of the time-frequency resources and transmit beamforming is exploited to serve multiple users to accomplish different C&S tasks in ISAC systems. Therefore, resource allocation and beam management are two key design issues in ISAC systems, as discussed in the following.

Resource allocation: In conventional communication systems, the collocated allocation of time-frequency resources inherently constrains the achievable delay and Doppler resolution for individual users. In ISAC systems, sparse resource allocation [153], [155] emerges as a promising alternative, enhancing sensing resolution by expanding the effective time and frequency spans. Sparse allocation can be broadly categorized into uniform and non-uniform approaches. Uniform sparse allocation employs fixed-interval resource distribution, which improves sensing resolution at the expense of a significantly reduced unambiguous range. Non-uniform sparse allocation maintains the unambiguous range while enhancing resolution, albeit at the cost of elevated sidelobes in delay/Doppler periodograms, potentially obscuring weak targets. Pre-storing non-uniform sparse patterns with suppressed sidelobes via traversal search and structured allocation based on nested [156], coprime [154], or minimum redundancy [157], coupled with differential coarray processing, are feasible non-uniform sparse allocation strategies. However, traversal search methods suffer from prohibitive computational complexity, while fixed allocation structures may conflict with functional reference signals in mobile communication standards. Consequently, critical challenges remain in optimizing time-frequency resources and subcarrier power allocation [40], necessitating further investigation into trade-offs between fairness and efficiency, as well as between C&S performance.

Beam management: Beam management [158], [159] can be categorized as referring to beam alignment, beam sweeping, or beam tracking, based on different transmission or reception methods. Beam alignment involves aligning the transmission and reception beams between a transmitter and a receiver to maximize the signal transmission efficiency. Beam sweeping is the most commonly used method for achieving beam alignment, which involves transmitting or receiving beams from a predefined codebook over a given time interval to cover a given spatial area. Beam tracking aims to achieve beam alignment by predicting and tracking the positions or angles of moving users or targets. These various beam management techniques alter the transmit beamforming vectors for different time-frequency resources across different CPIs and affect the duration of a CPI. The signal processing methods discussed in this paper are applicable within a given CPI for each beam management technique. However, multiple beams under different management schemes may be transmitted or received simultaneously. Orthogonal multiple access (OMA) [160] can separate them via orthogonal time-frequency resources, whereas nonorthogonal multiple access (NOMA) [161] presents additional challenges. Furthermore, multi-beam transmission requires power allocation among the beams, which is another critical consideration.

D. Sparse MIMO ISAC

Looking ahead to future 6G mobile communication networks, MIMO systems are expected to provide finer spatial resolution, not only enhancing the SE of wireless communication but also enabling more precise wireless sensing. To tackle higher power consumption and signal processing complexity of MIMO systems, sparse arrays have been proposed as an alternative configuration to achieve a large array aperture with a small number of antennas [52] [162]. Sparse MIMO arrays allow for a large virtual MIMO system for sensing [163], which still enables underdetermined target parameter estimation. However, sparse MIMO also leads to a number of challenging issues. For example, since sparse antenna geometries result in antenna spacing that exceeds half wavelength, undesired grating lobes are generated. When users or targets are located within these lobes, severe communication inter-user interference and radar angle ambiguity occur. For sparse MIMO sensing, differential or sum co-arrays have been demonstrated to effectively mitigate grating lobes and prevent angular ambiguity. However, the application of these techniques to communication or ISAC systems remains unclear. Moreover, in near-field scenarios, the the spatially nonstationary phase that results from spherical wave propagation does not allow formation of a virtual array as in the far-field case [164]. Therefore, developing methods to enhance DoFs in the near-field is a critical challenge. Sparse array design itself is a challenging problem, and more work is needed to develop designs that provide an effective balance between C&S performance.

E. NLoS Sensing and Localization in ISAC

The vision of pervasive sensing and connectivity for future 6G wireless networks relies not only on LoS signals but also

on NLoS sensing and localization. Nevertheless, achieving highly accurate localization, particularly for NLoS components, still remains an open issue. The parameter ambiguity introduced by NLoS propagation presents both challenges and opportunities in ISAC. Although it is challenging to directly perform sensing and localization based on NLoS components, hybrid localization schemes (e.g., RSS-ToA [165] and AoA-ToA [166]) integrated with prior information (e.g., CSI [167] and CKM) make NLoS sensing possible. NLoS components can significantly contribute to more precise localization services and transmission coverage. Thus, IRS [70], [71] aided ISAC is regarded as a promising approach that can enhance C&S performance by creating additional NLoS paths [168].

F. Mutualism between ISAC and CKM

More comprehensive and environment-aware ISAC networks can be designed by establishing a feedback loop in which ISAC facilitates the construction, updating, and calibration of CKM. CKM [73] is a site-specific database, marked with transmitter and/or receiver locations and containing channel-related information to enable environment-awareness. CKM provides a structured approach for directly incorporating environmental characteristics into communication and sensing processes, allowing for a deeper understanding of the relationship between the physical environment and wireless channel properties. This breaks the heavy reliance of current ISAC systems on geometric assumptions and LoS conditions, enabling more robust and accurate communication and sensing. For ISAC, CKM can leverage prior knowledge of CSI to facilitate environment-aware ISAC. This means ISAC systems can actively adapt to the surrounding environment, reducing frequent real-time channel estimation, optimizing data transmission, while enabling environmental clutter rejection, assisting in target detection and tracking, enabling NLoS localization and sensing, and enhancing localization and sensing performance. On the other hand, ISAC can utilize CSI captured through sensing to achieve initial CKM construction, enabling dynamic updates and long-term calibration of CKM. The mutualism between CKM and ISAC warrants more in-depth exploration for future wireless networks.

G. Prototyping and Experimental Validation

Proof-of-concept prototype design and experimental verification are prerequisites for realizing ISAC in practical wireless networks. Numerous efforts have been devoted to addressing this issues. Early experiments [169], [170] focused on validating and analyzing the performance of using OFDM for radar sensing, which later evolved into system prototype designs for monostatic OFDM ISAC [171], [172]. In [25], the authors developed an OFDM ISAC prototype based on the 5G NR frame structure. A bistatic ISAC prototype system using an OFDM frame-based approach for sensing tasks was validated in [37]. OFDM-based simultaneous localization and mapping (SLAM) [9], synthetic aperture radar (SAR) imaging [173], and ISAC-enabled CKM prototype [32] systems have been successful implemented. In contrast, while there are many mature experimental platforms for MIMO OFDM ISAC in

far-field scenarios, more prototype designs and experimental validations are needed for massive MIMO and near-field ISAC systems as well as the network-level implementations.

VII. CONCLUSION

MIMO-OFDM is the most likely wireless technology to realize ISAC in future 6G wireless networks due to its predominant use in existing wireless systems. This paper has provided a comprehensive tutorial overview on ISAC systems enabled by MIMO-OFDM. A unified approach to modeling MIMO-OFDM ISAC systems and their enabling signal processing algorithms were presented, under both far-field and near-field channel conditions. Open problems for MIMO-OFDM ISAC that deserve further investigation were also discussed. We hope that this tutorial will provide a useful guide for researchers working on ISAC, towards efficiently integrating it into future 6G wireless networks via efficient synergy with MIMO and OFDM.

REFERENCES

- [1] RECOMMENDATION, ITUR, “Framework and overall objectives of the future development of IMT for 2030 and beyond,” International Telecommunication Union (ITU) Recommendation (ITU-R), Tech. Rep., 2023.
- [2] 3GPP TR 22.837, “Feasibility study on integrated sensing and communication,” 3rd Generation Partnership Project (3GPP), Tech. Rep., 2022.
- [3] Z. Xiao and Y. Zeng, “Waveform design and performance analysis for full-duplex integrated sensing and communication,” *IEEE J. Sel. Areas Commun.*, vol. 40, no. 6, pp. 1823–1837, 2022.
- [4] A. Liu, Z. Huang, M. Li, Y. Wan, W. Li, T. X. Han, C. Liu, R. Du, D. K. P. Tan, J. Lu *et al.*, “A survey on fundamental limits of integrated sensing and communication,” *IEEE Commun. Surv. Tut.*, vol. 24, no. 2, pp. 994–1034, 2022.
- [5] Z. Wei, H. Qu, Y. Wang, X. Yuan, H. Wu, Y. Du, K. Han, N. Zhang, and Z. Feng, “Integrated sensing and communication signals toward 5G-A and 6G: A survey,” *IEEE Internet Things J.*, vol. 10, no. 13, pp. 11 068–11 092, 2023.
- [6] F. Liu, Y. Cui, C. Masouros, J. Xu, T. X. Han, Y. C. Eldar, and S. Buzzi, “Integrated sensing and communications: Toward dual-functional wireless networks for 6G and beyond,” *IEEE J. Sel. Areas Commun.*, vol. 40, no. 6, pp. 1728–1767, 2022.
- [7] J. A. Zhang, M. L. Rahman, K. Wu, X. Huang, Y. J. Guo, S. Chen, and J. Yuan, “Enabling joint communication and radar sensing in mobile networks—A survey,” *IEEE Commun. Surveys. Tuts.*, vol. 24, no. 1, pp. 306–345, 2021.
- [8] Y. Cui, F. Liu, C. Masouros, J. Xu, T. X. Han, and Y. C. Eldar, “Integrated sensing and communications: Background and applications,” in *Integrated Sensing and Communications*. Springer, 2023, pp. 3–21.
- [9] J. Yang, C.-K. Wen, X. Yang, J. Xu, T. Du, and S. Jin, “Multi-domain cooperative SLAM: The enabler for integrated sensing and communications,” *IEEE Wireless Commun.*, vol. 30, no. 1, pp. 40–49, 2023.
- [10] Z. Xiao and Y. Zeng, “An overview on integrated localization and communication towards 6G,” *Sci. China Inf. Sci.*, vol. 65, pp. 1–46, 2022.
- [11] D. Ma, N. Shlezinger, T. Huang, Y. Liu, and Y. C. Eldar, “Joint radar-communication strategies for autonomous vehicles: Combining two key automotive technologies,” *IEEE Signal Process. Mag.*, vol. 37, no. 4, pp. 85–97, 2020.
- [12] W. Tong and P. Zhu, *6G the Next Horizon: From Connected People and Things to Connected Intelligence*, 1st ed. Cambridge, UK: Cambridge University Press, 2021.
- [13] R. Saruthirathanaworakun, J. M. Peha, and L. M. Correia, “Opportunistic sharing between rotating radar and cellular,” *IEEE J. Sel. Areas Commun.*, vol. 30, no. 10, pp. 1900–1910, 2012.
- [14] A. J. Paulraj and T. Kailath, “Increasing capacity in wireless broadcast systems using distributed transmission/directional reception (DTDR),” Sep. 6 1994, U.S. Patent 5,345,599.
- [15] E. Fishler, A. Haimovich, R. Blum, D. Chizhik, L. Cimini, and R. Valenzuela, “MIMO radar: An idea whose time has come,” in *Proc. IEEE Radar Conf.*, Philadelphia, PA, USA, Apr. 2004, pp. 71–78.
- [16] C. Sturm and W. Wiesbeck, “Waveform design and signal processing aspects for fusion of wireless communications and radar sensing,” *Proc. IEEE*, vol. 99, no. 7, pp. 1236–1259, 2011.
- [17] A. Hassani, M. G. Amin, E. Abutianios, and B. Himed, “Dual-function radar communication systems: A solution to the spectrum congestion problem,” *IEEE Signal Process. Mag.*, vol. 36, no. 5, pp. 115–126, 2019.
- [18] R. M. Mealey, “A method for calculating error probabilities in a radar communication system,” *IEEE Trans. Space Electron. Telem.*, vol. 9, no. 2, pp. 37–42, 1963.
- [19] J. Chapin, “Shared spectrum access for radar and communications (SSPARC),” *Defense Advanced Research Projects Agency*, 2013.
- [20] Y. Xiong, F. Liu, Y. Cui, W. Yuan, T. X. Han, and G. Caire, “On the fundamental tradeoff of integrated sensing and communications under Gaussian channels,” *IEEE Trans. Inf. Theory*, vol. 69, no. 9, pp. 5723–5751, Sep. 2023.
- [21] Z. Wei, J. Piao, X. Yuan, H. Wu, J. A. Zhang, Z. Feng, L. Wang, and P. Zhang, “Waveform design for MIMO-OFDM integrated sensing and communication system: An information theoretical approach,” *IEEE Trans. Commun.*, vol. 72, no. 1, pp. 496–509, 2024.
- [22] H. Wang, Z. Xiao, and Y. Zeng, “Cramér-Rao bounds for near-field sensing with extremely large-scale MIMO,” *IEEE Trans. Signal Process.*, vol. 72, pp. 701–717, 2024.
- [23] Z. Xiao and Y. Zeng, “Integrated sensing and communication with delay alignment modulation: Performance analysis and beamforming optimization,” *IEEE Trans. Wireless Commun.*, vol. 22, no. 12, pp. 8904–8918, 2023.
- [24] F. Dong, F. Liu, Y. Xiong, Y. Cui, W. Wang, and S. Jin, “Communication-assisted sensing systems: Fundamental limits and ISAC waveform design,” *arXiv preprint arXiv:2409.03561*, 2024.
- [25] C. Zhang, Z. Zhou, H. Wang, and Y. Zeng, “Integrated super-resolution sensing and communication with 5G NR waveform: Signal processing with uneven CPs and experiments: (Invited Paper),” in *Proc. of 2023 21st Int. Symp. on Model. and Optim. in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, 2023, pp. 681–688.
- [26] W. Yuan, Z. Wei, S. Li, J. Yuan, and D. W. K. Ng, “Integrated sensing and communication-assisted orthogonal time frequency space transmission for vehicular networks,” *IEEE J. Sel. Topics Signal Process.*, vol. 15, no. 6, pp. 1515–1528, 2021.
- [27] S. Bhattacharjee, K. V. Mishra, R. AnnavaJJala, and C. R. Murthy, “Evaluation of orthogonal chirp division multiplexing for automotive integrated sensing and communications,” in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP)*, 2022, pp. 8742–8746.
- [28] A. Bemani, N. Ksairi, and M. Kountouris, “Affine frequency division multiplexing for next generation wireless communications,” *IEEE Trans. Wireless Commun.*, vol. 22, no. 11, pp. 8214–8229, 2023.
- [29] F. Uysal, “Phase-coded FMCW automotive radar: System design and interference mitigation,” *IEEE Trans. Veh. Technol.*, vol. 69, no. 1, pp. 270–281, 2020.
- [30] S. Lu, F. Liu, F. Dong, Y. Xiong, J. Xu, Y.-F. Liu, and S. Jin, “Random ISAC signals deserve dedicated precoding,” *IEEE Trans. Signal Process.*, vol. 72, no. 1, pp. 3453–3469, 2024.
- [31] S. P. Chepuri, N. Shlezinger, F. Liu, G. C. Alexandropoulos, S. Buzzi, and Y. C. Eldar, “Integrated sensing and communications with reconfigurable intelligent surfaces: From signal modeling to processing,” *IEEE Signal Process. Mag.*, vol. 40, no. 6, pp. 41–62, 2023.
- [32] C. Zhang, Z. Zhou, Y. Yang, Y. Qiu, Z. Yu, X. Xu, and Y. Zeng, “Real-time demo of ISAC-based channel knowledge map prototyping system,” in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC)*, 2024, pp. 295–296.
- [33] C. Zhang, Z. Zhou, X. Xu, Y. Zeng, Z. Zhang, and S. Jin, “Prototyping and experimental results for ISAC-based channel knowledge map,” *IEEE Trans. Veh. Technol.*, Feb. 2025, Early Access.
- [34] Z. Feng, Z. Fang, Z. Wei, X. Chen, Z. Quan, and D. Ji, “Joint radar and communication: A survey,” *China Commun.*, vol. 17, no. 1, pp. 1–27, 2020.
- [35] W. Chen, L. Li, Z. Chen, T. Quek, and S. Li, “Enhancing THz/mmwave network beam alignment with integrated sensing and communication,” *IEEE Commun. Lett.*, vol. 26, no. 7, pp. 1698–1702, 2022.
- [36] K. Ji, Q. Zhang, Z. Wei, Z. Feng, and P. Zhang, “Networking based ISAC hardware testbed and performance evaluation,” *IEEE Commun. Mag.*, vol. 61, no. 5, pp. 76–82, 2023.

[37] D. Brunner, L. G. de Oliveira, and C. Muth, "Bistatic OFDM-based ISAC with over-the-air synchronization: System concept and performance analysis," *IEEE Trans. Microw. Theory Techn.*, Nov. 2024, Early Access.

[38] P. Li, M. Li, R. Liu, Q. Liu, and A. Lee Swindlehurst, "MIMO-OFDM ISAC waveform design for range-Doppler sidelobe suppression," *IEEE Trans. Wireless Commun.*, vol. 24, no. 2, pp. 1001–1015, 2025.

[39] Y. Niu, Z. Wei, L. Wang, H. Wu, and Z. Feng, "Interference management for integrated sensing and communication systems: A survey," *IEEE Internet Things J.*, vol. 12, no. 7, pp. 8110–8134, 2025.

[40] F. Dong, F. Liu, Y. Cui, W. Wang, K. Han, and Z. Wang, "Sensing as a service in 6G perceptive networks: A unified framework for ISAC resource allocation," *IEEE Trans. Wireless Commun.*, vol. 22, no. 5, pp. 3522–3536, 2023.

[41] Z. Xiao, R. Liu, M. Li, W. Wang, and Q. Liu, "Sparsity exploitation via joint receive processing and transmit beamforming design for MIMO-OFDM ISAC systems," *IEEE Trans. Commun.*, Oct. 2024, Early Access.

[42] J. A. Zhang, F. Liu, C. Masouros, R. W. Heath, Z. Feng, L. Zheng, and A. Petropulu, "An overview of signal processing techniques for joint communication and radar sensing," *IEEE J. Sel. Topics Signal Process.*, vol. 15, no. 6, pp. 1295–1315, 2021.

[43] S. Kay, *Fundamentals of Statistical Signal Processing, Volume II: Detection Theory*. Prentice Hall, Feb. 1998.

[44] M. Jiang, G. Xu, H. Pei, Z. Feng, S. Ma, H. Zhang, and W. Hong, "4D high-resolution imagery of point clouds for automotive mmwave radar," *IEEE Trans. Intell. Transp. Syst.*, vol. 25, no. 1, pp. 998–1012, 2024.

[45] R. Zhu, J. Zhou, B. Cheng, Q. Fu, and G. Jiang, "Sequential frequency-domain imaging algorithm for near-field MIMO-SAR with arbitrary scanning paths," *IEEE J. Sel. Topics Appl. Earth Observ. Remote Sens.*, vol. 12, no. 8, pp. 2967–2975, 2019.

[46] R. Li, Z. Xiao, and Y. Zeng, "Toward seamless sensing coverage for cellular multi-static integrated sensing and communication," *IEEE Trans. Wireless Commun.*, vol. 23, no. 6, pp. 5363–5376, 2024.

[47] L. Han and K. Wu, "Joint wireless communication and radar sensing systems—state of the art and future prospects," *IET Microw. Antennas Propag.*, vol. 7, no. 11, pp. 876–885, 2013.

[48] C. Shi, F. Wang, M. Sellathurai, J. Zhou, and S. Salous, "Power minimization-based robust OFDM radar waveform design for radar and communication systems in coexistence," *IEEE Trans. Signal Process.*, vol. 66, no. 5, pp. 1316–1330, 2017.

[49] C. D'Andrea, S. Buzzi, and M. Lops, "Communications and radar coexistence in the massive MIMO regime: Uplink analysis," *IEEE Trans. Wireless Commun.*, vol. 19, no. 1, pp. 19–33, 2019.

[50] H. Lu and Y. Zeng, "Communicating with extremely large-scale array/surface: Unified modeling and performance analysis," *IEEE Trans. Wireless Commun.*, vol. 21, no. 6, pp. 4039–4053, 2021.

[51] H. Lu, Y. Zeng, C. You, Y. Han, J. Zhang, Z. Wang, Z. Dong, S. Jin, C.-X. Wang, T. Jiang, X. You, and R. Zhang, "A tutorial on near-field XL-MIMO communications towards 6G," *IEEE Commun. Surveys. Tuts.*, vol. 26, no. 4, pp. 2213–2257, 2024.

[52] X. Li, H. Min, Y. Zeng, S. Jin, L. Dai, Y. Yuan, and R. Zhang, "Sparse MIMO for ISAC: New opportunities and challenges," *IEEE Wirel. Commun.*, Mar. 2025, Early Access.

[53] H. Wang, C. Feng, Y. Zeng, S. Jin, C. Yuen, B. Clerckx, and R. Zhang, "Enhancing spatial multiplexing and interference suppression for near- and far-field communications with sparse MIMO," *arXiv preprint arXiv:2408.01956*, 2024.

[54] X. Li, H. Lu, Y. Zeng, S. Jin, and R. Zhang, "Near-field modeling and performance analysis of modular extremely large-scale array communications," *IEEE Commun. Lett.*, vol. 26, no. 7, pp. 1529–1533, 2022.

[55] X. Li, Z. Dong, Y. Zeng, S. Jin, and R. Zhang, "Near-field beam focusing pattern and grating lobe characterization for modular XL-array," in *Proc. IEEE Glob. Commun. Conf., GLOBECOM*, 2023.

[56] K.-K. Wong, A. Shoaieifard, K.-F. Tong, and Y. Zhang, "Fluid antenna systems," *IEEE Trans. Wireless Commun.*, vol. 20, no. 3, pp. 1950–1962, 2021.

[57] L. Zhu, W. Ma, and R. Zhang, "Movable antennas for wireless communication: Opportunities and challenges," *IEEE Commun. Mag.*, vol. 62, no. 6, pp. 114–120, 2024.

[58] Z. Dong, Z. Zhou, Z. Xiao, C. Zhang, X. Li, H. Min, Y. Zeng, S. Jin, and R. Zhang, "Movable antenna for wireless communications: prototyping and experimental results," 2024. [Online]. Available: <https://arxiv.org/abs/2408.08588>

[59] H. Lu, Y. Zeng, S. Jin, and R. Zhang, "Group movable antenna with flexible sparsity: Joint array position and sparsity optimization," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 3573–3577, 2024.

[60] X. Shao, Q. Jiang, and R. Zhang, "6D movable antenna based on user distribution: Modeling and optimization," *IEEE Trans. Wireless Commun.*, vol. 24, no. 1, pp. 355–370, 2025.

[61] F. Liu, L. Zhou, C. Masouros, A. Li, W. Luo, and A. Petropulu, "Toward dual-functional radar-communication systems: Optimal waveform design," *IEEE Trans. Signal Process.*, vol. 66, no. 16, pp. 4264–4279, 2018.

[62] F. Liu, C. Masouros, A. Li, H. Sun, and L. Hanzo, "MU-MIMO communications with MIMO radar: From co-existence to joint transmission," *IEEE Trans. Wireless Commun.*, vol. 17, no. 4, pp. 2755–2770, 2018.

[63] X. Liu, T. Huang, N. Shlezinger, Y. Liu, J. Zhou, and Y. C. Eldar, "Joint transmit beamforming for multiuser MIMO communications and MIMO radar," *IEEE Trans. Signal Process.*, vol. 68, pp. 3929–3944, 2020.

[64] H. Hua, J. Xu, and T. X. Han, "Optimal transmit beamforming for integrated sensing and communication," *IEEE Trans. Veh. Technol.*, vol. 72, no. 8, pp. 10588–10603, 2023.

[65] S. D. Liyanarachchi, C. B. Barneto, T. Riihonen, M. Heino, and M. Valkama, "Joint multi-user communication and MIMO radar through full-duplex hybrid beamforming," in *Proc. IEEE Int. Online Symp. on Joint Commun. & Sens. (JC&S)*, 2021.

[66] Z. Xiao, S. Chen, and Y. Zeng, "Simultaneous multi-beam sweeping for mmwave massive MIMO integrated sensing and communication," *IEEE Trans. Veh. Technol.*, vol. 73, no. 6, pp. 8141–8152, 2024.

[67] G. Jing, J. Hong, X. Yin, J. Rodríguez-Piñeiro, and Z. Yu, "Measurement-based 3-D channel modeling with cluster-of-scatterers estimated under spherical-wave assumption," *IEEE Trans. Wireless Commun.*, vol. 22, no. 9, pp. 5828–5843, 2023.

[68] R. Yang, C.-X. Wang, J. Huang, E.-H. M. Aggoune, and Y. Hao, "A novel 6G ISAC channel model combining forward and backward scattering," *IEEE Trans. Wireless Commun.*, vol. 22, no. 11, pp. 8050–8065, 2023.

[69] T. Liu, K. Guan, D. He, P. T. Mathiopoulos, K. Yu, Z. Zhong, and M. Guizani, "6G integrated sensing and communications channel modeling: Challenges and opportunities," *IEEE Veh. Technol. Mag.*, vol. 19, no. 2, pp. 31–40, 2024.

[70] B. Zheng, C. You, W. Mei, and R. Zhang, "A survey on channel estimation and practical passive beamforming design for intelligent reflecting surface aided wireless communications," *IEEE Commun. Surveys. Tuts.*, vol. 24, no. 2, pp. 1035–1071, 2022.

[71] Z. Liu, H. Zhang, T. Huang, F. Xu, and Y. C. Eldar, "Hybrid RIS-assisted MIMO dual-function radar-communication system," *IEEE Trans. Signal Process.*, vol. 72, pp. 1650–1665, 2024.

[72] Z. Yu, C. Feng, Y. Zeng, T. Li, and S. Jin, "Wireless communication using metal reflectors: Reflection modelling and experimental verification," in *Proc. IEEE Int. Conf. Commun.*, 2023, pp. 4701–4706.

[73] Y. Zeng, J. Chen, J. Xu, D. Wu, X. Xu, S. Jin, X. Gao, D. Gesbert, S. Cui, and R. Zhang, "A tutorial on environment-aware communications via channel knowledge map for 6G," *IEEE Commun. Surveys. Tuts.*, vol. 26, no. 3, pp. 1478–1519, 2024.

[74] Z. Xu, Z. Zhou, D. Wu, and Y. Zeng, "Channel knowledge map-enhanced clutter suppression for integrated sensing and communication," in *Proc. IEEE/CIC Int. Conf. Commun. China - Workshops (ICCC Workshops)*, 2024, pp. 90–95.

[75] Y. Liu, P. Roblin, X. Quan, W. Pan, S. Shao, and Y. Tang, "A full-duplex transceiver with two-stage analog cancellations for multipath self-interference," *IEEE Trans. Microw. Theory Techn.*, vol. 65, no. 12, pp. 5263–5273, 2017.

[76] C. B. Barneto, T. Riihonen, M. Turunen, L. Anttila, M. Fleischer, K. Stadius, J. Ryynänen, and M. Valkama, "Full-duplex OFDM radar with LTE and 5G NR waveforms: Challenges, solutions, and measurements," *IEEE Trans. Microw. Theory Techn.*, vol. 67, no. 10, pp. 4042–4054, 2019.

[77] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57, no. 8, pp. 1408–1418, 1969.

[78] J. Li and P. Stoica, "An adaptive filtering approach to spectral estimation and SAR imaging," *IEEE Trans. Signal Process.*, vol. 44, no. 6, pp. 1469–1484, 1996.

[79] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 276–280, 1986.

[80] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 7, pp. 984–995, 1989.

[81] D. Cohen, D. Cohen, Y. C. Eldar, and A. M. Haimovich, "SUMMeR: Sub-Nyquist MIMO radar," *IEEE Trans. Signal Process.*, vol. 66, no. 16, pp. 4315–4330, 2018.

[82] E. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489–509, 2006.

[83] L. Zheng and X. Wang, "Super-resolution delay-doppler estimation for ofdm passive radar," *IEEE Trans. Signal Process.*, vol. 65, no. 9, pp. 2197–2210, 2017.

[84] S.-J. Kim, K. Koh, M. Lustig, S. Boyd, and D. Gorinevsky, "An interior-point method for large-scale ℓ_1 -regularized least squares," *IEEE J. Sel. Top. Signal Process.*, vol. 1, no. 4, pp. 606–617, 2007.

[85] D. Malioutov, M. Cetin, and A. Willsky, "A sparse signal reconstruction perspective for source localization with sensor arrays," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3010–3022, 2005.

[86] Z. Yang and L. Xie, "Enhancing sparsity and resolution via reweighted atomic norm minimization," *IEEE Trans. Signal Process.*, vol. 64, no. 4, pp. 995–1006, 2016.

[87] Y. Chi and M. Ferreira Da Costa, "Harnessing sparsity over the continuum: Atomic norm minimization for superresolution," *IEEE Signal Process. Mag.*, vol. 37, no. 2, pp. 39–57, 2020.

[88] J. Xu, H. Wang, Y. Zeng, and X. Xu, "Little pilot is needed for channel estimation with integrated super-resolution sensing and communication," in *Proc. IEEE Wirel. Commun. Netw. Conf. (WCNC)*, 2024.

[89] J. Xu, H. Wang, Y. Zeng, X. Xu, Q. Wu, F. Yang, Y. Chen, and A. Jamalipour, "Efficient channel estimation for millimeter wave and terahertz systems enabled by integrated super-resolution sensing and communication," 2024. [Online]. Available: <https://arxiv.org/abs/2407.20607>

[90] R. Saadia and N. M. Khan, "Single carrier-frequency division multiple access radar: Waveform design and analysis," *IEEE Access*, vol. 8, pp. 35 742–35 751, 2020.

[91] Y. Tang, W. Shieh, and B. S. Krongold, "DFT-spread OFDM for fiber nonlinearity mitigation," *IEEE Photon. Technol. Lett.*, vol. 22, no. 16, pp. 1250–1252, 2010.

[92] Z. Xiao, Y. Zeng, F. Wen, Z. Zhang, and D. W. K. Ng, "Integrated sensing and channel estimation by exploiting dual timescales for delay-Doppler alignment modulation," *IEEE Trans. Wireless Commun.*, vol. 24, no. 1, pp. 415–429, 2025.

[93] M. A. Herman and T. Strohmer, "High-resolution radar via compressed sensing," *IEEE Trans. Signal Process.*, vol. 57, no. 6, pp. 2275–2284, 2009.

[94] W. Johannes, S. Stanko, and I. Kallfass, "Efficient joint broadband radar and single carrier communication system in frequency division multiplexing for high-range applications," in *Proc. 24th Int. Radar Symp. (IRS)*, 2023, pp. 1–10.

[95] M. Bekar, C. J. Baker, E. G. Hoare, and M. Gashinova, "Joint MIMO radar and communication system using a PSK-LFM waveform with TDM and CDM approaches," *IEEE Sensors J.*, vol. 21, no. 5, pp. 6115–6124, 2021.

[96] Z. Wang, J. Jiang, K. Han, and L. Chen, "Low-complexity transceiver beamforming for DFRC with MIMO radar and MU-MIMO communication," in *Proc. International Wireless Communications and Mobile Computing (IWCMC)*, 2022, pp. 32–37.

[97] K. Chen, Y. Liu, and W. Zhang, "Study on integrated radar-communication signal of OFDM-LFM based on FRFT," in *IET Int. Radar Conf. 2015*, 2015, pp. 1–6.

[98] G. Lelouch, R. Pribic, and P. van Genderen, "Frequency agile stepped OFDM waveform for HRR," in *2009 Int. Waveform Divers. Des. Conf.*, 2009, pp. 90–93.

[99] G. Lelouch, A. K. Mishra, and M. Inggs, "Stepped OFDM radar technique to resolve range and doppler simultaneously," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 2, pp. 937–950, 2015.

[100] X. Yuan, Z. Feng, J. A. Zhang, W. Ni, R. P. Liu, Z. Wei, and C. Xu, "Spatio-temporal power optimization for MIMO joint communication and radio sensing systems with training overhead," *IEEE Trans. Veh. Technol.*, vol. 70, no. 1, pp. 514–528, 2021.

[101] B. Manjunath, J. J. Zhang, A. Papandreou-Suppappola, and D. Morrell, "Sensor scheduling with waveform design for dynamic target tracking using MIMO radar," in *Proc. Conf. Rec. Asilomar Conf. Signals Syst. Comput.*, 2009, pp. 141–145.

[102] Y. Huang, S. Hu, S. Ma, Z. Liu, and M. Xiao, "Designing low-PAPR waveform for OFDM-based radcom systems," *IEEE Trans. Wireless Commun.*, vol. 21, no. 9, pp. 6979–6993, 2022.

[103] Z. Ni, J. A. Zhang, K. Yang, X. Huang, and T. A. Tsiftsis, "Multi-metric waveform optimization for multiple-input single-output joint communication and radar sensing," *IEEE Trans. Commun.*, vol. 70, no. 2, pp. 1276–1289, 2022.

[104] T. Wild, V. Braun, and H. Viswanathan, "Joint design of communication and sensing for beyond 5G and 6G systems," *IEEE Access*, vol. 9, pp. 30 845–30 857, 2021.

[105] 3GPP TSG RAN #101,RP-231798, "Rel-19 channel modeling for ISAC and new spectrum (7-24GHz) (RAN1-led)," vivo, Tech. Rep., September 11-15, 2023.

[106] X. Xu, Z. Zhou, and Y. Zeng, "How does CP length affect the sensing range for OFDM-ISAC?" 2025. [Online]. Available: <https://arxiv.org/abs/2503.08062>

[107] A.-J. van der Veen, M. Vanderveen, and A. Paulraj, "Joint angle and delay estimation using shift-invariance techniques," *IEEE Trans. Signal Process.*, vol. 46, no. 2, pp. 405–418, 1998.

[108] Z. Xiao, R. Liu, M. Li, Q. Liu, and A. L. Swindlehurst, "A novel joint angle-range-velocity estimation method for MIMO-OFDM ISAC systems," *IEEE Trans. Signal Process.*, vol. 72, pp. 3805–3818, 2024.

[109] W. L. Stutzman and G. A. Thiele, *Antenna Theory and Design*. John Wiley & Sons, 2012.

[110] J. Sherman, "Properties of focused apertures in the fresnel region," *IRE Trans. Antennas Propagat.*, vol. 10, no. 4, pp. 399–408, 1962.

[111] R. Martin, "Noise power spectral density estimation based on optimal smoothing and minimum statistics," *IEEE Trans. Speech Audio Process.*, vol. 9, no. 5, pp. 504–512, 2001.

[112] C. A. Balanis, *Antenna Theory: Analysis and Design*. John Wiley & Sons, 2016.

[113] Z. Zhou, Z. Xiao, and Y. Zeng, "Single-BS simultaneous environment sensing and UE localization without los path by exploiting near-field scatterers," *IEEE Commun. Lett.*, vol. 28, no. 9, pp. 2071–2075, 2024.

[114] H. Yamada, M. Ohmiya, Y. Ogawa, and K. Itoh, "Superresolution techniques for time-domain measurements with a network analyzer," *IEEE Trans. Antennas Propag.*, vol. 39, no. 2, pp. 177–183, 1991.

[115] T.-H. Liu and J. Mendel, "Azimuth and elevation direction finding using arbitrary array geometries," *IEEE Trans. Signal Process.*, vol. 46, no. 7, pp. 2061–2065, 1998.

[116] J.-F. Gu and P. Wei, "Joint SVD of two cross-correlation matrices to achieve automatic pairing in 2-D angle estimation problems," *IEEE Antennas Wireless Propag. Lett.*, vol. 6, pp. 553–556, 2007.

[117] S. Rouquette and M. Najim, "Estimation of frequencies and damping factors by two-dimensional ESPRIT type methods," *IEEE Trans. Signal Process.*, vol. 49, no. 1, pp. 237–245, 2001.

[118] B. Rao and K. Hari, "Performance analysis of root-MUSIC," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 12, pp. 1939–1949, 1989.

[119] M. Pesavento, A. Gershman, and M. Haardt, "Unitary root-MUSIC with a real-valued eigendecomposition: A theoretical and experimental performance study," *IEEE Trans. Signal Process.*, vol. 48, no. 5, pp. 1306–1314, 2000.

[120] S. Marcos, A. Marsal, and M. Benidir, "Performances analysis of the propagator method for source bearing estimation," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, vol. iv, 1994, pp. IV/237–IV/240 vol.4.

[121] N. Tayem and H. Kwon, "L-shape 2-dimensional arrival angle estimation with propagator method," *IEEE Trans. Antennas Propag.*, vol. 53, no. 5, pp. 1622–1630, 2005.

[122] L. Chen, W. Qi, P. Liu, E. Yuan, Y. Zhao, and G. Ding, "Low-complexity joint 2-D DOA and TOA estimation for multipath OFDM signals," *IEEE Signal Process. Lett.*, vol. 26, no. 11, pp. 1583–1587, 2019.

[123] Y. Pati, R. Rezaifar, and P. Krishnaprasad, "Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition," in *Proc. Asilomar Conf. Signals Syst. Comput.*, 1993, pp. 40–44 vol.1.

[124] J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4655–4666, 2007.

[125] O. Bar-Shalom and A. Weiss, "Efficient direct position determination of orthogonal frequency division multiplexing signals," *IET Radar, Sonar Navig.*, vol. 3, no. 2, pp. 101–111, 2009.

[126] Q. Dai, Z. Zhou, Z. Xiao, Y. Zeng, F. Yang, and Y. Chen, "Low-complexity integrated super-resolution sensing and communication with signal decimation and ambiguity removal," *ZTE Commun.*, vol. 22, no. 3, 2024.

[127] X. Li, Z. Dong, Y. Zeng, S. Jin, and R. Zhang, "Multi-user modular XL-MIMO communications: Near-field beam focusing pattern and user grouping," *IEEE Trans. Wireless Commun.*, vol. 23, no. 10, pp. 13 766–13 781, 2024.

[128] K. Abed-Meraim, Y. Hua, and A. Belouchrani, "Second-order near-field source localization: algorithm and performance analysis," in *Proc. Asilomar Conf. Signals Syst. Comput.*, 1996, pp. 723–727.

[129] J. Liang and D. Liu, "Passive localization of mixed near-field and far-field sources using two-stage MUSIC algorithm," *IEEE Trans. Signal Process.*, vol. 58, no. 1, pp. 108–120, 2010.

[130] N. Yuen and B. Friedlander, "Performance analysis of higher order ESPRIT for localization of near-field sources," *IEEE Trans. Signal Process.*, vol. 46, no. 3, pp. 709–719, 1998.

[131] X. Zhang, W. Chen, W. Zheng, Z. Xia, and Y. Wang, "Localization of near-field sources: A reduced-dimension MUSIC algorithm," *IEEE Commun. Lett.*, vol. 22, no. 7, pp. 1422–1425, 2018.

[132] J. Xie, H. Tao, X. Rao, and J. Su, "Passive localization of noncircular sources in the near-field," in *Proc. 16th Int. Radar Symp. (IRS)*, 2015, pp. 493–498.

[133] Y. Wu, H. Wang, and Y. Zeng, "FFT-enhanced low-complexity near-field super-resolution sensing," 2024. [Online]. Available: <https://arxiv.org/abs/2411.15532>

[134] J. He, M. N. S. Swamy, and M. O. Ahmad, "Efficient application of MUSIC algorithm under the coexistence of far-field and near-field sources," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 2066–2070, 2012.

[135] B. Friedlander, "Localization of signals in the near-field of an antenna array," *IEEE Trans. Signal Process.*, vol. 67, no. 15, pp. 3885–3893, 2019.

[136] Y.-D. Huang and M. Barkat, "Near-field multiple source localization by passive sensor array," *IEEE Trans. Antennas Propag.*, vol. 39, no. 7, pp. 968–975, 1991.

[137] E. Grosicki, K. Abed-Meraim, and Y. Hua, "A weighted linear prediction method for near-field source localization," *IEEE Trans. Signal Process.*, vol. 53, no. 10, pp. 3651–3660, 2005.

[138] W. Zhi and M. Y.-W. Chia, "Near-field source localization via symmetric subarrays," *IEEE Signal Process. Lett.*, vol. 14, no. 6, pp. 409–412, 2007.

[139] B. Wang, F. Gao, S. Jin, H. Lin, and G. Y. Li, "Spatial- and frequency-wideband effects in millimeter-wave massive MIMO systems," *IEEE Trans. Signal Process.*, vol. 66, no. 13, pp. 3393–3406, 2018.

[140] J. Du, H. Ma, Z. Yu, X. Li, S. Mumtaz, and C. Yuen, "DL-based ISAC via tensor analysis in massive MIMO-OFDM systems with spatial-frequency wideband effects," *IEEE Internet Things J.*, pp. 1–1, 2024.

[141] R. Zhang, L. Cheng, S. Wang, Y. Lou, C. Miao, W. Wu, and D. W. K. Ng, "Target sensing in wideband massive MIMO-ISAC systems in the presence of beam squint," in *Proc. IEEE Int. Conf. Commun. Workshops (ICC Workshops)*, 2023, pp. 931–936.

[142] S. Haykin, *Adaptive Filter Theory*. Prentice Hall, 2002.

[143] L. S. Cochran and B. J. Daniels, "A review of adaptive filtering techniques for clutter cancellation in radar," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 3, pp. 720–731, 2005.

[144] C. M. Bishop, *Pattern Recognition and Machine Learning*. Springer, 2006.

[145] Y. Zhang, R. Wang, and J. Liu, "Machine learning techniques for clutter identification in OFDM radar systems," *IEEE Trans. Wireless Commun.*, vol. 19, no. 3, pp. 1882–1895, 2020.

[146] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Trans. Commun.*, vol. 45, no. 12, pp. 1613–1621, 1997.

[147] L. G. de Oliveira, D. Brunner, A. Diewald, C. Muth, L. Schmalen, T. Zwick, and B. Nuss, "Bistatic OFDM-based joint radar-communication: Synchronization, data communication and sensing," in *Proc. 20th Eur. Radar Conf. (EuRAD)*, 2023, pp. 359–362.

[148] P.-Y. Tsai, H.-Y. Kang, and T.-D. Chiueh, "Joint weighted least-squares estimation of carrier-frequency offset and timing offset for OFDM systems over multipath fading channels," *IEEE Trans. Veh. Technol.*, vol. 54, no. 1, pp. 211–223, 2005.

[149] M. G. Floyd, "Interpolation in digital modems—part I: Fundamentals," *IEEE Trans. Commun.*, vol. 41, no. 3, pp. 501–507, 1993.

[150] L. Eruo, F. Gardner, and F. Harris, "Interpolation in digital modems part-II: Fundamentals and performance," *IEEE Trans. Comm.*, vol. 41, no. 6, pp. 998–1008, 1993.

[151] C. Farrow, "A continuously variable digital delay element," in *1988, IEEE Int. Symp. on Circuits and Syst.*, 1988, pp. 2641–2645 vol.3.

[152] Y. L. Sit, B. Nuss, and T. Zwick, "On mutual interference cancellation in a MIMO OFDM multiuser radar-communication network," *IEEE Trans. Veh. Technol.*, vol. 67, no. 4, pp. 3339–3348, 2017.

[153] Z. Xiao, Z. Zhou, Q. Dai, Y. Zeng, F. Yang, and Y. Chen, "Achieving full-bandwidth sensing performance with partial bandwidth allocation for ISAC," *arXiv preprint arXiv:2412.20083*, 2024.

[154] X. Wang, Z. Yang, F. Liu, P. Chu, and J. Zheng, "Coprime-based frame-level sensing OFDM waveform for 5G NR integrated radar sensing and communication via downlink sensing," *IEEE Sensors J.*, vol. 24, no. 15, pp. 24 421–24 437, 2024.

[155] H. Wang and Y. Zeng, "Can Sparse Arrays Outperform Collocated Arrays for Future Wireless Communications?" in *Proc. IEEE Glob. Commun. Conf. Workshops (GC Wkshps)*, Dec. 2023.

[156] P. Pal and P. P. Vaidyanathan, "Nested arrays: A novel approach to array processing with enhanced degrees of freedom," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4167–4181, 2010.

[157] A. Moffet, "Minimum-redundancy linear arrays," *IEEE Trans. Antennas Propag.*, vol. 16, no. 2, pp. 172–175, 1968.

[158] Q. Xue, C. Ji, S. Ma, J. Guo, Y. Xu, Q. Chen, and W. Zhang, "A survey of beam management for mmwave and THz communications towards 6G," *IEEE Commun. Surveys. Tuts.*, vol. 26, no. 3, pp. 1520–1559, 2024.

[159] M. Giordani, M. Polese, A. Roy, D. Castor, and M. Zorzi, "A tutorial on beam management for 3GPP NR at mmwave frequencies," *IEEE Commun. Surveys. Tuts.*, vol. 21, no. 1, pp. 173–196, 2019.

[160] Y. Liu, T. Huang, F. Liu, D. Ma, W. Huangfu, and Y. C. Eldar, "Next-generation multiple access for integrated sensing and communications," *Proc. IEEE*, vol. 112, no. 9, pp. 1467–1496, 2024.

[161] Y. Liu, Z. Qin, M. Elkashlan, Z. Ding, A. Nallanathan, and L. Hanzo, "Nonorthogonal multiple access for 5G and beyond," *Proc. IEEE*, vol. 105, no. 12, pp. 2347–2381, 2017.

[162] C. Zhou, C. You, H. Zhang, L. Chen, and S. Shi, "Sparse array enabled near-field communications: Beam pattern analysis and hybrid beamforming design," 2024. [Online]. Available: <https://arxiv.org/abs/2401.05690>

[163] P. Pal and P. P. Vaidyanathan, "Coprime sampling and the MUSIC algorithm," in *Proc. Digit. Signal Process. and Signal Process. Educ. Meeting (DSP/SPE)*, 2011, pp. 289–294.

[164] H. Cheng, C. You, and C. Zhou, "Near-field localization with coprime array," in *Proc. 30th Annu. Int. Conf. Mobile Comput. Netw. (ACM MOBICOM)*, 2024, p. 2209–2215.

[165] B. T. Sieskul, F. Zheng, and T. Kaiser, "A hybrid SS-ToA wireless NLoS geolocation based on path attenuation: ToA estimation and CRB for mobile position estimation," *IEEE Trans. Veh. Technol.*, vol. 58, no. 9, pp. 4930–4942, 2009.

[166] Y. Fu and Z. Tian, "CramerRao bounds for hybrid TOA/DOA-based location estimation in sensor networks," *IEEE Signal Process. Lett.*, vol. 16, no. 8, pp. 655–658, 2009.

[167] Z. Gao, X. Zhou, B. Ning, Y. Su, T. Qin, and D. Niyato, "Integrated location sensing and communication for ultra-massive MIMO with hybrid-field beam-squint effect," *arXiv preprint arXiv:2411.05278*, 2024.

[168] X. Shao, C. You, W. Ma, X. Chen, and R. Zhang, "Target sensing with intelligent reflecting surface: Architecture and performance," *IEEE J. Sel. Areas Commun.*, vol. 40, no. 7, pp. 2070–2084, Jul. 2022.

[169] M. Fuhr, M. Braun, C. Sturmz, L. Reichardtz, and F. K. Jondral, "An SDR-based experimental setup for OFDM-based radar," in *Proc. 7th Karlsruhe Workshop on Softw. Radio*, 2012, p. 176.

[170] Y. L. Sit, B. Nuss, S. Basak, M. Orzol, W. Wiesbeck, and T. Zwick, "Real-time 2D+ velocity localization measurement of a simultaneous-transmit OFDM MIMO radar using software defined radios," in *Proc. Eur. Radar Conf. (EuRAD)*, 2016, pp. 21–24.

[171] Y. Liu, Z. Du, F. Zhang, Z. Zhang, and W. Yu, "Implementation of radar-communication system based on GNU-Radio and USRP," in *Proc. Comput., Commun. and IoT Appl. (ComComAp)*, 2019, pp. 417–421.

[172] Z. Zhou, C. Zhang, and Y. Zeng, "Prototype of real-time integrated sensing and communication with millimeter-wave OFDM," in *Proc. IEEE/CIC Int. Conf. Commun. China (ICCC)*, 2023.

[173] J. Wang, X.-D. Liang, L.-Y. Chen, L.-N. Wang, and K. Li, "First demonstration of joint wireless communication and high-resolution SAR imaging using airborne MIMO Radar system," *IEEE Trans. Geosci. Remote Sens.*, vol. 57, no. 9, pp. 6619–6632, 2019.