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Radar and Communication Coexistence: An Overview

A review of recent methods

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Increased amounts of bandwidth are required to guarantee both high-quality/high-rate wireless services (4G and 5G) and reliable sensing capabilities, such as for automotive radar, air traffic control, earth geophysical monitoring, and security applications. Therefore, coexistence between radar and communication systems using overlapping bandwidths has come to be a primary investigation field in recent years. Various signal processing techniques, such as interference mitigation, precoding or spatial separation, and waveform design, allow both radar and communications to share the spectrum.

This article reviews recent work on coexistence between radar and communication systems, including signal models, waveform design, and signal processing techniques. Our goal is to survey contributions in this area to provide a primary starting point for new researchers interested in these problems.

Introduction

The use of radar has been widened to numerous civilian applications, including traffic control, remote sensing, car cruise control, and collision avoidance. On a parallel track, the quest for ever-increasing rates in wireless communications has pushed the carrier frequencies toward bands traditionally assigned to radar systems. This, along with the need to limit electromagnetic pollution, has resulted in the scenario of coexisting radar and communication systems [1], [2]. Emerging technologies in this field rely on such concepts as passive sensing, waveform diversity, codesign, and the so-called bioinspired strategies, wherein each part of a given architecture is seen as a subsystem whose design choices must be negotiated with the other constituent subsystems. To this last philosophy belongs the class of cognitive systems, which are, in turn, intimately linked to the concept of Bayesian learning as a means to facilitate and sometimes enable individual decision making [1], [3], [4].

In recent years, vibrant industrial and academic interest has grown regarding the convergence of sensing and communication functions. This has been affirmed by the announcement of the Shared Spectrum Access for Radar and Communication program by the U.S. Defense Advanced Research Projects

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Agency [5] and the demands of sensing and communication for self-driving cars [6]. As a result, a number of studies have been conducted, based on a variety of scenarios, degrees of cooperation between the coexisting systems, and design strategies.

The goal of this article is to review the existing results in this context and define a taxonomy of the different philosophies proposed so far. Three major architectures for coexistence can be defined as follows:
1) coexistence in spectral overlap
2) coexistence via cognition
3) functional coexistence.

Category 1 includes architectures wherein both radar and communication systems are equipped with active transmitters using the same frequency spectrum. Here, the major problem is to eliminate or mitigate mutual interference while guaranteeing satisfactory performance for both functions. Different degrees of cooperation between the active systems have been so far considered. For example, in [7] and [8], the inherent resilience to interference of properly designed coherent multiple-input, multiple-output (MIMO) radars is exploited, and attention is paid only to the performance of the radar system. A similar radar-centric philosophy is adopted in [9] and [10], wherein coexisting communication users are safeguarded by limiting the amount of interference produced by the radar on given subbandwidths. In a symmetrical fashion, uncooperative, communication-centric approaches have been suggested in a number of more recent studies, wherein countermeasures against the radar-induced interference are taken either at the communication receiver [11] or, in the presence of some prior information, directly at the transmitter [12], [13].

Cooperation between the active systems, possibly operating in full spectral overlap, to negotiate the respective transmit policies and adjust the corresponding detection/demodulation strategies is the idea underlying codesign, first introduced in [3] and further developed in [14]–[19]. In this approach, which we define as *holistic*, the coexisting systems are seen as constituent parts of a whole, so that the degrees of freedom under the designer's control are both the waveforms transmitted by the sensing systems and the codebooks employed by the communication systems. These are jointly optimized so as to guarantee that both the communication and the radar performance are satisfactory. Codesign allows taking into account in the transceiver design such effects as reverberation produced by the radar due to clutter or targets moving in close proximity to the communication receiver, range ambiguities, and (random) Doppler frequencies. It is important to underline that these schemes are heavily knowledge based and rely on information exchange between the constituent systems. This presupposes, on the one hand, the presence of a fusion center accessible to both systems and, on the other, the accessibility of a common database, wherein the basic channel parameters are made available.

In dynamic scenarios, codesign may greatly benefit from cognitive paradigms. Here, the channel state is learned through suitable algorithms, which is conducive to the philosophy of coexistence via channel sensing put forth in [4] and, more generally, to category 2 of the previously discussed classification. In fact, category 2 comprises systems wherein spectral overlap between the communication and radar transmitters is avoided through cognition, so that the corresponding channels are interference free. Starting from the idea (proposed in [14] and borrowed from cognitive radio networks) of using pilot signals to estimate the channels and share the channel information between the subsystems, new approaches have been recently proposed wherein the radar and/or the communication system is able to learn the environment without transmitting pilots or avoiding the need for coordination [20]–[23]. In [4], for example, the SpeCX system combines sub-Nyquist multiband sensing with sub-Nyquist radar [24] to enable the radar to sense the communication channel at very low rates.

Category 3 comprises architectures in which there is only one active transmitter, whereby coexistence is functional but no interference is produced and no real resource negotiation takes place. Dual-function radar communication systems rely on combining radar and communication transmitters in the same hardware platform, which is designed to guarantee the performances of both systems. The information is embedded [25]–[28] in the radar signal, and a MIMO radar transmitter uses a combination of beamforming and waveform diversity to direct information bits toward multiple communication receivers without affecting the performance of the sensing function and while guaranteeing satisfactory bit error rate performance.

Opportunistic sensing systems instead consist of a receiver colocated with the communication transmitter and a dedicated software chain aimed at processing the received signal. The receiver can avail itself of some side information, such as timing and transmitted data. This architecture has been proposed and theoretically assessed with reference to the 802.11ad format used in conjunction with a sensing system in an automotive environment [29], [30]. Passive radar systems also can be thought of as belonging to category 3 because they exploit other transmissions (communications or broadcast) rather than having their own dedicated radar transmitter [1], [31].

Coexistence in spectral overlap

System model

In the discussion hereinafter, we unify the single-input, single-output and MIMO settings, as they are amenable to similar approaches. Thus, to keep the discussion as general as possible, we consider a scenario wherein a MIMO radar with M_T transmit and M_R receive antennas (typically, but not necessarily, colocated) should coexist with a MIMO communication system equipped with N_T transmit and N_R receive antennas, respectively, as illustrated in Figure 1.

The MIMO radar transmits M_T signals, where the signal sent from the ith transmit element is characterized by a fast-time code $\mathbf{c}_i = [c_i(0), \ldots, c_i(P_r - 1)] \in \mathbb{C}^{P_r}$. The continuous-time waveform at the ith transmit element is then given by

$$\tilde{c}_i(t) = \sum_{p=0}^{P_r - 1} c_i(p)\,\psi(t - pT_r). \tag{1}$$

Here, $\psi(\cdot)$ is a Nyquist waveform of bandwidth $B=1/T_r$, i.e., such that its autocorrelation $R_\psi(\cdot)$ satisfies the condition $R_\psi(kT_r)=\delta(k)$, with $\delta(\cdot)$ denoting the Kronecker delta and $1/T_r$ the fast-time coding rate. (Nyquist waveforms with bandwidth $B=1/T_r$ are strictly band limited and therefore not time limited. In practice, they are generated by the truncation of an ideal waveform, whereby the discretization may incur some degree of aliasing; however, by allowing some excess bandwidth, this effect can be kept under control. A detailed discussion can be found in [32].) The product of the bandwidth and the effective duration of these coded pulses is typically much larger than one. Therefore, these signals are sometimes referred to as *sophisticated waveforms*, as opposed to conventional unsophisticated signals whose bandwidth is on the order of the inverse of their duration.

In the architecture of Figure 1, every radiating element is allowed to transmit a train of N coded pulses of the form of (1), spaced apart by the pulse repetition time T and amplitude modulated by a slow-time code $\mathbf{g}_i=[g_i(0), g_i(1), \ldots, g_i(N-1)]^T \in \mathbb{C}^N$. Thus, the ith element transmits the signal

$$s_i(t)=\sum_{n=0}^{N-1} g_i(n)\tilde{c}_i(t-nT). \tag{2}$$

Some special cases of the radar signal model (2) are as follows:

- *Case 1*: In this instance, a single-antenna transmitter uses a single signal with fast-time code $\mathbf{c}=[c(0),\ldots,c(P_r-1)]^T$, corresponding to $N=M_T=1$.
- *Case 2*: Here, a single-antenna transmitter employs an amplitude-modulated train of pulses, corresponding to $M_T=1$, $P_r=1$. The train is uniquely determined by the slow-time code $\mathbf{g}=[g(0),\ldots,g(N-1)]^T \in \mathbb{C}^N$. The usual pulsed radar corresponds to an all-one slow-time code.
- *Case 3*: In this scenario, there is a multiantenna transmitter, wherein each antenna transmits a single sophisticated signal. As a consequence, $N=1$, $s_i(t)=c_i(t)$, and the $P_r \times M_T$ space–time code matrix $\mathbf{C}=[\mathbf{c}_1,\ldots,\mathbf{c}_{M_T}]$ is the degree of freedom to be employed at the transmitter side [33].
- *Case 4*: Here, there exists a multiantenna transmitter wherein each antenna transmits a train of unsophisticated signals, amplitude modulated by the slow-time code. In this case, $P_r=1$, and the $N \times M_T$ space–time code matrix $\mathbf{G}=[\mathbf{g}_1,\ldots,\mathbf{g}_{M_T}]$ is the degree of freedom at the transmitter side [17].

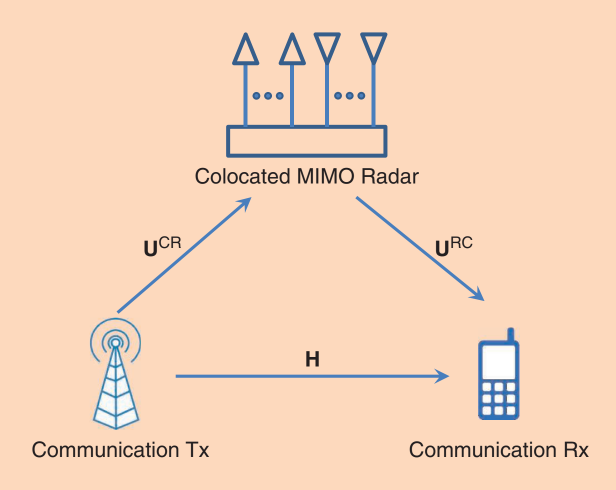

FIGURE 1. A MIMO communication system sharing spectrum with a MIMO radar system. Tx: transmitter; Rx: receiver.

Radars use radio waves to determine the range, angle, or velocity of objects. The operation of a typical MIMO radar receive chain is summarized in "The Stages of Multiple-Input, Multiple-Output Radar Processing." For a given signal-to-noise ratio (SNR), the radar range resolution is dictated by the transmit bandwidth, i.e., $1/T_r$ in (1). The velocity resolution is determined by the duration of coherent integration, i.e., NT in (2). In cases 1 and 3, no Doppler processing is undertaken, mainly because typical single-pulse durations are too short to allow the measuring of the Doppler shift induced by targets in moderate radial motion. In cases 2 and 4, moving objects that generate steering vectors

The Stages of Multiple-Input, Multiple-Output Radar Processing

Classic colocated multiple-input, multiple-output (MIMO) radar processing traditionally includes the following stages.

- *Sampling*: At each radar receiver $1 \le j \le M_R$, the signal $r_j(t)$ is projected onto the orthonormal system $\{\psi(t-mT_r)\}_{m=0}^{P_r-1}$ and sampled at its Nyquist rate $B=1/T_r$, creating the samples $r_j(m), 0 \le m \le P_r-1$.
- *Matched filter*: The sampled signal is convolved with the transmitted radar codes $\mathbf{c}_i, 1 \le i \le M_T$. The time resolution attained in this step is $1/B$.
- *Beamforming*: The correlations between the observation vectors from the previous step and the steering vectors corresponding to each azimuth on the grid defined by the array aperture are computed.
- *Doppler detection*: The correlations between the resulting vectors and Doppler vectors, with Doppler frequencies lying on the grid defined by the number of pulses, are computed. The Doppler resolution is $1/NT$.
- *Peak detection*: This is a heuristic detection process, in which knowledge of the number of targets, the targets' powers, clutter location, and so on may help in discovering the targets' positions. For example, if we know there are κ targets, then we can choose the κ-strongest points in the map. Alternatively, constant false alarm rate detectors determine a power threshold, above which a peak is considered to originate from a target so that a required false alarm probability is achieved.

and Doppler shifts up to $1/T$ can be unambiguously measured. Likewise, pulse trains with pulse repetition time T generate range ambiguities whereby scatterers located at distances corresponding to delays that are integer multiples of T contribute to the same range cell.

The signal model for the communication system is simpler, in that we just have to distinguish between the case of single and multiple transmit antennas. In particular, we assume that the communication system operates on the same frequency band as the radar, occupying a fraction B/L of its dedicated bandwidth. Setting $T_c = L/B$, the signal radiated by the ith transmit element is written as

$$x_i(t) = \sum_{p=-\infty}^{\infty} v_i(p)\psi_L(t - pT_c), \tag{3}$$

where $v_i(p)$ is the data sequence to be transmitted, and $\psi_L(\cdot)$ satisfies the Nyquist criterion with respect to $T_c = LT_r$. The situation of full spectral overlap corresponds to $L = 1$. We note that there may be a multiplicity of narrow-band communication systems, each occupying a fraction of the radar bandwidth.

Assume that the radar and the communication receivers are equipped with M_R and N_R receive antennas, respectively. The signal at the jth antenna of the radar receiver can be cast in the form

$$r_j(t) = \sum_{i=1}^{M_T} a_{i,j}s_i(t - \tau_{i,j}) + \sum_{i=1}^{N_T} (u_{i,j}^{\mathrm{CR}} * x_i)(t) + \sum_{i=1}^{M_T} (a_{i,j}^{\mathrm{I}} * s_i)(t) + n_{\mathrm{R},j}(t), \tag{4}$$

where $a_{i,j}$ is the target complex backscattering coefficient, including the path loss and the phase shift due to the target angle and position with respect to the transmit and receive antennas; $u_{i,j}^{\mathrm{CR}}(t)$ is the response of the channel from the communication transmitter to the radar receiver; $\tau_{i,j}$ is the delay of the target from the ith transmitter to the jth receiver; $a_{i,j}^{\mathrm{I}}(t)$ is the response of the clutters; $*$ is the convolution operation; and $n_{\mathrm{R},j}(t)$ denotes the noise at the jth receiver antenna. Likewise, the signal received at the jth antenna of the communication receiver is given by

$$y_j(t) = \sum_{i=1}^{N_T} (h_{i,j} * x_i)(t) + \sum_{i=1}^{M_T} (u_{i,j}^{\mathrm{RC}} * s_i)(t) + n_{\mathrm{C},j}(t), \tag{5}$$

where $h_{i,j}(t)$ is the channel response from the ith communication transmitter to the jth communication receiver; $u_{i,j}^{\mathrm{RC}}(t)$ is the response of the interfering channel from the radar transmitter to the communication receiver; and $n_{\mathrm{C},j}(t)$ denotes the noise of the jth communication receiver antenna.

In (4), the transmitted signal $s_i(t)$ is known, and $u_{i,j}^{\mathrm{CR}}(t)$ can be estimated via pilot training. On the other hand, $x_i(t)$ and $a_{i,j}^{\mathrm{I}}(t)$ are unknown at the radar receiver. The radar needs to detect the presence of the target, i.e., $a_{i,j} = 0$ for $\mathcal{H}_0$ and $a_{i,j} \neq 0$ for $\mathcal{H}_1$, and estimate the parameters $\tau_{i,j}$ and $a_{i,j}^{\mathrm{I}}(t)$. For the communication system given by (5), $h_{i,j}(t)$ can be estimated via pilot training. In coordinated architectures, where the radar transmits pilots and communicates with the communication receiver, $u_{i,j}^{\mathrm{RC}}$ and $s_i(t)$ are known at the communication receiver, while, in uncoordinated scenarios, $u_{i,j}^{\mathrm{RC}}$ and $s_i(t)$ are both unknown.

Based on the models (4) and (5), different coexistence scenarios can be analyzed. In the section "Uncoordinated Design: Radar Centric" a radar-centric approach wherein a single-antenna radar transmits a single sophisticated signal with fast-time code, i.e., case 1, is discussed. The section "Uncoordinated Design: Communication Centric" reviews some communication-centric approaches, assuming different degrees of prior knowledge as to the radar interference (i.e., cases 2 and 3). The section "Coordinated Design" focuses on the coordinated design of the radar waveforms and the communication codebooks, assuming the most general scenario (i.e., cases 3 and 4) of multiple transmit and receive antennas for both systems, with either slow-time or fast-time coding.

Uncoordinated design: Radar centric

We begin by discussing a radar-centric approach in which the radar function is considered primary while unlicensed users are allowed to transmit in partial spectral overlap on the same bandwidth. Following [9] and [10], we assume N_I interferers of the form (3). Their presence is acknowledged by limiting the amount of interference the radar produces on the shared bandwidths. The focus is on the design of the radar system, assumed to employ a single coded pulse according to case 1 in the previous section, designed so as to guarantee the maximum possible signal-to-interference-plus-noise ratio (SINR) at the radar receiver.

Assume that the radar receiver is equipped with a single antenna and the interference is dominated by the direct path between the radar and the communication. The subscript j can thus be removed from the variables in (4). Therefore, $r_j(t)$ becomes $r(t)$, and $u_i^{\mathrm{CR}}(t) = \delta(t - \tau_i^{\mathrm{RC}})$, with τ_i^{RC} dictated by the distance between the ith communication transmitter and the radar receiver. Such a model holds for narrow-band systems, where the flat-fading assumption is valid [14], and can be extended to more sophisticated situations by using different forms of channel responses [34]. For simplicity, we assume there is only one target and let the target delay be $\tau = 0$.

Plugging (2) into (4) and projecting the equation onto the orthonormal system $\{\psi(t - mT_r)\}_{m=0}^{P_r-1}$ leads to

$$\begin{aligned} r(m) &= \langle r(t), \psi(t - mT_r)\rangle \\ &= \left\langle a\sum_{p=0}^{P_r-1} c_i(p)\psi(t - pT_r), \psi(t - mT_r)\right\rangle \\ &\quad + \sum_{k=1}^{N_T} u_k \underbrace{\langle x(t - \tau_k^{\mathrm{CR}}), \psi(t - mT_r)\rangle}_{x_k(m)} \\ &\quad + \underbrace{\left\langle \sum_{i=1}^{M_T} (a_i^{\mathrm{I}} * s_i)(t), \psi(t - mT_r)\right\rangle}_{n_{\mathrm{I}}(m)} \\ &\quad + \underbrace{\langle n_{\mathrm{R}}(t), \psi(t - mT_r)\rangle}_{n_{\mathrm{R}}(m)} \end{aligned} \tag{6}$$

with a the target complex backscattering coefficient, including the path loss, and u_k the coefficient of the interfering channel for user k. Denoting $\mathbf{r} = [r(0), r(1), \ldots, r(P_r-1)]^T$, we have

$$\mathbf{r} = a\mathbf{c} + \sum_{k=1}^{N_I} u_k \mathbf{x}_k + \mathbf{n}_\mathrm{I} + \mathbf{n}_\mathrm{R} \in \mathbb{C}^{P_r} \tag{7}$$

with $\mathbf{x}_k = [x_k(0), x_k(1), \ldots, x_k(P_r-1)]^T$ the kth communication user occupying the bandwidth, $\mathbf{n}_\mathrm{I} = [n_\mathrm{I}(0), n_\mathrm{I}(1), \ldots, n_\mathrm{I}(P_r-1)]^T \in \mathbb{C}^{P_r}$ the clutter, and $\mathbf{n}_\mathrm{R} = [n_\mathrm{R}(0), n_\mathrm{R}(1), \ldots, n_\mathrm{R}(P_r-1)]^T \in \mathbb{C}^{P_r}$ the noise term.

Equation (7) describes the model for the signal in the radar receiver. Next, we discuss the interference from the radar to the communication users, i.e., the second term in (5). As to the communication users coexisting with the radar of interest, we suppose that each of them is operating over a frequency band $[f_1^k, f_2^k]$, where f_1^k and f_2^k denote the lower and upper normalized frequencies for the kth system, respectively. Following (2) and (3) in [9], the interfering energy produced on the kth communication user is given by $\mathbf{c}^H \mathbf{R}_k \mathbf{c}$, where

$$\mathbf{R}_k(m,n) = \begin{cases} f_2^k - f_1^k, & \text{if } m = n \\ \dfrac{e^{j2\pi f_2^k (m-n)} - e^{j2\pi f_1^k (m-n)}}{j2\pi(m-n)}, & \text{if } m \neq n \end{cases}$$
$$(m,n) \in \{1, 2, \ldots, P_r\}^2. \tag{8}$$

The covariance matrix $\mathbf{M}$ of the exogenous interference, i.e., of the signal-independent component of the overall interference $\sum_{k=1}^{N_I} u_k \mathbf{x}_k + \mathbf{n}_\mathrm{R}$, is assumed to be known or perfectly estimated.

The objective thus becomes to design the radar code $\mathbf{c}$ so as to maximize the SINR at the radar receiver while ensuring that the interference produced on the coexisting communication users is smaller than a constrained value. Additional constraints to be enforced are an energy constraint on the radar code $\mathbf{c}$ and its closeness to some reference code $\mathbf{c}_0$ with prescribed correlation properties [9], [10]. The latter is also referred to as a *similarity constraint*. The design then reduces to solving the following constrained maximization problem:

$$\begin{aligned} &\max_{\mathbf{c} \in \mathbb{C}^{N\times 1}} \mathrm{SINR} = a^2 \mathbf{c}^H \mathbf{M}^{-1} \mathbf{c} \\ &\text{s.t.} \quad \sum_{k=1}^{N_R} \omega_k \mathbf{c}^H \mathbf{R}_k \mathbf{c} \leq E_\mathrm{I}, \\ &\qquad (1-\eta)\rho \leq \mathbf{c}^H \mathbf{c} \leq \rho, \\ &\qquad \|\mathbf{c} - \mathbf{c}_0\|_2 \leq \epsilon. \end{aligned} \tag{9}$$

In this equation, the terms $\mathbf{c}^H \mathbf{R}_k \mathbf{c}$ represent the interference produced onto the kth communication receiver, $k = 1, 2, \ldots, N_\mathrm{R}$; E_I is the maximum interference that can be tolerated by the coexisting communication networks; $\omega_k \geq 0$ for $k = 1, 2, \ldots, N_R$ are weights that can be assigned to the coexisting wireless users, based, for instance, on their distance from the radar and their tactical importance; $0 \leq \eta \leq 1$ is a design parameter that introduces some tolerance on the nominal interference level; and ρ is the transmit energy of the radar. With relaxation, the optimization problem (9) can be transformed into a convex optimization amenable to semidefinite programming, which entails polynomial computational complexity [10].

The scenario leading to problem (9) holds true only when the clutter is either absent or has a rank one covariance matrix, i.e., is modeled as a specular image of the transmitted signal reflected toward the receiver by a point-like scatterer. Conversely, if more complex channel models are considered and the clutter covariance has a rank larger than one (i.e., the point-like model does not carry over to reverberation), then constrained maximization of the SINR results in a fractional nonconvex problem [16].

Uncoordinated design: Communication centric

The approach of optimizing radar waveforms, although theoretically well established, is not always applicable, mainly because government and military agencies are unwilling to make major changes in their radar deployments, which may impose huge costs. Thus, coexisting communication systems must be equipped with proper countermeasures to guarantee the required quality of service (QoS) when the radar systems do not modify their transmission policy. Attention is thus shifted back to the communication transceiver, which explains the name *communication-centric* design. The approaches so far available in the literature focus either on the receiver [11], when prior information on the radar signals is not available, or on the transmitter [13], when the structure of the radar transmitted waveform is known.

Assume first the scenario considered in [11], wherein a multiplicity of radars may be potentially active in full spectral overlap with a communication system. Each radar is allowed to transmit a sophisticated waveform, but no prior knowledge is available as to the number of active systems, their distance from the communication receiver, or the channel gains. The scenario is thus akin to the one outlined in case 3 in the "System Model" section, wherein M_T now plays the role of the maximum number of potentially active emitters. The antennas of such a multiple input system are widely spaced, so that the delays with which their signals arrive at the communication receiver are all different and unknown.

As to the communication signal, the scenario assumed in [11] is fairly general. The transmitted symbols are assumed to undergo suitable precoding, where the choice of the precoding matrix dictates the type of system, ranging from code-division multiple access (CDMA) to orthogonal frequency-division multiplexing (OFDM). In particular, suppose the communication and radar systems have the same bandwidth, i.e., $L = 1$, $T_c = T_r$, and $\psi_L(t) = \psi(t)$. The signal transmitted by the communication system in the interval $[0, P_r T_r]$ is assumed to have the form

$$x(t) = \sum_{p=0}^{P_r-1} v(p)\psi(t - pT_r).$$

In this equation, $\mathbf{v} = [v(0), \ldots, v(P_r-1)]^T \in \mathbb{C}^{P_r}$ is tied to a generic P-dimensional data vector $\mathbf{b}_0 = [b_0(0), \ldots, b_0(P-1)]^T$ to be transmitted as $\mathbf{v} = \mathbf{A}\mathbf{b}_0$, with $\mathbf{A} \in \mathbb{C}^{P_r \times P}$ a suitable matrix. Relevant special cases of this model are the OFDM transmission format (wherein $P_r = P$ and $\mathbf{A}$ take on the form of an inverse discrete Fourier transform matrix) and a CDMA system with P active users (wherein $\mathbf{A}$ contains the users' signatures) [11]. Here, to keep the discussion simple, we confine our attention to the case of direct transmission of the constellation points in full spectral overlap, so that $P = P_r$, $\mathbf{b}_0 = \mathbf{b} \in \mathbb{C}^{P_r}$, and $\mathbf{A} = \mathbf{I}_{P_r}$ ($\mathbf{I}_{P_r}$ denotes the identity matrix of order P_r).

Suppose a single antenna communication receiver and a single-tap model for both communication and interference channels. It is also assumed that the typically high-power radar transmitter is not saturating the front end of the communication receiver. The communication signal in (5) can thus be rewritten as

$$y(t) = h \sum_{p=0}^{P_r-1} b(p)\psi(t - pT_r) + \sum_{m=1}^{M_T} \sum_{p=0}^{P_r-1} u_m c_m(p)\psi(t - pT_r - \tau_m) + n_C(t). \quad (10)$$

Here, a flat-fading channel is assumed for the communication network, where h is the channel coefficient, and τ_m and u_m denote the unknown delay and complex coupling coefficient for the mth radar, respectively. When $u_m = 0$, the mth transmitter is idle. We also assume that, in each frame, P_r symbols are transmitted and that the frame synchronization between the radar and communication is guaranteed, i.e., the communication system is made aware of the beginning of the radar train pulse. This is low-rate information that can be shared once and for all and regularly updated to account for possible timing drifts.

The communication receiver has to accomplish jointly the two tasks of interference estimation/removal and data demodulation. For interference removal, we need to estimate τ_m and $u_m c_m(p)$ so as to subtract the second term from (10). Obviously, data demodulation and interference estimation are inherently

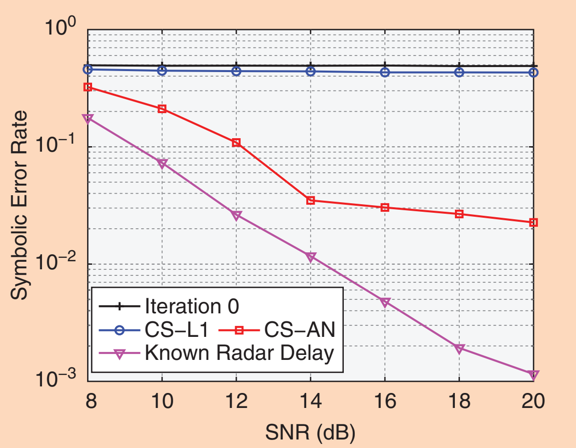

FIGURE 2. A comparison of the algorithm symbolic error rates. L1: ℓ_1 norm; AN: atomic norm. (From [11].)

coupled. In [11], an iterative procedure is proposed for joint data demodulation and interference estimation, and a direct demodulation function $\hat{\mathbf{b}}^{(0)} = \Psi(\{y(t)\}_{0 \le t \le P_r T_r})$ is used as the initial step.

In a general uncoordinated scenario, the communication receiver may not know the exact form of the interfering radar signals but rely only on the coarse information of the family to which they belong. A viable means to account for this uncertainty is to assume that $\mathbf{c}_m$ lives in a low-dimensional subspace of $\mathbb{C}^{P_r}$, spanned by the columns of a known $P_r \times K$ matrix $\boldsymbol{\Phi} = [\phi_0, \phi_1, \ldots, \phi_{P_r-1}]^T \in \mathbb{C}^{P_r \times K}$, with $K \ll P_r$, i.e., $\mathbf{c}_m = \boldsymbol{\Phi}\boldsymbol{\alpha}_m$ for some unknown $\boldsymbol{\alpha}_m \in \mathbb{C}^K$, tied to the corresponding minimal and maximum distances of all of the potential radar transmitters from the receiver.

Following [11], the signal $z^{(\ell)}(t) = y(t) - h\sum_{p=0}^{P_r-1} \hat{b}^{(\ell)}(p)\psi(t - pT_r)$ contains the superposition of the residual communication signal (due to demodulation errors), the residual radar interference, and noise. To understand the joint interference removal and symbol demodulation algorithm proposed in [11], let us refer to the first iteration:

$$z^{(1)}(t) = h \sum_{p=0}^{P_r-1} \underbrace{(b(p) - \hat{b}^{(0)}(p))}_{\beta^{(0)}(p)} \psi(t - pT_r) + \underbrace{\sum_{m=1}^{M_T} \sum_{p=0}^{P_r-1} \phi_m^T u_m \boldsymbol{\alpha}_m \psi(t - pT_r - \tau_m)}_{X(t)} + n_C(t), \quad (11)$$

where τ_m for $m = 1, 2, \ldots, M_T$ are the desired unknown delays. In (11), the quantities ϕ_m and h are known, while the objects of interest to be estimated are τ_m, $\beta^{(0)}(p)$, and $u_m\boldsymbol{\alpha}_m$.

Define $\beta^{(\ell)} = [\beta^{(\ell)}(0), \beta^{(\ell)}(1), \ldots, \beta^{(\ell)}(P_r - 1)]^T$. Notice that building up iterations may rely on two types of sparsity: $X(t)$ in (11) is a combination of at most M_T components with unknown modulation $u_m\boldsymbol{\alpha}_m$, and $M_T \ll P_r$; and $\|\beta^{(\ell)}\|_0$ has to be as small as possible. The problem can be solved by using the recently developed mathematical theory of continuous sparse recovery for superresolution and, in particular, by employing atomic-norm minimization techniques [11]. Figure 2 illustrates the achievable results in terms of the symbol error rate for atomic norm-based and compressed sensing- (CS)-based methods, and it allows assessing the loss due to the lack of prior knowledge as to the delays of the radar systems.

A fairly different scenario is the one considered in [13], where it is assumed that the radar transmits a pulse train, possibly amplitude modulated (according to the transmit format of case 2 in the "System Model" section). Perfect channel state information concerning the attenuation and delay of the radar signal in its travel to the communication receiver is assumed. Thus, the interference generated by the radar onto the communication system is intermittent and presents a large peak-to-average-power ratio because it consists of pulses with large amplitudes.

If the radar transmit code is a phase-only one (or if, more realistically, the pulse complex amplitudes vary significantly only in the phase), then a narrow-band communication system experiences an interference that is approximately a constant-envelope additive signal. Specifically, the interference is $(u_{i,j}^{\mathrm{RC}} * s_i)(t) = \sqrt{I}\, e^{j\theta(t)},\ t \in \Xi$, where $\theta(t)$ is the interference phase, assumed uniform in $[0, 2\pi)$; $I = |ug|^2$ denotes the average power of the radar interference, assumed known; and Ξ designates the time intervals where the communication system is interfered. The communication transmitter, in turn, randomly selects the symbols to be transmitted from the set $\mathcal{B} = \{\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_Q\}$ of unit-energy and equally likely points. Exploiting the statistical independence between these symbols and $\theta(t)$, the optimal decoding regions can be obtained, and the constellation $\mathcal{B}$ can be designed to maximize the transmission rate and/or minimize the error rate.

Coordinated design

The major drawback of the previous approaches is that they rely on a simplified scenario wherein several important phenomena are not accounted for.

- The radar system, especially when operating in search mode, generates reverberation from the surrounding environment, so-called clutter, which impairs not only its own performance but that of the communication system.
- The scattering centers creating clutter could have radial motion with respect to both the radar and the communication receivers, thus generating Doppler shifts that should be accounted for if slow-time coding is considered.

Cooperation between the active systems, possibly operating in full spectral overlap, to negotiate the respective transmit policies and adjust the corresponding detection/demodulation strategies is the idea underlying codesign, first introduced in [3] and further developed in [14], [16], and [17]. It is generally assumed that the radar and the communication system may exchange information. The availability of large databases accurately mapping the scattering characteristics of large areas has allowed the development of cognitive systems (see, e.g., [35] and [36]). Joint design of the radar waveforms and the communication system codebook thus appears as a natural means to allow coexistence by preserving the performance of both.

Consider an $N_T \times N_R$ communication system coexisting in full spectral overlap with an $M_T \times M_R$ MIMO radar with closely spaced antennas and colocated transmitter and receiver. We denote by $\mathbf{D}$ the space–time code matrix of the radar. If fast-time coding is adopted, then $\mathbf{D} = \mathbf{C}$, with $\mathbf{C}$ defined in case 3. If, instead, slow-time coding is undertaken, then $\mathbf{D} = \mathbf{G}$, and case 4 occurs. Denote by $\mathbf{V}$ the signal matrix of the communication system, composed of the N_T spatial codewords emitted in successive epochs. Specifically, $\mathbf{V} = [\mathbf{v}(0), \mathbf{v}(1), \ldots, \mathbf{v}(P_r - 1)] \in \mathbb{C}^{N_T \times P_r}$, where $\mathbf{v}(p) = [v_1(p), v_2(p), \ldots, v_{N_T}(p)]^T$ is the spatial codeword transmitted at epoch p. Projecting the received signal (4) and (5) onto the orthonormal system $\{\psi(t - mT_r)\}_{m=0}^{P_r - 1}$ leads to

$$\mathbf{R} = \mathbf{A}\mathbf{D} + \mathbf{U}^{\mathrm{CR}}\mathbf{V} + \mathbf{A}^{\mathrm{I}}\mathbf{D} + \mathbf{N}_{\mathrm{R}}, \tag{12}$$

$$\mathbf{Y} = \mathbf{H}\mathbf{V} + \mathbf{U}^{\mathrm{RC}}\mathbf{D} + \mathbf{N}_{\mathrm{C}}, \tag{13}$$

where $\mathbf{A} \in \mathbb{C}^{M_R \times M_T}$ is the response of the target to be detected, $\mathbf{A}^{\mathrm{I}} \in \mathbb{C}^{M_R \times M_T}$ is the response of the clutters, $\mathbf{N}_{\mathrm{R}}$ is the noise at the radar receiver, $\mathbf{N}_{\mathrm{C}}$ is the noise at the communication receiver, $\mathbf{U}^{\mathrm{CR}} \in {}^{M_R \times N_T}$ is the interfering channel from the communication transmitter to the radar receiver, $\mathbf{U}^{\mathrm{RC}} \in {}^{N_R \times M_T}$ is the interfering channel from the communication transmitter to the radar receiver, and $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix from the communication transmitter to the radar receiver. In (13), the MIMO communication system is assumed to have perfect channel state information, i.e., knowledge of $\mathbf{V}$, to be periodically shared with the radar system through a dedicated channel.

In (12), the purpose of the MIMO radar is to detect the presence of a target ($\mathbf{A} = \mathbf{0}$ for $\mathcal{H}_0$ and $\mathbf{A} \neq \mathbf{0}$ for $\mathcal{H}_1$) and estimate the matrix $\mathbf{A}$ related to the target parameters, such as angle and velocity. An important additional degree of freedom is the space–time filter that can be applied to the radar signal $\mathbf{R}$ in (12). Let $\tilde{\mathbf{r}} = \mathrm{vec}(\mathbf{R}) = [\mathbf{r}(0)^T, \mathbf{r}(1)^T, \ldots, \mathbf{r}(P_r - 1)^T]^T$ with $\mathbf{r}(p)$ the $(p+1)$th column of $\mathbf{R}$. The filtered signal becomes

$$\bar{r} = \tilde{\mathbf{w}}^T \tilde{\mathbf{r}} \tag{14}$$

with $\tilde{\mathbf{w}} \in \mathbb{C}^{M_R P_r \times 1}$. We recall here that the receive filter is of fundamental importance in coherent MIMO radar because time filtering regulates the transmit beamwidth, while space filtering controls the receive beampattern.

A possible criterion to exploit transmitter coordination for a coherent MIMO radar coexisting with a communication system is to force the radar waveforms $\mathbf{D}$ to live in the null space of the interference channel $\mathbf{U}^{\mathrm{RC}}$ via a spatial approach [18]. The MIMO structure indeed provides the degrees of freedom to suitably design the space–time code matrix determining the probing signal. To illustrate further, assume that the case 3 model in the "System Model" section is in force, and that the fast-time space–time code matrix $\mathbf{C}$ is to be designed. To this end, we regroup the signals transmitted by the MIMO radar in the vectors $\mathbf{c}(p) = [c_1(p), c_2(p), \ldots, c_{M_T}(p)]^T$, encapsulating the spatial codeword transmitted for the pth subpulse.

Consider the situation in which $\bar{N}$ communication receivers exist, and let the interference channels of the communication receivers be $\{\mathbf{U}^{(1)}, \mathbf{U}^{(2)}, \ldots, \mathbf{U}^{(\bar{N})}\}$. In [15], where the idea is fully developed, these abstract communication receivers are actually clusters of base stations. The interference that would be produced onto the nth communication receiver is $\mathbf{U}^{(n)}\mathbf{c}(p)$. At the MIMO radar, the channel state information can be estimated using a blind null-space learning algorithm [37].

Our goal here is to assure zero interference to one of the communication receivers with minimum degradation in the radar performance. Suppose we want no interference at the nth communication receiver. The communication signal can be projected onto the null space of the channel $\mathbf{U}^{(n)}$. The null space $\mathcal{N}(\mathbf{U}^{(n)}) = \{\mathbf{c} \in \mathbb{C}^{M_T} : \mathbf{U}^{(n)}\mathbf{c} = \mathbf{0}\}$ can then be calculated using the singular value decomposition. Specifically, letting $\mathbf{U}^{(n)} = \boldsymbol{\Upsilon}_1 \boldsymbol{\Sigma} \boldsymbol{\Upsilon}_2^H$, the right singular vectors corresponding to vanishing singular values are collected in $\bar{\boldsymbol{\Upsilon}}_2$ for the formation of the projection matrix $\mathbf{P}_{\bar{\boldsymbol{\Upsilon}}_2}^{(n)} = \bar{\boldsymbol{\Upsilon}}_2 (\bar{\boldsymbol{\Upsilon}}_2^H \bar{\boldsymbol{\Upsilon}}_2)^{-1} \bar{\boldsymbol{\Upsilon}}_2^H$. The transmitted radar signal is thus the projection of $\mathbf{c}(p)$ onto the null space, i.e.,

$$\tilde{\mathbf{c}}(p) = P_{\bar{\boldsymbol{\Upsilon}}_2}^{(n)} \mathbf{c}(p). \tag{15}$$

The precoder $\mathbf{P}_{\bar{\boldsymbol{\Upsilon}}_2}^{(n)}$ inevitably introduces correlation among the signals emitted by the different transmit elements, thus generating some performance loss for target direction estimation. Note that the radar waveform is orthogonal to one communication channel but not to all. The MIMO radar selects the best interference channel, defined as

$$\mathbf{U}_{\text{Best}} = \mathbf{U}^{(i_{\max})}, \quad \text{with} \quad i_{\max} = \arg \max_{1 \le i \le \bar{N}} \dim [\mathcal{N}(\mathbf{U}^{(i)})], \tag{16}$$

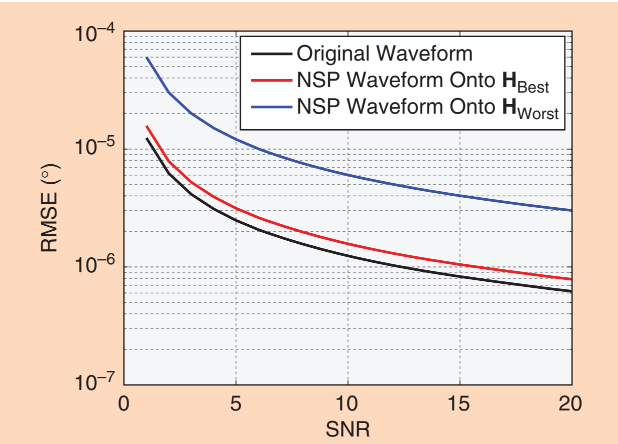

FIGURE 3. The Cramer–Rao bound on the target direction RMSE as a function of the SNR, when $\mathbf{U}_{\text{Best}}$ and $\mathbf{U}_{\text{Worst}}$ (marked as $\mathbf{H}_{\text{Best}}$ and $\mathbf{H}_{\text{Worst}}$, respectively) channels are selected. NSP: null-space projection. (From [18].)

and avoids the worst channel, defined as

$$\mathbf{U}_{\text{Worst}} = \mathbf{U}^{(i_{\min})}, \quad \text{with} \quad i_{\min} = \arg \min_{1 \le i \le \bar{N}} \dim [\mathcal{N}(\mathbf{U}^{(i)})]. \tag{17}$$

In general, in the fully cooperative scenario outlined in [15], the radar can take a snapshot of the interference situation for each cluster and broadcast it to allow proper users assignment protocols. Users may then be assigned to less- or more-interfered base stations based on priority order.

In Figure 3, we compare the root-mean-square error (RMSE) of the target direction estimation under different radar waveforms. Note that the estimation performance using the null-space projection (NSP) waveform onto $\mathbf{U}_{\text{Best}}$ is closer to the performance of the original radar waveform, in the RMSE sense. Thus, by an appropriate selection of the interference channel, the degradation in the radar performance due to the waveform's NSP can be reduced.

A MIMO radar may operate without creating interference at any of the communication receivers if the number of radar transmit antennas is greater than the sum of the requested degrees of freedom of all of the communication receivers [38]. Cooperation between all of the base stations and radar allows forming the interference matrix $\bar{\mathbf{U}} = [\mathbf{U}^{(1)T}, \mathbf{U}^{(2)T}, \dots, \mathbf{U}^{(\bar{N})T}]^T \in \mathbb{C}^{\bar{N}N_R \times M_T}$. Applying the previous strategy yields $\tilde{\mathbf{c}}(p) \in \mathcal{N}(\bar{\mathbf{U}})$. Other alternative strategies may rely on forcing the radar waveform to be designed according to a minimum mean square error criterion (rather than the aforementioned zero-forcing strategy).

More general approaches to the coordinated design of radar and communication are based on optimization methods (illustrated in Figure 4). We assume that the radar uses a $P_r \times M_T$ space–time code matrix $\mathbf{C}$. The extension to slow-time coding can be undertaken by changing the time scale, considering the Doppler effect in the signal model of (12) and (13), and solving for the slow-time space–time matrix $\mathbf{G}$

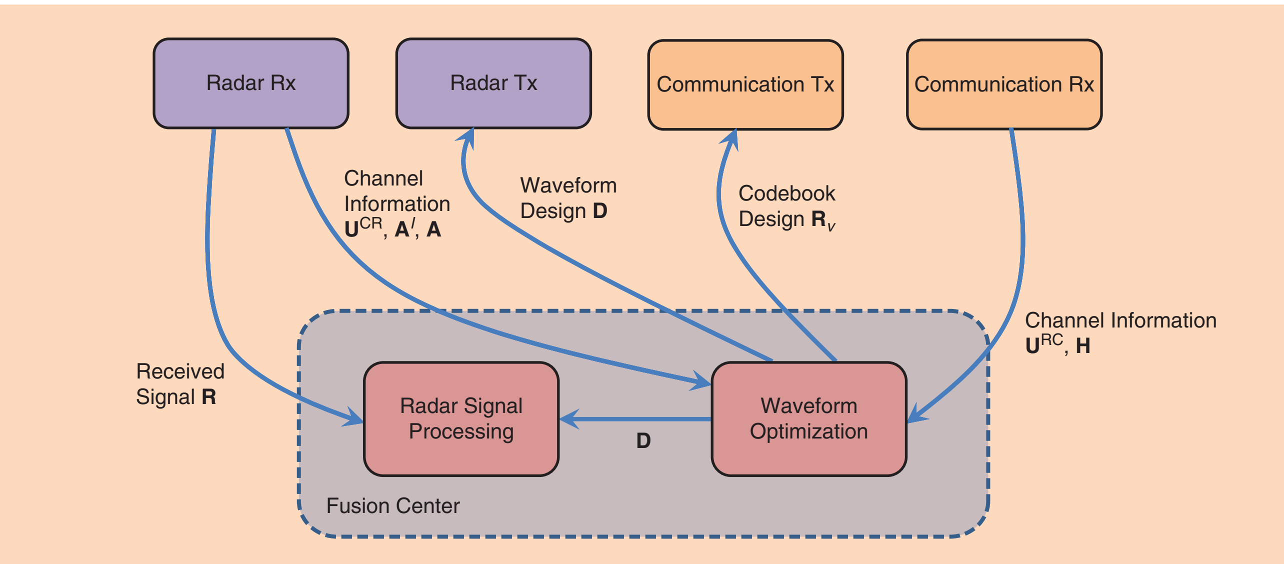

FIGURE 4. The schematic structure of a coordinated design of radar and communication waveforms based on optimization.

[16]. The space–time filter $\tilde{\mathbf{w}}$ in (14) can also be optimized to improve radar performance.

Assume that the SINR is the figure of merit of interest to the radar, and let Q be the figure of merit chosen for the communication system. They depend on $\mathbf{D}$, on the symbol matrix $\mathbf{V}$ (or on some statistical feature thereof, if random coding is undertaken), and on a number of channel parameters tied to the reverberation that we combine in an unspecified array $\mathbf{Z}$. A suitable figure of merit guaranteeing the performance of the communication system is the mutual information between the input symbol stream and the observations [3], [16]. In particular, the mutual information averaged over P_r time slots, assuming Gaussian interference, is

$$C = \frac{1}{P_r}\sum_{p=0}^{P_r-1}\log_2 \det\left(\mathbf{I}_{N_R L} + \mathbf{R}_{\text{Cin}}^{-1}\mathbf{H}\mathbf{R}_v(p)\mathbf{H}^H\right), \tag{18}$$

where $\mathbf{R}_v(p) = \mathbb{E}[\mathbf{v}(p)\mathbf{v}(p)^H]$ is the covariance matrix of the communication codebook, and $\mathbf{R}_{\text{Cin}} \in \mathbb{C}^{N_R \times N_R}$ is the covariance of interference plus noise, assumed to be either known or perfectly estimated. The transceivers are designed to guarantee a prescribed QoS to both systems.

A possible optimization problem can be formulated as

$$\mathcal{P}\begin{cases} \max\limits_{D,\{R_v(p)\},\tilde{w}} \; \text{SINR}(\mathbf{D}, \{\mathbf{R}_v(p)\}, \mathbf{Z}, \tilde{\mathbf{w}}), & \\ \text{s.t.} \;\; Q(\mathbf{D}, \{\mathbf{R}_v(p)\}, \mathbf{Z}) \geq Q_0, & \text{QoS of Comm. Syst.} \\ \qquad g_i(\mathbf{D}) \leq 0, i = 1, \ldots, I_R, & \text{Rad. Wav. Constr.} \\ \qquad f_i(\{\mathbf{R}_v(p)\}) \leq 0, i = 1, \ldots, I_T, & \text{Comm. Codes. Constr.} \end{cases}, \tag{19}$$

where $\text{SINR}(\cdot)$ is the SINR at the output of the radar receiver, and $g_i(\cdot)$ and $f_i(\cdot)$ are a set of constraints forced on the radar and communication transmitted signals, respectively. [Note that the expansions of the abbreviations in (19) are as follows: QoS of Comm. Syst.: QoS of communication system; Rad. Wav. Const.: radar waveform constraint; and Comm. Codes. Constr.: communication codes constraint.] The problem in (19) is typically nonconvex. Alternating maximization techniques have been proposed and implemented in [14] and [16] through decompositions into subproblems that are either convex or solvable through fractional programming methods. In [16], for example, (19) has been reformulated for slow-time coding, explicitly accounting for Doppler shifts of both the target to be detected and the environmental reverberation.

Coexistence via cognition

Environment-sensing techniques

The idea of knowledge-based design is central for spectrum-sharing systems [4], [7], [14], [17]–[19], [38]. The communication and/or the radar system undertakes suitable environment-sensing phases to determine the transmit policies. Inspired by cooperative methods in cognitive radio networks, [14] uses pilot signals to estimate the channels and feed back the channel information between the subsystems, possibly assigning to one of them a functional priority, as, e.g., in [39] and [40], where the radar is considered primary. These approaches rely on a centralized architecture, namely, a strict coordination between the active players, to allow coexistence.

More recently, approaches wherein the radar and/or the communication system is able to learn the environment without transmitting pilots or avoiding the need for coordination have been proposed. These advanced approaches are discussed in the following two scenarios:

1) *Environment sensing at the communication receiver*: A communication system shares its spectrum with an ensemble of potential interferers, i.e., a set of radar/sensing systems. The interfering waveforms from the radars lie in the subspace of a known dictionary and impinge on the communication receiver with unknown, possibly time-varying delays and coupling coefficients.
2) *Environment sensing at the radar receiver*: A sparse target scene is assumed, allowing the reduction of the radar sampling rate without sacrificing delay and Doppler resolution. The Xampling framework can be adopted, where the system architecture is designed for sampling and processing of analog inputs at rates far below Nyquist, whose underlying structure can be modeled as a union of subspaces [24].

The former situation has been described in the section "Uncoordinated Design: Communication Centric." The communication receiver must be made adaptive to jointly accomplish the two tasks of interference estimation/removal and data demodulation. For the latter situation, the SpeCX system (shown in Figure 5) was proposed in [4]; it combines sub-Nyquist multiband sensing with sub-Nyquist radar to enable the radar to sense the communication channel at very low rates. Compared to other works, SpeCX presents a complete solution that shows the recovery of both the radar and communication signal with minimal information known about the spectrum.

More specifically, a sub-Nyquist cognitive radio is first implemented to sense the communication channel and determine which bands are occupied. This can be done using the modulated wideband converter (MWC), a sub-Nyquist communication receiver developed specifically for this task, which is capable of detecting sparse signals at very low rates [20]–[23]. Once the empty bands in the spectrum are identified, a cognitive radar receiver is employed that transmits a wideband signal that consists of several narrow-band signals in the vacant frequency bands [41].

Using the radar Xampling paradigm, it can be shown that high-resolution delay and Doppler can be performed from such a multiband, wideband radar signal by combining the methods of sub-Nyquist sampling and compressed beamforming [22], [42], [43]. This allows the detection of targets with high resolution while using a transmit signal that consists of several narrow bands spread over a wide frequency regime. The advantage of such a system is that the total bandwidth occupied is small while still allowing for high resolution. This enables the transmission of an adaptive radar signal that can coexist

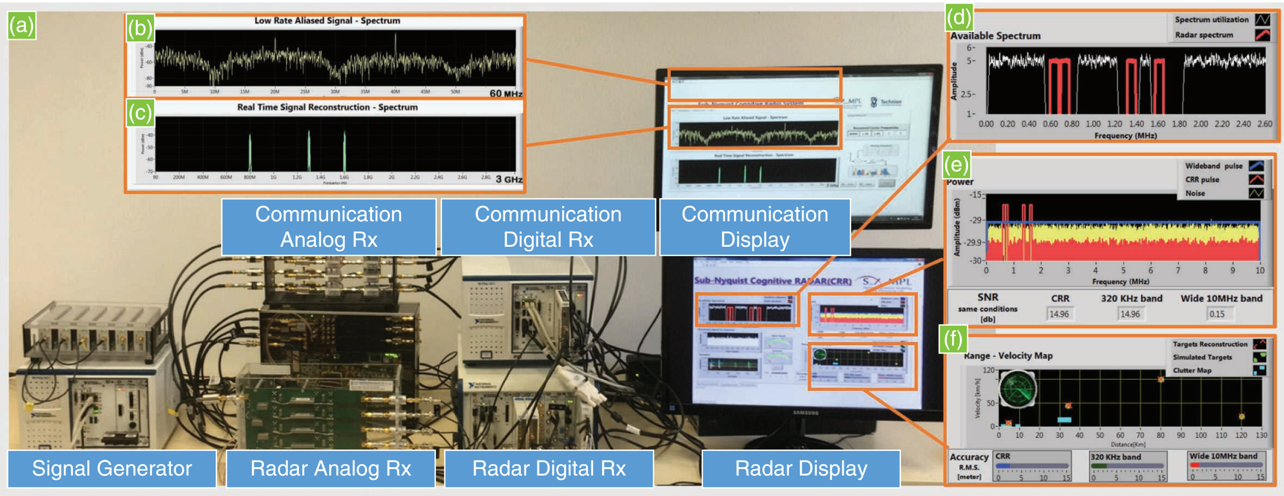

FIGURE 5. (a) A SpeCX prototype. The system consists of a signal generator, a cognitive radio receiver based on the MWC, a communication digital receiver, and a cognitive radar. The SpeCX communication system display shows (b) low-rate samples acquired from one MWC channel at a rate of 120 MHz and (c) a digital reconstruction of the entire spectrum from sub-Nyquist samples. The SpeCX radar display shows (d) the coexisting communication and cognitive radar, (e) the cognitive radar spectrum compared with the full-band radar, and (f) the range–velocity display of the detected and true locations of the targets. (From [4].)

with a standard communication channel and also leads to low-rate, low-power receivers.

Knowledge-based design

In this subsection, we survey knowledge-based radar transmission designs based on environment sensing. For example, in some settings, the radar interference can be eliminated by forcing the radar waveforms to live in the null space of the interference channel between the radar transmitters and the communication receiver [18]. This idea has been well studied in the cognitive radio research community and has been applied to spectrum-sharing systems. Typical approaches include exploiting the spatial degrees of freedom granted by a MIMO radar [7], [18], [19], [38] and adaptive transmit/receive strategies to test the occupancy of the frequency bands [4].

In [4] and [22], the bands selected by the radar are chosen to optimize the radar probability of detection. More specifically, after the communication signal support is identified, denoted as $\mathcal{F}_c$, the communication receiver provides a spectral map of occupied bands to the radar. Equipped with the detected spectral map and known radio environment map, denoted as $\mathcal{F}_r$, the objective of the radar is to identify an appropriate transmit frequency set that does not overlap with the union of $\mathcal{F}_c$ and $\mathcal{F}_r$ and maximizes the probability of correct detection. This probability increases with the SINR when the probability of false alarm is fixed. Therefore, it is proposed to maximize the SINR or minimize the spectral power in the undesired parts of the spectrum. This is achieved by using a structured sparsity framework [44]. Additional requirements of transmit energy constraints, range sidelobe levels, and a minimum separation between the bands can also be imposed. Once the optimal radar support is identified, a suitable waveform code may be designed over it.

There is another approach to waveform design. Rather than creating a waveform that avoids interference, one can base a design on spectral notching that minimizes the transmit energy in specific frequency bands while maintaining desirable envelope and sidelobe characteristics [45]. A waveform designed to avoid transmitting in specific bands, a spectrally disjoint waveform, must be characterized using other metrics because interference is not driving the design, and, thus, no such SINR can be calculated. Such metrics include average power levels in the undesired frequency bands, peak sidelobe levels, and integrated sidelobe levels.

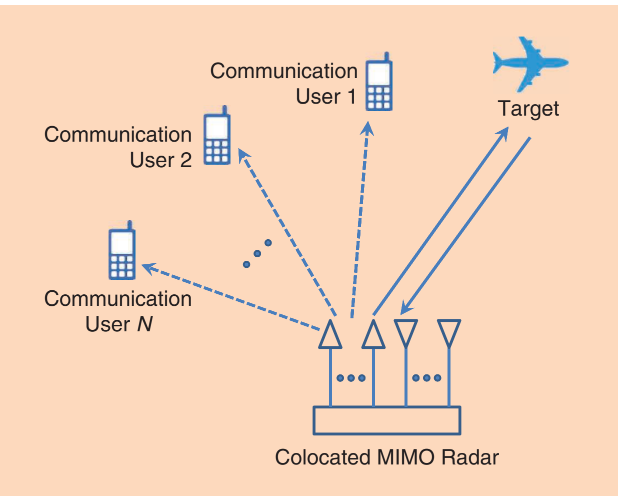

FIGURE 6. Dual-function radar communications.

Functional coexistence

Embedding data into radar waveforms

A fairly natural evolution of radar and communication coexistence is to use radar to perform communication, also known as *dual-function radar communication* [46]. This approach is illustrated in Figure 6, wherein radar and communication systems are combined in the same hardware platform, usually with the same waveform or transmitter, which should be designed so as to guarantee the performance of both systems. In these architectures, as echoed by the name itself, coexistence is basically functional, and no spectrum overlap or resource negotiation takes place. This philosophy relies on the strategy of information embedding. Consider a joint radar communication platform equipped with M_T transmit antennas arranged as a uniform linear array. The radar receiver employs an array of M_R receive antennas with an arbitrary linear configuration. Without loss of generality, a single-element communication receiver is assumed to be located in the direction θ_c, which is known to the transmitter.

A fairly natural evolution of radar and communication coexistence is to use radar to perform communication, also known as *dual-function radar communication*.

Let $\mathbf{s}(t)=[s_1(t), s_2(t), \ldots, s_{M_T}(t)]^T \in \mathbb{C}^{M_T\times 1}$ be the baseband equivalent of the signal transmitted by a MIMO radar. Suppose a target is located at θ with delay τ. The received signal is then given by

$$\mathbf{r}(t)=\gamma \mathbf{a}_r(\theta)\mathbf{a}_t(\theta)^T\mathbf{s}(t-\tau)+\mathbf{n}_{\mathrm{R}}(t), \tag{20}$$

where $\mathbf{a}_t(\theta)$ and $\mathbf{a}_r(\theta)$ are the steering vectors of the transmit and receive array, and γ is the coefficient accounting for both target reflection and propagation loss. The radar needs to detect the presence of the target, i.e., $\gamma=0$ for $\mathcal{H}_0$ and $\gamma\neq 0$ for $\mathcal{H}_1$, and estimate the parameters θ and τ. Assuming a single-antenna communication receiver and considering a sophisticated single-pulse MIMO radar, the baseband signal at the output of the communication receiver can be expressed as

$$\begin{aligned} y(t) &= u\mathbf{a}_t(\theta_c)^T\mathbf{s}(t)+n_{\mathrm{C}}(t)\\ &= u\mathbf{a}_t(\theta_c)^T\sum_i \tilde{\mathbf{s}}_i(t)+n_{\mathrm{C}}(t), \end{aligned}\tag{21}$$

where u is the channel coefficient of the received signal encapsulating the propagation environment between the transmit array and the communication receiver, and $\tilde{\mathbf{s}}_i(t)$ is the transmitted radar signal in the ith subpulse.

The fine structure of the transmitted signal $\mathbf{s}(t)$ dictates the information-embedding method. Proposed strategies include the following.

- *Waveform diversity-based information embedding [47]*: Here, N_b bits of information per pulse are embedded by selecting the radar waveform on a pulse-to-pulse basis from a set of $K=2^{N_b}$ waveforms [46]. Assume that the kth communication symbol is embedded in the ith pulse. Then the corresponding transmit signal vector can be expressed as

$$\tilde{\mathbf{s}}_i(t)=\sqrt{P_t}\,\mathbf{1}_{M_T}\psi_k(t-iT_r), \tag{22}$$

where P_t is the transmitting power, $\mathbf{1}_{M_T}$ is the $M_T\times 1$ vector of 1, and $\psi_k(t)$ for $k=1,2,\ldots,K$ are orthogonal waveforms.

- *Phase modulation-based information embedding [48]–[50]*: Information is embedded by controlling the phase of the signal. Assume that the kth communication symbol $b(k)$ is transmitted through the phase information of the constant-envelope vector $\mathbf{v}=[v(0), v(1), \ldots, v(P_r-1)]^T$. Suppose the symbol $\mathbf{v}$ is, in turn, embedded in a single antenna radar waveform. The total transmit signal is then given by

$$s(t)=\sum_{p=0}^{P_r-1} v(p)c(p)\psi(t-pT_r), \tag{23}$$

where the radar phase modulation $c(p)$ enables direct control of the degree of range sidelobe modulation (RSM). RSM occurs because of the changing waveform structure during the coherent processing interval [51] by trading off bit error rate and/or data throughput. When not properly addressed, RSM translates to residual clutter in the range/Doppler response and hence degraded target visibility [2], [52]. Receive filter design to mitigate RSM is addressed for this type of information embedding in [48] and [52]. Design methods focus on the realization of a common filter response and exploit the inherent commonality among the radar/communication waveforms. It is worth noting that phase modulation will also inevitably lead to spectrum alteration of the radar waveform, which may result in energy leakage outside the assigned bandwidth [53].

- *Sidelobe amplitude modulated-based communications [54]–[56]*: To embed the kth communication symbol $b(k)$ during the ith pulse, the beamforming weight vector $\mathbf{c}_k$ should be associated with that symbol. The amplitude modulated-based method models the transmit signal during the ith pulse as

$$\tilde{\mathbf{s}}_i(t)=\sqrt{P_t}\,\mathbf{c}_k\psi(t-iT_r). \tag{24}$$

The design of $\mathbf{c}_k$ is formulated as the following optimization problem [26]:

$$\begin{aligned} &\min_{\mathbf{c}_k}\max_{\theta}\big|\,G(\theta)-|\mathbf{c}_k^H\mathbf{a}_t(\theta)|\,\big|,\ \theta\in\Theta,\\ &\text{s.t.}\quad |\mathbf{c}_k^H\mathbf{a}_t(\theta)|\le\epsilon,\ \theta\in\bar{\Theta},\ \mathbf{c}_k^H\mathbf{a}_t(\theta_c)=\Delta_k, \end{aligned}\tag{25}$$

where $G(\theta)$ is the desired transmit beampattern, Θ is the spatial sector the radar keeps under surveillance, $\bar{\Theta}$ is the sidelobe region for communication, ϵ is a positive number of users' choice for controlling the sidelobe levels, and Δ_k is the kth sidelobe level toward the communication direction θ_c. Several other variations of the sidelobe-modulating approach are discussed in [57] and [58].

- *Multiwaveform amplitude shift keying-based information embedding [26]*: This method uses multiple waveforms and two transmit beamforming weight vectors $\mathbf{c}_H$ and $\mathbf{c}_L$. The method requires N_b orthogonal waveforms to embed N_b bits per radar pulse. Then, N_b waveforms are transmitted simultaneously, where the total transmit energy P_t is divided equally among the N_b waveforms. Every transmitted waveform is used to deliver one information bit, and the waveform $\psi_k(t)$, $k = 1, 2, \ldots, N_b$, is radiated either via $\mathbf{c}_H$ for $b_i(k) = 0$ or $\mathbf{c}_L$ for $b_i(k) = 1$ [46]. The transmit signal is then

$$\tilde{\mathbf{s}}_i(t) = \sqrt{\frac{P_t}{N_b}} \sum_{k=1}^{N_b} ((1 - b_i(k))\mathbf{c}_H + b_i(k)\mathbf{c}_L)\psi_k(t - iT_r). \quad (26)$$

Radar employing communication waveforms

Another evolution of functional coexistence is to exploit the waveforms transmitted by a communication network to perform sensing (radar) functions. Without loss of generality, we assume a single-element communication transmitter (or a phased array with an extremely directional beampattern). The baseband signal at the communication transmitter is given by (3), with $x_i(t)$ and $v_i(p)$ replaced by $x(t)$ and $v(p)$, respectively.

Suppose the radar is equipped with M_R antennas and the communication transmitter is located at angle θ_c. There are a number of scattering centers (targets), the ith of which is with path delay τ_i, Doppler shift ν_i, and angle θ_i. Let γ_i be the coefficient accounting for both the target reflection and propagation loss of the ith target. The response from the communication transmitter to the radar receiver in (4) can be rewritten as

$$u_j^{\mathrm{CR}}(t) = u a_{r,j}(\theta_c)\delta(t - \tau_c) + \sum_i \gamma_i a_{r,j}(\theta_i) e^{j2\pi\nu_i t}\delta(t - \tau_i),$$

where $a_{r,j}(\theta)$ is the angle response of the jth radar receiver, u is the coefficient of the direct path between the communication transmitter and radar receiver, and τ_c is the delay of the direct path. As no radar transmitter is used, the baseband equivalent signal at the radar receiver can be obtained from (4), with $\Sigma_{i=1}^{M_T} a_{i,j}\boldsymbol{s}_i(t - \tau_{i,j})$ and $\Sigma_{i=1}^{M_T}(a_{i,j}^{\mathrm{I}} * \boldsymbol{s}_i)(t)$ removed:

$$\mathbf{r}(t) = u\mathbf{a}_r(\theta_c)x(t - \tau_c) + \sum_i \gamma_i e^{j2\pi\nu_i t}\mathbf{a}_r(\theta_i)x(t - \tau_i) + \mathbf{n}_{\mathrm{R}}(t), \quad (27)$$

where $\mathbf{a}_r(\theta) = [a_{r,1}(\theta), a_{r,2}(\theta), \ldots, a_{r,M_R}(\theta)]^T \in \mathbb{C}^{M_R}$ is the receive steering vector.

One option for using a communication waveform $x(t)$ for sensing is the opportunistic radar based on the 802.11ad standard proposed in [29] and [30]. The adoption of the 802.11ad standard for 5G wireless systems and the exploitation of millimeter waves (mm-waves) in the 28-GHz and 60-GHz bandwidths [59] immediately raised interest in utilizing some key characteristics of the proposed standard for sensing applications. Indeed, mm-waves suffer from heavy atmospheric attenuation, resonance in the oxygen molecule, absorption by rain, and almost complete shadowing by obstacles, thus requiring line-of-sight paths between transmitter and receiver. This, in turn, is achievable thanks to extremely directional beampatterns and frequent scanning procedures during which the surrounding space is swept in search of nodes willing to establish directional links.

As a consequence, the so-called sector level sweep phase of the beamforming training protocol provides signals of opportunity that can be exploited for short-range obstacle detection, typically in automotive applications [29]. In such a phase, the transmitted signal consists of a preamble, containing concatenated complementary Golay codes, and a payload, containing data. The proposed architectures rely on the presence of a receiver, colocated with the wireless transmitter and accessing some key information, such as the timing, as well as part, if not all, of the transmitted signal. With reference to (27), $\tau_c = 0$ and $u = 0$ because there is no direct path, and $x(t)$ is either partially known, because the preamble has a fixed structure, or completely known, if the transmitted data are communicated to the radar receiver.

Suppose there is one target in each sector. We denote by γ its unique complex scattering coefficient. A number of receiving structures have been proposed for target detection and localization in the range/Doppler domain in [29] and [30], mostly based on the generalized likelihood ratio test (GLRT) [60] and assuming different degrees of prior knowledge and cooperation between the radar receiver and the communication transmitter.

1) *GLRT-1*: Everything but the triplet (γ, ν, τ) in (27) is known.
2) *GLRT-1, simplified version* (*simpl.*): The receiver is as in GLRT-1 but processes only the preamble.
3) *GLRT-2, Swerling* (*SW*)*-1*: This is like GLRT-1, but γ is a nuisance parameter, modeled as complex Gaussian.
4) *GLRT-3*: The payload data are not available to the radar receiver.
5) *GLRT-4 SW-1*: This is as in GLRT-3 but with γ a nuisance parameter.
6) *Preamble detector*: This is the preamble detector of [29].

We underline here that the GLRT strategy is aimed at solving composite hypotheses tests, namely, those wherein the densities under the two alternatives contain unknown parameters. In practice, these parameters are replaced by the corresponding maximum-likelihood estimates, performed with the same set of data used to make the final decision. Consequently, the GLRT considers, as a by-product, an estimate of the unknown parameters. Figures 7 and 8 represent examples of what can be achieved with such opportunistic structures in terms of both detection and localization of an obstacle in short-range applications.

Notwithstanding the encouraging results so far available, a number of problems still remain before claims can be made on the feasibility of such structures. The channel models underlying the results of Figures 7 and 8 are very simple, assuming that either a single object is present or that it absorbs all of the radiation, thus shielding other obstacles. Moreover, because the range resolution is on the order of decimeters, most objects are typically range spread, a situation not accounted for so far in the open literature.

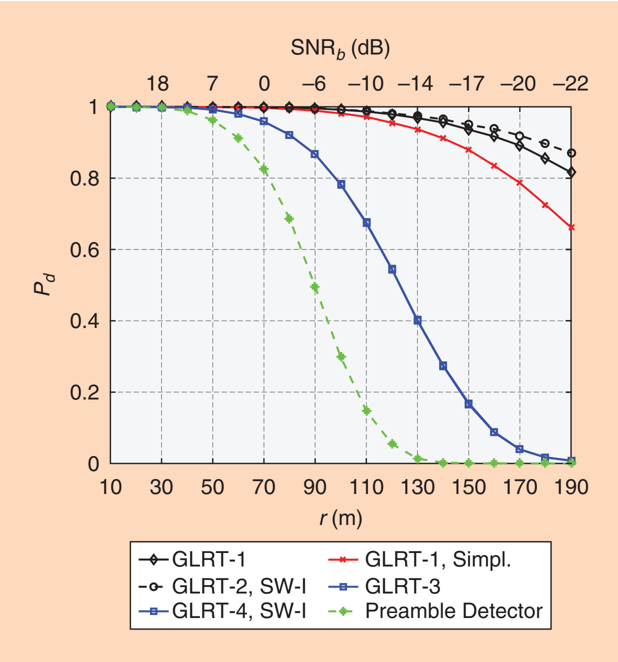

FIGURE 7. The detection probability as a function of the target range and the SNR per bit. The false alarm probability is set at $P_{\text{fa}} = 10^{-4}$. (Used with permission from [30].)

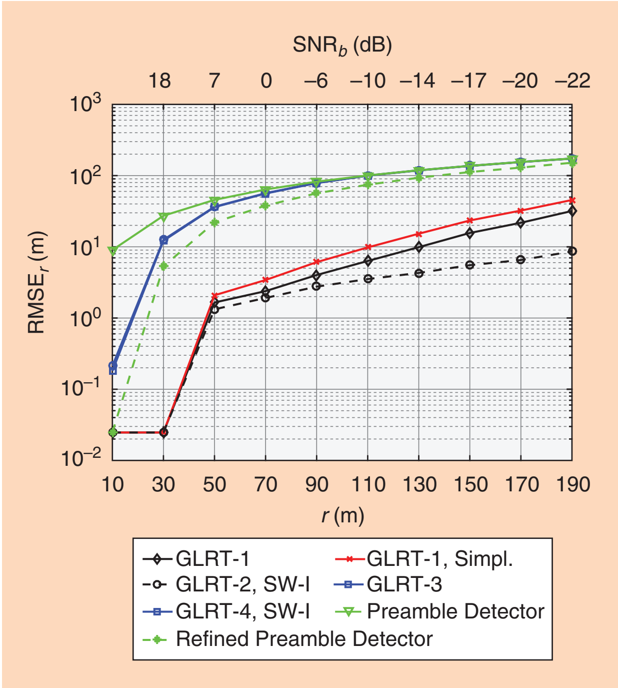

FIGURE 8. The ranging accuracy as a function of the target range and the SNR per bit. (Used with permission from [30].)

Passive radar is another option that exploits other transmissions (communications, broadcast, or radio navigation) rather than having its own dedicated radar transmitter [1], [31]. It is generally necessary to have a reference channel (RC) dedicated to acquiring the direct path signal as the reference waveform for matched filtering and for there to be surveillance channels (SCs) from which the target reflections are acquired. For a communication transmitter with a known position, θ_c in (27) can be obtained [61]. The signal in the RC is given by

$$z_{\text{RC}}(t) = \mathbf{a}_r(\theta_c)^H \mathbf{z}(t) = ux(t-\tau_c) + n_{\text{RC}}(t), \tag{28}$$

where $n_{\text{RC}}(t) = \mathbf{a}_r(\theta_c)^H(\Sigma_i \gamma_i \mathbf{a}_r(\theta_i)x(t-\tau_i) + \mathbf{n}_{\text{R}}(t))$.

The SC signal is obtained via beamforming on direction $\tilde{\theta}$:

$$\begin{aligned} z_{\text{SC}}(t) &= \mathbf{a}_r(\tilde{\theta})^H \mathbf{z}(t) \\ &= u\zeta_c x(t-\tau_c) + \sum_i \gamma_i e^{j2\pi\nu_i t}\zeta_i x(t-\tau_i) + n_{\text{SC}}(t), \end{aligned} \tag{29}$$

where $\zeta_c = \mathbf{a}_r(\tilde{\theta})^H \mathbf{a}_r(\theta_c)$, $\zeta_i = \mathbf{a}_r(\tilde{\theta})^H \mathbf{a}_r(\theta_i)$, and $n_{\text{SC}}(t) = \mathbf{a}_r(\tilde{\theta})^H \mathbf{n}_{\text{R}}(t)$. To detect the target at delay τ and Doppler ν, the signal is match-filtered via [62]

$$r(\tau) = \int z_{\text{SC}}(t) e^{-j2\pi\nu t} z^*_{\text{RC}}(t - \tau + \tau_c)\, dt. \tag{30}$$

The surveillance signal $z_{\text{SC}}(t)$ contains the signal from the direct path, which causes strong interference. Another issue is that the RC is not very clean in many practical cases, and the performance of the radar is significantly degraded when there is a great deal of interference, clutter, and noise.

To improve the performance of passive radar, one can make use of the structural information of the underlying communication signal. In particular, because the type of modulation is typically known, we can first estimate the data symbols by demodulation. As demodulation provides better accuracy than directly using the signal in the RC, detection and estimation performance of such radar systems may improve [61], [62]. It is worth noting that passive radar operation is generally inferior to active radar operation because of nonoptimal waveforms, spatial beampatterns, and transmit power [2]. Some recent works proposed what they called *commensal radar* [63], [64], in which the communication signal is designed with the double purpose of transferring information and improving target localization (through a careful autocorrelation function shaping) for a coexisting passive sensing system.

Conclusions

We reviewed some of the main ideas and techniques to allow the coexistence of sensing and communication functions in sharing the same frequency spectrum. The strategies so far proposed are grouped into three major categories. The first allows spectral overlap between the signal transmitted by the radar and communication systems, while the other two avoid mutual interference either by cognitively assigning disjoint subbands to the different services or allowing just one transmitter to be active and guaranteeing functional coexistence.

For each of these categories, the basic ideas are outlined, discussing advantages and disadvantages and offering some examples to illustrate their performance. In the future, hardware prototypes should be built and deployed to be tested on real data. This would permit assessing their performance under real-world conditions, including different types of noise, clutter, and interference.

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