Joint Precoding Design for Multi-antenna Multi-user ISAC Systems

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Abstract In this chapter, we consider the joint transmit design for a multiple input multiple output (MIMO) based integrated sensing and communications (ISAC) system, which simultaneously probes multiple radar targets and communicates with multiple downlink users. The joint transmitter radiates the sum of precoded communication symbols for multiple users and dedicated radar waveform. The precoding weights are jointly designed with optimization problems under certain power constraints, such that performance of both radar and communication is optimized or guaranteed. Typical optimization objectives for radar and communication functions are discussed, including the Cramér-Rao lower bound (CRB) on target estimation, and signal-to-interference-plus-noise ratio (SINR) at each communication user. The formulated optimization problems are generally non-convex, and are therefore often addressed via convex relaxation techniques. We also theoretically analyze the Shannon capacity of the joint transmit under radar performance constraints, which reveals the inherent trade-off between communication and radar functions. Future research directions are also discussed.

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1 Introduction

The integration between radar sensors and communication systems, which is often referred to as integrated sensing and communications (ISAC), has received considerable attention from both industry and academia [24]. By sharing use of the spectrum and hardware platform, ISAC techniques reduce costs and improve spectral-, energy-, and hardware- efficiency [16].

It is a major challenge in ISAC to design a dual-functional waveform simultaneously serving target sensing and information delivering. The design methodology can be generally split into three categories: radar-centered, communication-centered, and joint design [16, 24]. Radar-centered approaches are built on the basis of a radar probing signal, where the communication data are modulated onto the signal by varying the parameters of radar pulses like amplitude, phase and frequencies (e.g., [27]). Communication-centered schemes rely on existing communication waveforms [31] and/or standard-compatible protocols, while the returns of the transmit signals are received and processed for probing aims. In the joint design approach, the ISAC waveform is usually designed by solving some optimization problem that considers both functionalities, rather than based on existing radar or communication waveforms. While existing radar (or communication) waveforms generally result in performance loss for the communication (or radar) function [18], joint design can usually lead to better trade-off between radar and communication.

Recent works on joint-design-based ISAC methods employ multiple-input multiple-output (MIMO) systems, which provide higher spatial degrees of freedom (DoF), and can simultaneously synthesize multiple beams towards several communication users and radar targets. In contrast, ISAC schemes with a single antenna suffer from a common problem that these systems usually form a single directional beam, which illuminates only the radar target inside the beam [22]. Therefore, single-antenna methods are not able to illuminate multiple targets and communicate with multiple users simultaneously as in Fig. 1. This leads to notable degradation in signal-to-noise ratio (SNR) when the communication receivers are not physically located within the radar main lobe.

Existing joint transmit designs for ISAC MIMO radar and communications have achieved favorable performance for both radar and communications [18]. These signaling strategies are essentially based on transmit beamforming. The goal of transmit beamforming is to generate multiple beams in the spatial domain without interfering with each other, and each beam either conveys desired information to a down-link communication user or points towards the targets in the intended direction.

However, when one optimizes the transmit beams, the radar and communication functions often have conflicted criteria due to their inherent difference in operating principles, resulting in challenges in ISAC waveform design. An intuitive example is that the communication function demands the waveform to be random, which enables carrying information, while radar prefers certainty to avoid fluctuation of probing performance, e.g., change of the sidelobe level and variation of SNR. In this chapter, we review the following challenges:

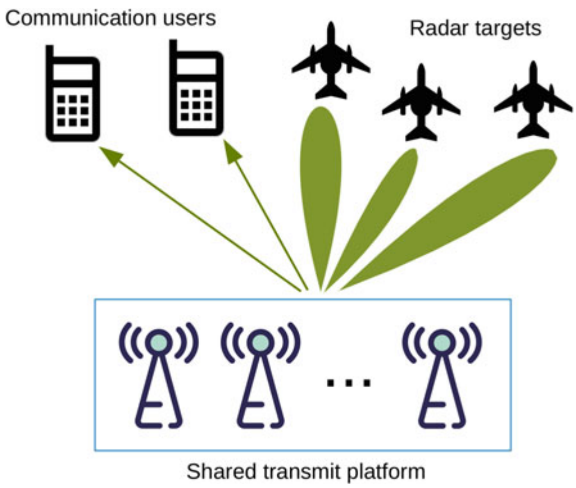


Fig. 1 A dual function system in which communication and radar share the transmit platform

- Signaling strategy. Early works [18] use communication-only signals to realize both communicating and probing functions, however, such transmit signal lacks in DoF and hence restricts the capability of probing multiple targets. This requires a more general and flexible signaling strategy such that the DoF of both radar and communication functions can be guaranteed.
- Formulating and solving the waveform design problems. Based on the signaling strategy, the next step is to consider objective functions that quantitatively evaluate the performance of radar and/or communication, and to formulate the waveform design tasks as optimization problems. These optimization problems are generally non-convex and thus difficult to solve. Therefore, it is critical to design effective and efficient solutions to these problems, and explore whether these solutions have provable guarantees.
- Theoretical performance limitation. A fundamental problem of joint transmit design is to investigate the theoretical capability of both radar and communication functions regardless of any specific waveform under certain constraints (e.g., restricted transmit power), which also reveals the inherent compromise between radar and communication. As theoretical analyses for either radar or communication is usually challenging, considering both functions based on a joint transmit signal is even harder.

In this chapter, we review some recent progress in addressing the challenges above. In particular, in Sect. 2, we introduce a general signaling strategy that extends the communication-only signals by adding a dedicated radar signal. Based on the general signaling strategy, in Sect. 3, we present some optimization problems formulated for designing the joint transmit signals as per typical radar and communication performance metrics, as well as some solvers for these optimization problems. In Sect. 4, we show some preliminary results on analyzing the communication capacity under constraints on radar performance. Sections 5 and 6 are devoted to numerical results and conclusions, respectively.

2 Transmit Signal Model

Consider an antenna array shared by a colocated monostatic MIMO radar system and a multiuser MIMO communication transmitter as depicted in Fig. 1. A dual-function MIMO transmitter realizes both functionalities simultaneously by designing the transmit waveforms of each antenna element. Particularly, multiuser communication is achieved by precoding digital messages that are intended to communication users, and radar performance metrics also need to be considered in the precoding design.

There are generally two kinds of precoding strategies, as shown in Fig. 2:

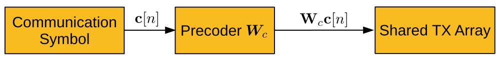
(1) Precoding pure communication symbols [17], where the transmitted signal is a weighted sum of communication symbols, as illustrated in Fig. 2a. For an antenna array of M elements, we let the discrete-time transmit signal of this array at time index n be given by

$$\mathbf{x}[n] = \mathbf{W}_c\mathbf{c}[n], \ n = 0, \ldots, N-1. \tag{1}$$

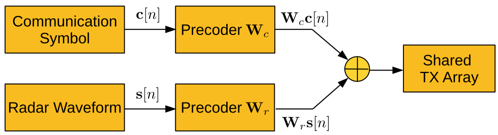
Here, $\mathbf{c}[n] = [c_1[n], \ldots, c_K[n]]^T$ is a $K \times 1$ vector including K parallel communication symbol streams to be communicated to K users, while the $M \times K$ matrix $\mathbf{W}_c$ is the precoding matrix for communications.

(2) Precoding both communication symbols and dedicated radar waveform [22, 26]. In this approach, as shown in Fig. 2b, the transmit signal is given by

Fig. 2 Two different precoding schemes for joint MIMO radar and communications



(a) The joint transmitter only utilizes precoded communication symbols.



(b) The joint transmitter jointly precodes both communication symbols and a dedicated radar waveform.

$$\mathbf{x}[n] = \mathbf{W}_c\mathbf{c}[n] + \mathbf{W}_r\mathbf{s}[n],\ n = 0, \ldots, N-1, \tag{2}$$

where $M \times 1$ vector $\mathbf{s}[n] = [s_1[n], \ldots, s_M(n)]^T$ denotes a dedicated radar waveform and particularly includes M individual waveforms, and the $M \times M$ matrix $\mathbf{W}_r$ is the beamforming (or precoding) matrix for the radar waveform.

Here, the radar waveform $\mathbf{s}[n]$ and communication symbol $\mathbf{c}[n]$ are given, and the goal of waveform design is to design the matrices $\mathbf{W}_c$ in (1) or $\{\mathbf{W}_c, \mathbf{W}_r\}$ in (2), such that both sensing and communication functions operate well. In both strategies, the communication signals can also be used for sensing, since the radar receiver has complete knowledge of the communication waveform. In this way, the communication signal is not regarded as interference at the radar receiver.

Comparing the two strategies, we note that the former scheme based on (1), which precodes only communication symbols for probing, can be regarded as a special case of the latter by letting the radar waveform in (2) be zero, namely transmitting only communication symbols. Since in the latter scheme the variables to design have larger DoF than those in the former, the latter approach always yields better performance when one uses the same optimization criterion for precoding design. Particularly, under the communication-only strategy (1), the available DoF for the MIMO radar waveform is restricted to no more than the number of communication users, K, affecting the radar beam pattern and especially the ability to form multiple main beams for different radar targets. Nevertheless, the latter strategy which jointly precodes individual communication symbols and radar waveforms extends the DoF of MIMO radar waveform to its maximal value, i.e., M, the number of transmit antennas, enhancing the performance of detecting multiple targets. In the following, we focus on the second scheme because it is more general and has significant advantages.

Without loss of generality, we make the following assumptions: (1) Both radar and communication signals are zero-mean, temporally-white and wide-sense stationary stochastic process; (2) The communication symbols are uncorrelated with the radar waveforms, i.e.,

$$\mathbb{E}\left(\mathbf{s}[n]\mathbf{c}^H[n]\right) = \mathbb{E}\left(\mathbf{s}[n]\right)\mathbb{E}(\mathbf{c}^H[n]) = \mathbf{0}_{M\times K}; \tag{3}$$

(3) Communication symbols intended to different users are uncorrelated, namely,

$$\mathbb{E}\left(\mathbf{c}[n]\mathbf{c}^H[n]\right) = \mathbf{I}_K; \tag{4}$$

(4) The individual radar waveforms are generated by pseudo random coding [8, 11, 13, 28, 29], and thus are uncorrelated with each other, resulting in

$$\mathbb{E}\left(\mathbf{s}[n]\mathbf{s}^H[n]\right) = \mathbf{I}_M. \tag{5}$$

Here, both signals are normalized to have unit power, and their real power is encapsulated in their corresponding precoders $\mathbf{W}_r$ and $\mathbf{W}_c$.

In practice, the precoders should satisfy some power constraints, e.g., per-antenna power or total power constraint. The per-antenna power constraint means that the transmit power of each antenna is identical. It settles with the common practice that radar waveforms should be transmitted with their maximal available power [30], and has also been applied in multi-antenna communication systems [23, 37, 44]. Total power constraints are also usually applied [41], depending on the hardware requirements.

Since the transmit signals are random, we consider the average transmit power of each antenna, relying on the covariance of the transmit waveform, defined as

$$\mathbf{R} = \mathbb{E}\left(\mathbf{x}[n]\mathbf{x}^H[n]\right). \tag{6}$$

Substituting (2)–(5) into (6) yields the covariance $\mathbf{R}$ as

$$\mathbf{R} = \mathbf{W}_r\mathbf{W}_r^H + \mathbf{W}_c\mathbf{W}_c^H. \tag{7}$$

The per-antenna power constraint implies that for each $m = 1, \ldots, M$ it holds that

$$\mathbf{R}_{m,m} = P/M, \tag{8}$$

where P is the total transmit power.

3 Precoding Design

3.1 General Optimization Models

To meet the requirements of both radar and communication functions, the precoding matrices $\{\mathbf{W}_c, \mathbf{W}_r\}$ are designed by solving optimization problems. Consider functions $L_c(\mathbf{W}_c, \mathbf{W}_r)$ and $L_r(\mathbf{W}_c, \mathbf{W}_r)$, which numerically evaluate the communication and radar performance, respectively. Then, we divide these optimization models into three categories:

(1) Optimize radar performance under communication performance constraint, represented by

$$\begin{aligned} \min_{\mathbf{W}_c,\mathbf{W}_r} \quad & L_r(\mathbf{W}_c, \mathbf{W}_r) \\ \text{s.t.} \quad & \mathbf{R}_{m,m} = P/M,\ m = 1, \ldots, M, && \text{(9a)} \\ & L_c(\mathbf{W}_c, \mathbf{W}_r) \le \Gamma,\ k = 1, \ldots, K, && \text{(9b)} \end{aligned}$$

where (9a) applies the per-antenna power constraint (8), and Γ denotes the threshold for communication performance.

(2) Optimize communication performance under radar performance constraint, which can be modeled as

$$\begin{aligned} \min_{\mathbf{W}_c, \mathbf{W}_r} \quad & L_c(\mathbf{W}_c, \mathbf{W}_r) \\ \text{s.t.} \quad & \mathbf{R}_{m,m} = P/M, \; m = 1, \ldots, M, \quad (10a) \\ & L_r(\mathbf{W}_c, \mathbf{W}_r) \leq \beta, \; k = 1, \ldots, K, \quad (10b) \end{aligned}$$

where β is a constant restricting the radar performance.

(3) Jointly optimize weighted radar and communication performance. This can be realized by solving

$$\begin{aligned} \min_{\mathbf{W}_c, \mathbf{W}_r} \quad & L_r(\mathbf{W}_c, \mathbf{W}_r) + \lambda L_c(\mathbf{W}_c, \mathbf{W}_r) \\ \text{s.t.} \quad & \mathbf{R}_{m,m} = P/M, \; m = 1, \ldots, M, \quad (11a) \end{aligned}$$

where λ is the regularization parameter.

Within these optimization models, evaluation functions L_r and L_c play the key roles. Different performance metrics lead to different optimization problems, yielding different precoding matrices and transmit waveforms. In addition to average power constraint, there are other restrictions that occur in practical systems, e.g., radars prefer constant envelop signals [21] or waveforms with a low peak to average power ratio, which can also be incorporated into the optimization models. In the following, we introduce several radar and communication performance metrics that are widely used.

3.2 MIMO Radar Performance Metrics

The goal of radar waveform design is to enhance the detection and estimation performance of targets. There are multiple quantitative metrics to evaluate the performance: (1) Choose the receive SNR of the target as the metric, because both detection and estimation performance essentially depends on it. (2) As the variance of an unbiased estimator for target parameters is lower bounded by the CRB, CRB is often applied to represent the estimation accuracy of target parameters. (3) The receive SNR or signal-to-clutter-plus-noise ratio (SCNR) of radar targets are determined by the transmit beam pattern, hence beam pattern is used as a performance metric. (4) Since the covariance matrix $\mathbf{R}$ controls the beam pattern, the similarity between designed and desired covariance matrix can be regarded as an indirect evaluation of radar performance. In the following, we introduce these metrics individually.

3.2.1 Signal Model of Radar Returns

Consider a target located at direction θ. Under far-field and narrow-band assumptions, the received radar returns are given by

$$\mathbf{y}_r(n) = \alpha \mathbf{a}^c(\theta)\mathbf{a}^H(\theta)\mathbf{x}(n) + \mathbf{v}_r(n). \tag{12}$$

Here, α denotes the signal amplitude, $\mathbf{a}(\theta)$ is the steering vector of the antenna array, $\mathbf{v}_r(n)$ is additive white Gaussian noise (AWGN) with variance $\sigma_r^2\mathbf{I}$, and $(\cdot)^c$ means element-wise conjugate operation. Without loss of generality, we ignore the delay of radar returns.

The received radar returns are then sequentially processed by time-domain and space-domain matched filtering. The time-domain filter calculates the correlation between the received and transmitted signals, given by

$$\mathbf{R}_{yx} = \sum_{n=0}^{N-1} \mathbf{y}_r(n)\mathbf{x}^H(n). \tag{13}$$

The space-domain matched filter aims to identify the angles of targets, by considering

$$\chi(\theta') = \mathbf{a}^T(\theta')\mathbf{R}_{yx}\mathbf{a}(\theta'), \tag{14}$$

where θ' denotes the matched angle. In an ideal case, it is assumed that the amplitude of $\chi(\theta')$ reaches its maximum when $\theta' = \theta$ and is small enough to avoid false alarm when $\theta' \neq \theta$.

Substituting (12) and (13) into (14), we find that the amplitude of the signal component in $\chi(\theta')$ is determined by

$$\chi(\theta') \propto \mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta'). \tag{15}$$

Here, it is assumed that the sample covariance matrix approximates the expected covariance matrix, i.e., $\sum_{n=0}^{N-1} \mathbf{x}(n)\mathbf{x}^H(n) \approx \mathbf{R}$. This term is essentially related to the SNR of targets and beam pattern, introduced next.

3.2.2 Beam Pattern

From the perspective of beamforming, radar aims to direct more energy to the desired angles and less to the rest. To enhance the radar performance [42], it is desired to reduce the correlation between radar echoes from different directions. The beam pattern, given by,

$$P(\theta; \mathbf{R}) = \mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta), \tag{16}$$

describes the distribution of transmit energy over direction θ. From (15), the correlation between echoes from angles θ_1 and θ_2 is given by [30]

$$P_c(\theta_1, \theta_2; \mathbf{R}) = \mathbf{a}^H(\theta_1)\mathbf{R}\mathbf{a}(\theta_2). \tag{17}$$

One may use the weighted sum of two parts to construct the objective function for radar [6, 30]. The first part is the mean-squared error (MSE) between the obtained and desired beam pattern, given by

$$L_{r,1}(\mathbf{R}, \alpha) = \frac{1}{L}\sum_{l=1}^{L} |\alpha d(\theta_l) - P(\theta_l; \mathbf{R})|^2, \tag{18}$$

where α is a scaling factor, $d(\theta)$ is the given desired beam pattern, and $\{\theta_l\}_{l=1}^{L}$ are sampled angle grids. The second part is the mean-squared cross correlation pattern, expressed as

$$L_{r,2}(\mathbf{R}) = \frac{2}{P^2 - P}\sum_{p=1}^{P-1}\sum_{q=p+1}^{P} \left|P_c(\overline{\theta}_p, \overline{\theta}_q; \mathbf{R})\right|^2, \tag{19}$$

where $\{\overline{\theta}_p\}_{p=1}^{P}$ are the given directions of the targets. The summation in (19) is normalized by $\frac{2}{P^2-P}$, as there exists $\frac{P^2-P}{2}$ pairs of distinct directions in the set $\{\overline{\theta}_p\}$. The loss function of radar is then

$$L_r(\mathbf{R}, \alpha) = L_{r,1}(\mathbf{R}, \alpha) + w_c L_{r,2}(\mathbf{R}), \tag{20}$$

where w_c is a weighting factor. As discussed in [6, 30], the loss function $L_r(\mathbf{R}, \alpha)$ can be written as a positive-semidefinite quadratic function of $\mathbf{R}$ and α.

3.2.3 SNR of Radar Targets

For a target located in direction θ, the receive SNR of the target is expressed as

$$P(\theta; \mathbf{R}) = \mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta),$$

where we ignore the scattering intensity of target and noise variance because they are constants with respect to the transmit waveform or precoding matrices. When one sets the goal as maximizing SNR, the opposite of the above term, $-P(\theta; \mathbf{R})$, can be used as an objective or constraint function L_r to evaluate the radar performance [4].

3.2.4 CRB on Radar Targets' Parameters

When there are Q targets with different angles in the same range cell, the received signal in (12) is extended to

$$\mathbf{y}_r(n) = \sum_{q=1}^{Q} \alpha_q \mathbf{a}^c(\theta_q)\mathbf{a}^H(\theta_q)\mathbf{x}(n) + \mathbf{v}_r(n), \; n = 0, \ldots, N-1. \tag{21}$$

In (21), the MIMO radar observation and unknown variables to estimate are

$$\mathbf{Y}_r = \left[\mathbf{y}_r[0], \ldots, \mathbf{y}_r[N-1]\right]$$

and

$$\boldsymbol{\theta} = [\theta_1, \ldots, \theta_Q, \Re(\alpha_1), \ldots, \Re(\alpha_Q), \Im(\alpha_1), \ldots, \Im(\alpha_Q)]^T,$$

respectively. When the estimation accuracy of targets' parameter θ is of concern, one may use the CRB as an evaluation function [15], because CRB is a lower bound on the variance for all unbiased estimators. By minimizing the CRB, one expects to improve the accuracy of measuring radar targets. The CRB of a specific target parameter is the corresponding diagonal element in the inverse of Fisher information matrix (FIM)

$$\mathbf{I}_{\boldsymbol{\theta}} = \mathbb{E}\left\{\nabla_{\boldsymbol{\theta}} p(\mathbf{Y}_r; \boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}}^T p(\mathbf{Y}_r; \boldsymbol{\theta}) \,|\, \boldsymbol{\theta}\right\},$$

where $p(\mathbf{Y}_r; \boldsymbol{\theta})$ is the probability density function for $\mathbf{Y}_r$ conditioned on the value of $\boldsymbol{\theta}$. In [15], the authors derive the FIM, which is a linear function of $\mathbf{R}$. The full expressions of FIM are complicated, and are therefore omitted here for conciseness. In the case of a single point target, the CRB of estimating the angle θ can be given in the form of

$$\text{CRB}(\theta) = \frac{\text{tr}\left(\mathbf{A}^H\mathbf{A}\mathbf{R}\right)}{\gamma_R\left[\text{tr}\left(\dot{\mathbf{A}}^H\dot{\mathbf{A}}\mathbf{R}\right)\text{tr}\left(\mathbf{A}^H\mathbf{A}\mathbf{R}\right) - \left|\text{tr}\left(\dot{\mathbf{A}}^H\mathbf{A}\mathbf{R}\right)\right|^2\right]}, \tag{22}$$

where $\mathbf{A} = \mathbf{a}^c(\theta)\mathbf{a}^H(\theta)$, $\dot{\mathbf{R}} = \frac{\partial \mathbf{A}}{\partial \theta}$, and $\gamma_R = \frac{\sigma_r^2}{2|\alpha|^2 N}$. Readers may refer to Eq. (13) in [15] for details.

3.2.5 Similarity to Desired Covariance Matrix

The above discussions lead to the common result that the radar performance relies on the covariance matrix $\mathbf{R}$. Therefore, another design criterion is to minimize or constrain the mismatch between the achieved and desired $\mathbf{R}$ [18], given by

$$\|\mathbf{R} - \mathbf{R}_o\|_F \le \varepsilon. \tag{23}$$

Here, the desired covariance $\mathbf{R}_o$ is pre-calculated. For example, $\mathbf{R}_o$ could be obtained by optimizing the radar-only setup, given by

$$\mathbf{R}_o = \arg\min_{\mathbf{R}} L_r(\mathbf{R}), \quad \text{s.t. } \mathbf{R}_{m,m} = P/M, \ m = 1, \ldots, M. \tag{24}$$

For an extreme case that the radar performance should be guaranteed in high priority and no sacrifice is tolerable, we set ε in (23) to 0, i.e., using the strong radar constraint $\mathbf{R} = \mathbf{R}_o$; see a recent work [19].

3.3 Multiuser MIMO Communication Performance Metrics

The down-link users receive the signals transmitted by the MIMO transmitter, including the desired communication information, and unwanted interference (i.e., signals for other users and radar waveforms) as well as noises. The communication performance depends on the signal-to-interference-plus-noise ratio (SINR), which is usually regarded as the performance metric for waveform design. To better eliminate the interference impinging on the down-link users, some non-linear precoding technique like dirty paper coding (DPC) can be applied, by encoding the communication signals to adapt to the interference. In this section, we first introduce the signal model of down-link users, followed by the definitions of SINR under both linear precoding and DPC precoding.

3.3.1 Signal Model of Down-link Communication Users

Consider $K < M$ down-link users, each of which has a single receive antenna and observes the output of a frequency flat Gaussian noise channel. The channel output at the K users at time instance n, represented via the $K \times 1$ vector $\mathbf{r}[n]$, is given by

$$\mathbf{r}[n] = \mathbf{H}\mathbf{W}_c\mathbf{c}[n] + \mathbf{H}\mathbf{W}_r\mathbf{s}[n] + \mathbf{v}[n], \tag{25}$$

where $\mathbf{H}$ is the $K \times M$ narrow-band channel matrix and $\mathbf{v}[n]$ is AWGN with covariance $\sigma^2\mathbf{I}_K$.

Here, it is assumed that the transmit array knows the instantaneous downlink channel $\mathbf{H}$. This knowledge can be obtained for example, by exploiting wireless channel reciprocity when operating in time-division duplex mode, i.e., the downlink channel is obtained via uplink channel estimation. Alternatively, in frequency-division duplex mode, the downlink channel can be obtained via channel feedback from the users; see, e.g., [43].

3.3.2 SINR in the Linear Precoding Scheme

Define the equivalent radar-to-user channel and equivalent inter-user channel matrices as

$$\mathbf{G} = \mathbf{H}\mathbf{W}_r, \tag{26a}$$

$$\mathbf{F} = \mathbf{H}\mathbf{W}_c, \tag{26b}$$

respectively. Since the users are generally not able to cooperate with each other, the off-diagonal elements of **F** lead to inter-user interference, which should be mitigated by precoding. At the same time, since the users generally do not have any prior information on the radar waveform, **G** leads to interference from the radar. At the k-th user, the signal power is

$$\mathbb{E}\left\{ \left|\mathbf{F}_{k,k} c_k(n)\right|^2 \right\} = \left|\mathbf{F}_{k,k}\right|^2, \tag{27}$$

the power of inter-user interference is

$$\mathbb{E}\left\{ \sum_{i \neq k} \left|\mathbf{F}_{k,i} c_i(n)\right|^2 \right\} = \sum_{i \neq k} \left|\mathbf{F}_{k,i}\right|^2, \tag{28}$$

and the power of interference from the radar is

$$\mathbb{E}\left\{ \sum_{i=1}^{M} \left|\mathbf{G}_{k,i} s_i(n)\right|^2 \right\} = \sum_{i=1}^{M} \left|\mathbf{G}_{k,i}\right|^2. \tag{29}$$

Therefore, the SINR at the k-th user is expressed as

$$\text{SINR}_k = \frac{|\mathbf{F}_{k,k}|^2}{\sum_{i \neq k} |\mathbf{F}_{k,i}|^2 + \sum_{i=1}^{M} |\mathbf{G}_{k,i}|^2 + \sigma^2}. \tag{30}$$

3.3.3 SINR in the DPC Scheme

To further improve the SINRs, one can perform non-linear precoding techniques, which eliminate the interference by encoding the communication signals to adapt the interference. In particular, we consider DPC [5], which reveals that the capacity of an AWGN channel corrupted by interference equals to the capacity of an interference-free AWGN channel if the interference is known at the transmitter. DPC was applied to downlink multiuser communications for inter-user interference elimination [3, 35, 36, 39], and was shown to achieve the capacity region of MIMO Gaussian broadcast channel (GBC) [40].

We apply DPC to the GBC in (25) by serially encoding the source signal of each user. The encoding operations are conducted in the order $\{1, \ldots, K\}$. When performing DPC for the k-th user, $\{c_1(n)\}, \ldots, \{c_{k-1}(n)\}$ are already encoded while $\{c_{k+1}(n)\}, \ldots, \{c_K(n)\}$ are not encoded yet. Thus, the interference from the $1, \ldots, (k-1)$-th user is known while the interference from the $k+1, \ldots, K$-th user is unknown at the transmitter. The radar interference is also known at the transmitter. Therefore, the effective SINR at the k-th user in the DPC regime is [36]

$$\mathrm{SINR}_k^{\mathrm{dpc}} = \frac{|\mathbf{F}_{k,k}|^2}{\sum_{i>k} |\mathbf{F}_{k,i}|^2 + 1}, \tag{31}$$

for $k = 1, \ldots, K$.

Note the SINRs in (31) are achievable when the channel state information (CSI) is perfectly known at the transmitter. When the CSI is not perfectly known, there exists a difference between the obtained CSI at the transmitter and the actual channel, leading to a residual between the known interference at transmitter and the actual interference, which is unknown and cannot be eliminated by DPC. Therefore, the achievable SINRs may become lower than that in (31). Although the precoding matrix design in the presence of CSI error [12, 38] can be meaningful, we only consider the case that the CSI is perfectly known in this chapter.

Comparing (30) and (31), it is observed that DPC improves the SINR compared with transmit beamforming by eliminating the interference.

3.4 Beamforming Design by Solving the Optimization Problems

Here, we introduce two kinds of optimization problems for joint MIMO radar and multiuser communications. The first optimizes the radar performance under individual SINR constraints at communication users [22]. The second is radar-centric, which performs SINR balancing for multiuser communication with a given transmit covariance for MIMO radar [19].

3.4.1 Radar Transmit Beamforming Design Under SINR Constraints for Communication Users

The goal of the optimization is to optimize the radar performance with the quality of service (QoS) guaranteed for communication users. In particular, we minimize the performance function $L_r(\mathbf{R})$ for radar, under the constraints on per-antenna power (8) and the achieved SINR of each user (higher than a given threshold). For linear precoding, the optimal precoding matrices can be obtained by solving the following optimization problem

$$\min_{\mathbf{W}_r,\mathbf{W}_c} \quad L_r(\mathbf{R}) \tag{32a}$$

$$\text{s.t.} \quad \mathbf{R} = \mathbf{W}_c\mathbf{W}_c^H + \mathbf{W}_r\mathbf{W}_r^H, \tag{32b}$$

$$\mathbf{R}_{m,m} = P/M, \ m = 1, \ldots, M, \tag{32c}$$

$$\text{SINR}_k \geq \Gamma, \ k = 1, \ldots, K, \tag{32d}$$

where (32d) follows from considering the SINR requirement and Γ is the given SINR threshold.

The trade-off between the communication quality and radar performance can be achieved by adjusting the threshold Γ. When $\Gamma = 0$, (32d) always holds, and the joint radar-communication optimization in (32) reduces to the radar-only optimization in (24). When $\Gamma > 0$, compared with the radar-only transmit beamforming problem in (24), there can be an inherent radar performance loss induced by the need to meet the communication performance guarantees, as compared to the radar-only case. If higher Γ is set, higher signal power and less interference is expected to be observed at the user side, further restricting the precoding matrices. As a result, the performance loss of MIMO radar becomes more significant.

The optimization problem (32) is non-convex because of the quadratic equality constraint in (32b) and is thus difficult to solve. Nonetheless, it can be recast using semi-definite relaxation (SDR) such that the solution to the solvable relaxed problem is also the global optimizer of the original non-convex problem (32), i.e., the relaxation is tight [22].

Let $\mathbf{W} = [\mathbf{W}_c, \mathbf{W}_r]$ and $\mathbf{w}_i$ be the i-th column in $\mathbf{W}$, for $i = 1, \ldots, K + M$. With SDR, we define the rank-one semi-definite matrices $\mathbf{R}_i = \mathbf{w}_i\mathbf{w}_i^H$, for $i = 1, \ldots, K + M$. Then, (32) is equivalently expressed as a semi-definite problem with rank-one constraints:

$$\min_{\mathbf{R},\mathbf{R}_1,\ldots,\mathbf{R}_K} \quad L(\mathbf{R}) \tag{33a}$$

$$\text{s.t.} \quad \mathbf{R} \succeq 0, \ \mathbf{R} \succeq \sum_{k=1}^{K} \mathbf{R}_k, \tag{33b}$$

$$\mathbf{R}_{m,m} = P/M, \ m = 1, \ldots, M, \tag{33c}$$

$$\mathbf{R}_k \succeq 0, \ \text{rank}(\mathbf{R}_k) = 1, \ k = 1, \ldots, K, \tag{33d}$$

$$\left(1 + \Gamma^{-1}\right)\mathbf{h}_k^H\mathbf{R}_k\mathbf{h}_k \geq \mathbf{h}_k^H\mathbf{R}\mathbf{h}_k + 1, \ k = 1, \ldots, K, \tag{33e}$$

where (33e) is obtained from the SINR constraint in (32d). The optimization problem (33) is still non-convex because of the rank-one constraints. By omitting these constraints, SDR converts (33) to a relaxed convex optimization

$$\min_{\mathbf{R},\mathbf{R}_1,\ldots,\mathbf{R}_K} \quad L(\mathbf{R}) \tag{34a}$$

$$\text{s.t.} \qquad \mathbf{R} \succeq \sum_{k=1}^{K} \mathbf{R}_k,\ \mathbf{R}_k \succeq 0,\ k = 1, \ldots, K,\ \text{(33c) and (33e)}. \tag{34b}$$

The relaxed problem (34) is convex because the objective function is convex and the constraints are either linear or semi-definite. The global optimum of (34) can be obtained in polynomial time with convex optimization toolboxes [32–34].

While SDR is generally not tight, it is shown in [22] that the SDR used in obtaining (34) from (33) is tight. In other words, once a pair of optimal solution $\hat{\mathbf{R}}, \hat{\mathbf{R}}_1, \ldots, \hat{\mathbf{R}}_K$ for (34) is obtained, the desired rank-one solution $\hat{\mathbf{R}}_1, \ldots, \hat{\mathbf{R}}_K$ can be computed by

$$\tilde{\mathbf{w}}_k = \left(\mathbf{h}_k^H \hat{\mathbf{R}}_k \mathbf{h}_k\right)^{-1/2} \hat{\mathbf{R}}_k \mathbf{h}_k,\ \tilde{\mathbf{R}}_k = \tilde{\mathbf{w}}_k \tilde{\mathbf{w}}_k^H, \tag{35}$$

for $k = 1, \ldots, K$. This solution is optimal for (33). Based on (35), we summarize in Algorithm 1 the procedure to compute the optimal $\mathbf{W}_c$, $\mathbf{W}_r$ by the relaxed optimization in (34).

Algorithm 1 Precoding design for joint radar and communications via SDR.

Input: Transmit power P, objective function for MIMO radar $L_r(\mathbf{R})$, communication channel $\mathbf{H}$, SINR threshold Γ.

1: Obtain the solution of $\hat{\mathbf{R}}, \hat{\mathbf{R}}_1, \ldots, \hat{\mathbf{R}}_K$ by solving (34).

2: Compute $\tilde{\mathbf{w}}_1, \ldots, \tilde{\mathbf{w}}_K$ via (35).

3: Compute $\tilde{\mathbf{w}}_{K+1}, \ldots, \tilde{\mathbf{w}}_{K+M}$ via

$$\left[\tilde{\mathbf{w}}_{K+1}, \ldots, \tilde{\mathbf{w}}_{K+M}\right] = \left[\hat{\mathbf{R}} - \sum_{k=1}^{K} \tilde{\mathbf{w}}_k \tilde{\mathbf{w}}_k^H\right]^{1/2},$$

4: Obtain the optimal precoding matrices $\mathbf{W}_c$ and $\mathbf{W}_r$ via

$$\mathbf{W}_c = \left[\tilde{\mathbf{w}}_1, \ldots, \mathbf{w}_K\right],\ \tilde{\mathbf{W}}_r = \left[\tilde{\mathbf{w}}_{K+1}, \ldots, \tilde{\mathbf{w}}_{K+M}\right].$$

3.4.2 SINR Balancing with Given Transmit Covariance for Radar

The optimization problem in (32) constrains the SINR of multiuser communications, but cannot guarantee the radar performance. Instead of constraining the performance of communication, in this section we consider a radar-centric design and optimize the performance of communication with given radar constraints. In particular, we solve the SINR balancing problem for multiuser communications with a given transmit covariance for radar, as the radar performance is determined by the transmit covariance. The SINR balancing problem maximizes the worst SINR among the users, which is expressed as

$$\max_{\mathbf{W}_c,\mathbf{W}_r,\gamma} \ \gamma, \ \ \text{s.t. } \mathbf{R}_o = \mathbf{W}_r\mathbf{W}_r^H + \mathbf{W}_c\mathbf{W}_c^H, \tag{36a}$$

$$\text{SINR}_k \geq \gamma, \ k = 1, \ldots, K, \tag{36b}$$

where $\mathbf{R}_o$ is the given transmit covariance for radar.

We next discuss solvers for (36). While the constraints in (36b) are non-convex, we can convert them to convex constraints, and hence reformulate (36) to a linear conic optimization problem. First, with the covariance constraint, the sum power of the desired signal and the interference at the k-th user equals to $[\mathbf{R}_h]_{k,k}$, where $\mathbf{R}_h = \mathbf{H}\mathbf{R}_o\mathbf{H}^H$, i.e.

$$\sum_{i=1}^{K} |\mathbf{F}_{k,i}|^2 + \sum_{i=1}^{M} |\mathbf{G}_{k,i}|^2 = [\mathbf{R}_h]_{k,k}. \tag{37}$$

Substituting (37) into (30), the SINR constraints in (36b) are simplified to

$$|\mathbf{F}_{k,k}| \geq \sqrt{\frac{\gamma}{1+\gamma}} s_k, \ k = 1, \ldots, K, \tag{38}$$

where $s_k = ([\mathbf{R}_h]_{k,k} + 1)^{1/2}$. In (38), the reformulated SINR constraint only involves $\mathbf{F}$ but not $\mathbf{G}$.

Similarly, the covariance constraint can also be rewritten as a constraint on $\mathbf{F}$, as stated by Theorem 1.

Theorem 1 ([19]) *Given* $\mathbf{F} \in \mathbb{C}^{K\times K}$*, there exists* $\{\mathbf{W}_c, \mathbf{W}_r\}$ *that obey*

$$\mathbf{F} = \mathbf{H}\mathbf{W}_c, \ \mathbf{R}_o = \mathbf{W}_r\mathbf{W}_r^H + \mathbf{W}_c\mathbf{W}_c^H, \tag{39}$$

if and only if $\mathbf{F}\mathbf{F}^H \preceq \mathbf{R}_h$.

We have thus reformulated (36) into the following optimization with respect to $\mathbf{F}, \gamma$:

$$\max_{\mathbf{F},\gamma} \gamma, \text{ s.t. } \mathbf{F}\mathbf{F}^H \preceq \mathbf{R}_h, \tag{40a}$$

$$|\mathbf{F}_{k,k}| \geq \sqrt{\frac{\gamma}{1+\gamma}} s_k, \ k = 1, \ldots, K. \tag{40b}$$

From Schur complement [7], the constraint $\mathbf{F}\mathbf{F}^H \preceq \mathbf{R}_h$ is equivalent to a convex semidefinite constraint. Note that for a feasible $\mathbf{F}$ in (40), multiplying its k-th column by a scalar phase factor $e^{j\theta_k}$ does not violate feasibility [1, 44]. Therefore, we only need to consider $\mathbf{F}$ with real diagonal elements. Introducing a new variable $t = \sqrt{\gamma/(1+\gamma)}$, (40) is equivalent to:

$$\max_{\mathbf{F},t} t, \quad \text{s.t. } \mathbf{F}\mathbf{F}^H \preceq \mathbf{R}_h, \tag{41a}$$

$$\Re\{\mathbf{F}_{k,k}\} \geq t s_k, \ k = 1, \dots, K, \tag{41b}$$

which is solvable by linear conic programming. While linear conic optimizations in (41) can be effectively solved by optimization softwares in polynomial time, it may be more efficient to solve the dual problem of (41) with gradient descent methods. Readers may refer to [19] for further details.

4 Theoretical Analyses

To facilitate the derivation of fundamental limits of a MIMO system jointly designed for radar and communications functions, we discuss the communication capacity under radar performance constraint. The communication capacity is widely used as a theoretical limit of communication performance. The capacity of a MIMO communication system is affected when the transmit signal is also restricted to bear a radar function, because radar constraints essentially reduce the transmit signal DoF.

4.1 Capacity Under Radar Constraint

We first consider a typical example, that is the sum-rate capacity of multiuser communications under some power and radar constraints. For a given covariance matrix $\mathbf{R}$ of the transmit signal, the sum rate capacity of the vector broadcast channel in (25) is given by [39]

$$C(\mathbf{R}) = \min_{\mathbf{Z}\succeq 0} \log \left|\mathbf{I}_K + \mathbf{Z}^\dagger \mathbf{H}\mathbf{R}\mathbf{H}^H\right|, \tag{42a}$$

$$\text{s.t. } \mathbf{Z}_{k,k} = \sigma^2, \ k = 1, \dots, K, \tag{42b}$$

where $\mathbf{Z}^\dagger$ is the Moore-Penrose inverse [25] of $\mathbf{Z}$. Here, the capacity equals the lower bound of the mutual information for a cooperative channel where the receivers cooperate and have a variable noise covariance $\mathbf{Z}$ constrained by (42b).

We impose the power constraint

$$\text{tr}(\mathbf{R}) \leq P, \tag{43}$$

where P is the maximal average transmit power, and impose the radar constraint based on similarity restriction of $\mathbf{R}$, given in (23). With these constraints, the maximized sum-rate capacity is given by

$$\max_{\mathbf{R}\succeq 0}\min_{\mathbf{Z}\succeq 0} \ \log\left|\mathbf{I}_K + \mathbf{Z}^{\dagger}\mathbf{H}\mathbf{R}\mathbf{H}^H\right|, \tag{44a}$$

$$\text{s.t.} \ \ \mathbf{Z}_{k,k} = \sigma^2, \ k = 1, \ldots, K, \tag{44b}$$

$$\|\mathbf{R} - \mathbf{R}_o\|_F \leq \varepsilon, \ \operatorname{tr}(\mathbf{R}) = P. \tag{44c}$$

The problem is to solve (44), which is generally a convex-concave saddle point (CCSP) problem with respect to $\mathbf{Z}$ and $\mathbf{R}$: The objective function is convex with respective to $\mathbf{Z}$ and is concave with respective to $\mathbf{R}$; The constraints in (44) are convex. In the special case when $\varepsilon = 0$, i.e., $\mathbf{R} = \mathbf{R}_o$, (44) becomes a convex optimization problem with respect to $\mathbf{Z}$ that can be solved via linear conic program [14].

By exchanging the order of minimization and maximization in this CCSP problem, it can be reformulated into an optimization problem with respect to $\mathbf{Z}$, and solved via gradient descend; details on calculating the gradient are in [20]. Numerical result in Sect. 5 show that tighter radar constraints reduce communication capacity, indicating the trade-off between these two functions. To analytically reveal the inherent trade-off between radar and communications in a MIMO transmitter, we consider a toy case, discussed in the sequel.

4.2 A Toy Example

We now consider a toy example where both the schemes of communicating and probing are simplified, with the goal of obtaining an analytical expression of the capacity and the waveform design strategy that achieves this capacity. In particular, we assume that there is only one down-link user with one antenna, i.e., $K = 1$. Then the channel reduces to $\mathbf{H} = \mathbf{h}^H$. Here, $\mathbf{h}$ is referred to as the channel vector. There is only one radar target at angle θ with the steering vector being $\mathbf{a}_t = \mathbf{a}(\theta)$. We use the SNR of the target to evaluate the radar performance. Hence, the capacity under radar constraint becomes

$$\begin{aligned} \max_{\mathbf{R}\succeq 0} \ & \log\left(1 + \mathbf{h}^H\mathbf{R}\mathbf{h}\right), \\ \text{s.t.} \ & \mathbf{a}_t^H\mathbf{R}\mathbf{a}_t \geq \beta, \\ & \operatorname{tr}(\mathbf{R}) = P, \end{aligned} \tag{45}$$

where β denotes SNR threshold for radar target. Since $\log\left(1 + \mathbf{h}^H\mathbf{R}\mathbf{h}\right)$ monotonically increases with $\mathbf{h}^H\mathbf{R}\mathbf{h}$, the above optimization problem is equivalent to

$$\begin{aligned} \max_{\mathbf{R}\succeq 0} \ & \mathbf{h}^H\mathbf{R}\mathbf{h}, \\ \text{s.t.} \ & \mathbf{a}_t^H\mathbf{R}\mathbf{a}_t \geq \beta, \\ & \operatorname{tr}(\mathbf{R}) = P. \end{aligned} \tag{46}$$

According to [37], there exists an optimal $\mathbf{R}$ with rank one and eigenvector being a linear combination of $\mathbf{h}$ and $\mathbf{a}_t$. Therefore, we denote two scalars to solve by $a, b \in \mathbb{C}$, and a vector by $\mathbf{c} = a\mathbf{h} + b\mathbf{a}_t \in \mathbb{C}^M$. Then, we rewrite $\mathbf{R}$ as

$$\mathbf{R} = \mathbf{c}\mathbf{c}^H. \tag{47}$$

Substituting (47) into (46) yields an optimization problem with respect to scalars a, b. Using Karush-Kuhn-Tucker (KKT) condition [2], we obtain the following solution [45], which depends on the predefined threshold β:

(i) When $0 \le \beta < \frac{P|\mathbf{h}^H\mathbf{a}_t|^2}{\|\mathbf{h}\|_2^2}$, the solution is

$$\begin{cases} |a| = \dfrac{\sqrt{P}}{\|\mathbf{h}\|_2}, \\ |b| = 0, \end{cases} \tag{48}$$

where the angle of the complex-valued scalar a is arbitrary.

(ii) When $\frac{P|\mathbf{h}^H\mathbf{a}_t|^2}{\|\mathbf{h}\|_2^2} \le \beta \le P\,\|\mathbf{a}_t\|_2^2$, the solution becomes

$$\begin{cases} |a| = \eta, \\ |b| = \dfrac{\sqrt{\beta}}{\|\mathbf{a}_t\|_2^2} - \dfrac{\left|\mathbf{h}^H\mathbf{a}_t\right|}{\|\mathbf{a}_t\|_2^2}\eta, \end{cases} \tag{49}$$

where

$$\eta = \sqrt{\frac{P\,\|\mathbf{a}_t\|_2^2 - \beta}{\|\mathbf{h}\|_2^2\,\|\mathbf{a}_t\|_2^2 - \left|\mathbf{h}^H\mathbf{a}_t\right|^2}},$$

and the angles of a and b should obey

$$\arg(a) - \arg(b) = \arg\left(\mathbf{h}^H\mathbf{a}_t\right) \tag{50}$$

if $\mathbf{h}^H\mathbf{a}_t \ne 0$ or could be arbitrary if $\mathbf{h}^H\mathbf{a}_t = 0$.

(iii) When $\beta > P\,|\mathbf{a}_t|^2$, there is no feasible solution.

The analytical solutions of this example facilitate discussing the characters of the optimal dual-function transmit signal and the achievable system performance. The formation of the eigenvector $\mathbf{c}$ indicates that the design of a dual-function signal is actually power allocation between the radar target and communication user. The coefficients a and b represent the amplitudes of resources allocated for communication and dedicated probing, respectively. Whether or how much of the communication energy, $|a|^2$, simultaneously benefits the probing function depends on the correlation between $\mathbf{h}$ and $\mathbf{a}_t$. In the extreme case where $\mathbf{h}$ is parallel to $\mathbf{a}_t$, the communication waveform also serves as the probing signal, and there is no need to transmit a dedicated radar signal. In the opposite case when $\mathbf{h}$ is orthogonal to $\mathbf{a}_t$ with $\mathbf{h}^H\mathbf{a}_t = 0$, radar and communication functions operate independently without energy sharing between each other. The sum of energies for communication user and radar target, i.e., $|a|^2 \|\mathbf{h}\|_2^2$ and $|b|^2 \|\mathbf{a}_t\|_2^2$, is exactly the whole energy, since it holds that $P = |a|^2 \|\mathbf{h}\|_2^2 + |b|^2 \|\mathbf{a}_t\|_2^2$.

In more general cases, partial energy allocated for communicating is shared with radar functionality. Particularly, in case (i), there is no dedicated radar signal, because the communication signal shed on the radar target is strong enough to meet the SNR requirement. Therefore, the capacity is equal to that of a communication-only MIMO system. In case (ii), there should be a dedicated radar signal to guarantee the target SNR, which reduces the energy for the down-link user and affects capacity. In case (iii), the SNR requirement is unachievable, even when all the available energy is allocated for probing.

4.3 Discussions on Trade-off Between Radar and Communications

Theoretical analyses with respect to capacity reveal the inherent trade-off between radar and communications performance, both of which depend on the covariance matrix of the transmit signal. Generally, there does not exist a single covariance matrix simultaneously achieving optimal radar and communication performance, resulting in a compromise between them.

From the perspective of power allocation, both radar and communication functions desire the transmitter to increase the gain of the beam towards the targets or users, respectively. This leads to conflicts between the two functions, when the down-link users and radar targets are located at different directions (or their channels are different).

From the perspective of spatial diversity, communication capacity increases with the DoF of transmit signals, defined by the rank of their covariance matrix $\mathbf{R}$. The DoF reaches the maximum if signals transmitted by different antennas are independent of each other. It generally yields higher capacity than the cases when signals are correlated with (restricted by) each other. The DoF reduces to 1 in the extreme case when the radar operates in the phased array model. In this scenario, the transmitted signal of each array is the same except a fixed phase difference that is determined by the desired direction of the beam. However, radar function typically prefers the phased array model because the beam is directional and the transmit energy is focused towards the target, leading to higher SNR or detection probability of the target. In the independent waveform model, the energy is spread over all directions, resulting in wider coverage but lower SNR of each target. In summary, communication function prefers randomness, providing higher communication capacity, while radar function tends towards certainty, guaranteeing higher SNR. This suggests unavoidable compromise between radar and communication functions when designing dual-function waveforms.

5 Numerical Results

In this section, we provide some numerical results regarding the performance of joint MIMO radar and multiuser communications. In the first experiment [22], we numerically evaluate the performance trade-off between radar and multiuser communications. In the second experiment [19], we consider the radar-centric design in (36) and demonstrate the balanced SINR of down-link users versus transmit power with a given transmit covariance for radar. The third experiment [20] computes the sum-rate capacity discussed in Sect. 4.1 under different covariance mismatch ε.

5.1 Performance Trade-off Between Radar and Multiuser Communications

To exploit the performance trade-off, we use the SDR based method in Algorithm 1 to solve (32). Here, we first show the tradeoff results where the radar performance is evaluated using the beam pattern MSE, defined as the MSE between the obtained MIMO radar transmit beam pattern and the optimal radar-only beam pattern, and is written as

$$\mathrm{MSE} = \frac{1}{L}\sum_{l=1}^{L} \left| P(\theta_l; \mathbf{R}_0) - P(\theta_l; \mathbf{R}) \right|^2, \tag{51}$$

where $P(\theta_l; \mathbf{R}_o)$ is the optimal radar-only beam pattern with $\mathbf{R}_o$ obtained from (24). The communication performance is encapsulated in the achieved SINRs defined in (30) and (31). We also compare the result with the precoding scheme proposed in [17] which only precodes communication symbols as shown in Fig. 2a.

In the experiments reported in this section, the transmit array is a uniform linear array with half wavelength spaced elements. The number of transmit antennas is $M = 10$ and the total transmit power $P = 100$. For MIMO radar transmit beamforming, the ideal beam pattern consists of three main beams, whose directions are $\overline{\theta}_1 = -40°$, $\overline{\theta}_2 = 0°$ and $\overline{\theta}_3 = 40°$. The width of each ideal beam is $\Delta = 10°$, and thus the desired beam pattern is

$$d(\theta) = \begin{cases} 1, \ \overline{\theta}_p - \frac{\Delta}{2} \le \theta \le \overline{\theta}_p + \frac{\Delta}{2}, \ p = 1, 2, 3, \\ 0, \ \text{otherwise}. \end{cases} \tag{52}$$

In (18), the direction grids $\{\theta_l\}_{l=1}^{L}$ are obtained by uniformly sampling the range of $-90°$ to $90°$ with resolution $0.1°$. The radar loss in (20) accounts for both objectives equally, namely, the weighting factor is set to $w_c = 1$. The multi-user communications channel obeys a Rayleigh fading model, i.e., the entries of $\mathbf{H}$ are i.i.d. standard complex normal random variables, and the channel output at each user is corrupted with an AWGN of variance $\sigma^2 = 1$.

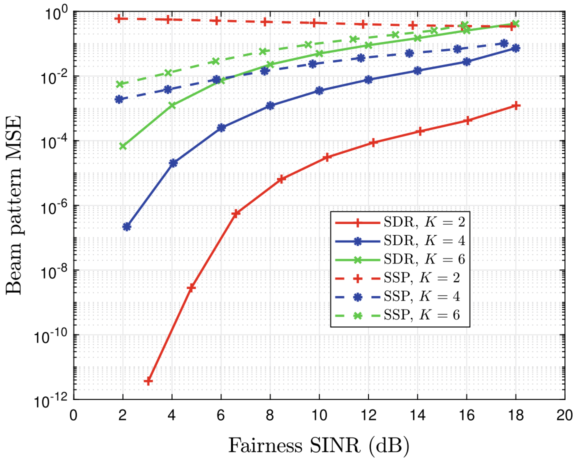


Fig. 3 Beam pattern MSE versus SINR threshold Γ

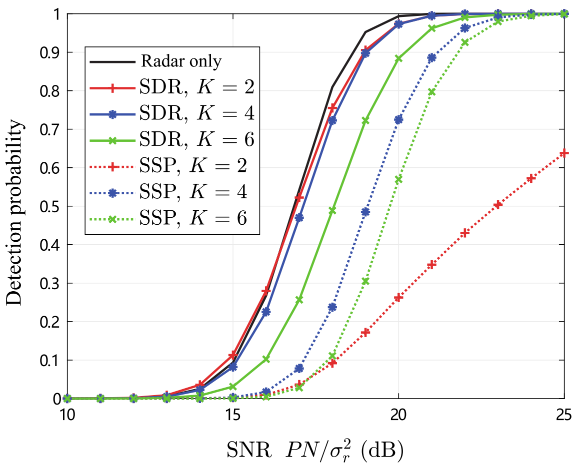
We simulate different Γ and K to test the impact of these parameters on the performance of the proposed joint design methods. We vary SINR threshold values Γ from 4dB to 24dB, and set the number of users as $K = 2, 4, 6$. For each value of Γ and K, the performance is evaluated by averaging over 1000 Monte Carlo tests. The individual radar waveform and communication symbols comprising the transmitted signal $\mathbf{x}[n]$ in (2) are generated as random quadrature-phase-shift-keying modulated sequences, and the transmit signal block size is set to $N = 1024$.

We compare the proposed joint design method with the method proposed in [17], in which only communication symbols are precoded. We use the MATLAB CVX toolbox [9, 10] to solve the optimization in (34), and apply gradient projection method to solve the sum-square penalty (SSP) problem which only precodes communication symbols under per-antenna constraint in [17]. In the sum-square penalty problem in [17], the weighing factors are $\rho_1 = 1$, $\rho_2 = 2$ and the given SINR at each user is equal to the SINR threshold Γ.

Figure 3 demonstrates the performance trade-off between radar and communications for the two methods, for $K = 2, 4, 6$. As clearly demonstrated in Fig. 3, the proposed SDR based method notably outperforms the SSP approach for $K = 2$. Since the SSP approach only precodes communication symbols, it cannot provide enough DoFs for MIMO radar transmit beamforming, and the obtained beam pattern MSE is significant. When $K = 4, 6$, our SDR based method still outperforms the SSP approach, although the gain is less notable compared to $K = 2$. The fact that the SDR method outperforms the SSP method in [17] even when the latter is capable of exploiting the full MIMO radar DoF stems from the following reasons: (1) The SSP problem is non-convex and the obtained solution may be a local optimum; (2) In the SSP problem, the radar loss function, defined as $\|\mathbf{R} - \mathbf{R}_o\|_F^2$, does not directly reflect the performance of the radiation beam pattern.

Since the beam pattern MSE is not the only performance measure for radar, we also analyze the sensing capabilities at the radar receiver. We simulate three radar

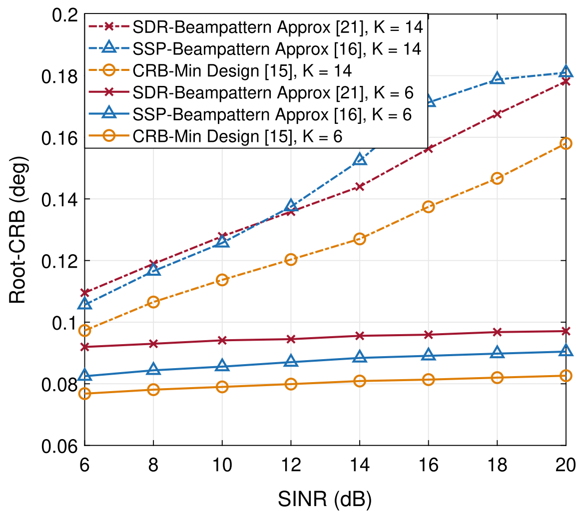
Fig. 4 Detection probability versus transmit SNR under false alarm probability $P_{fa} = 10^{-4}$, for $\Gamma = 12$ dB



targets located at directions $\overline{\theta}_1 = -40°$, $\overline{\theta}_2 = 0°$, and $\overline{\theta}_3 = 40°$, respectively. These targets are in the same range resolution bin and the complex amplitude of the targets are all 1. The targets' reflected signal is corrupted with AWGN whose covariance is $\mathbf{R}_r = \sigma_r^2 \mathbf{I}_M$, where $\sigma_r^2 = 1$. The numerically evaluated detection probability versus transmit SNR for the proposed SDR method, the SSP method in [17] and the radar-only case is depicted in Fig. 4, for $\Gamma = 12$ dB and $P_{fa} = 10^{-4}$. From [17], it is noted that there exists detection performance loss for simultaneous multiuser information transmission compared to the radar-only case. If $K = 2$, the detection performance of SDR beamforming notably outperforms that of the the SSP approach, because the SSP approach usually cannot provide enough DoF to form three beams to cover the three target. Hence, reflected signal from one of the targets may experience notable SNR loss, significantly reducing the detection probability. If $K = 4, 6$, the detection performance of SDR method is still better than the SSP method, since the SDR method can achieve better transmit beam patterns and higher beam gain, leading to higher SNR at the radar receiver.

We further show the radar-communication tradeoff performance between the single-target angle estimation CRB (22) and per-user SINR in Fig. 5, where the SDR and SSP based beampattern approximation approaches [17, 22] serve as benchmark techniques, with $M = 16$. It can be observed that the CRB minimization method outperforms the beamforming approximation designs in general, which improves the target estimation performance under a given SINR constraint for communication users. Moreover, we observe that for a smaller number of users, the CRB remains at a low level despite that the users' SINR is growing.

Fig. 5 CRB versus transmit SINR per user for $M = 16$, $K = 6, 14$



5.2 *Communication Performance Under Given Transmit Covariance for Radar*

In this experiment, the optimal covariance $\mathbf{R}_o$ for radar is given by $\mathbf{R}_o = P\mathbf{S}_o$, where P is the transmit power and $\mathbf{S}_o$ is the power normalized covariance. Two design goals for MIMO radar are considered, resulting in two different $\mathbf{S}_o$. In the first, the radar transmits orthogonal waveforms and forms an omni-directional beam pattern with $\mathbf{S}_o = (1/M)\mathbf{I}_M$. In the second, one follows the beam pattern matching design in the last experiment to form multiple beams towards $-40°, 0°, 40°$ with a beam width of $10°$, where $\mathbf{S}_o$ is obtained by a semidefinite quadratic optimization.

To show the communication performance under the covariance constraint, we solve the SINR balancing problem in (36) with different $\mathbf{S}_o$ and P. The balanced SINR versus transmit SNR for different $\mathbf{S}_o$ and K is displayed in Fig. 6. From Fig. 6, it is observed that the balanced SINR increases with transmit SNR, but the increment becomes slow when the SNR is high enough. The reason is that the interference to the down-link users cannot be effectively canceled via transmit beamforming under the transmit covariance constraint. To zero-force the interference, transmit beamforming requires $\mathbf{S}_h = \mathbf{H}\mathbf{S}_o\mathbf{H}^H$ to be a diagonal matrix, while this condition generally does not hold if $\mathbf{H}$ is Rayleigh fading. Despite this limitation from the covariance constraint, the communication performance can actually be further enhanced via multiuser interference elimination techniques such as dirty paper coding [5]; see [19].

When there are only a few users, the inter-user interference can be less serious, and an acceptable balanced SINR can be achieved under high SNR. With the increase of K, the balanced SINR may become very low, e.g. $K = 6$ for multi-beam patterns. Note that the rank of $\mathbf{S}_o$ is 4 for the multi-beam pattern. When K exceeds the rank,

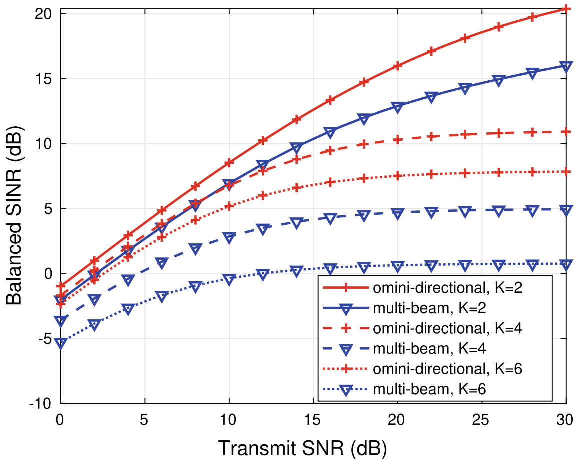


Fig. 6 Balanced SINR versus transmit SNR

the received signals at the users become linearly dependent, making it impractical to send independent data streams to the users. In other words, the meaningful operating regime for multiuser transmit beamforming is $K \leq \text{rank}(\mathbf{R}_o)$. For Rayleigh fading channels, the balanced SINR for the omni-directional pattern is better than that for multi-beam patterns, since its transmit covariance has a higher rank, and thus can provide more DoFs for communications.

5.3 Sum-rate Capacity Versus Covariance Mismatch for Radar

In this section, we numerically show the trade-off between the sum-rate capacity of multi-user communications and the covariance mismatch for radar. In the simulations, the number of antennas is $M = 10$, and the number of users is $K = 4$. The communication performance is compared under three different $\mathbf{R}_o$. The first is $\mathbf{R}_o = (P/M)\mathbf{I}_M$, which forms omni-directional radar transmit beam pattern. The second is for a phased-array radar that forms a transmit beam towards 0°, and thus $\mathbf{R}_o = (P/M)\mathbf{1}\mathbf{1}^T$. The third employs the beam pattern matching design to form multiple beams towards $-40°, 0°, 40°$ with a beam width of 10°.

The average sum-rate capacity under Rayleigh fading channel is compared for different ϵ and $\mathbf{R}_o$ in Fig. 7. As shown in Fig. 7, the sum-rate capacity becomes higher as ϵ becomes larger. When ϵ becomes larger, the restriction from radar is more relaxed, leading to better communication performance. By comparing the results for different $\mathbf{R}_o$, it is observed that the desired radar transmit beam pattern affects the communication performance. The sum-rate capacity is the highest for omni-directional radar transmit beam pattern, and is the lowest for phased-array single transmit beam. When radar intends to form a single or a few main transmit beams,

Fig. 7 Ergodic sum-rate capacity versus transmit SNR P/σ^2 for different ϵ and $\mathbf{R}_o$

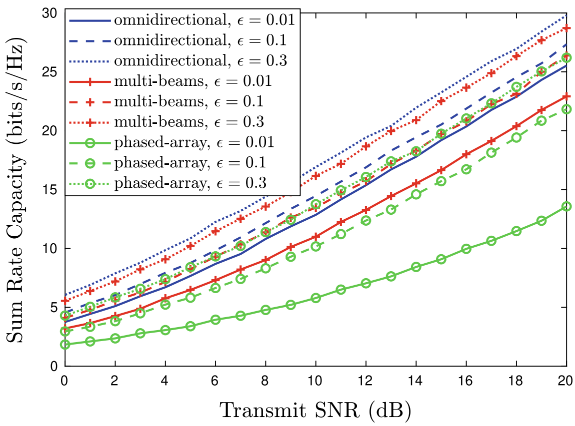
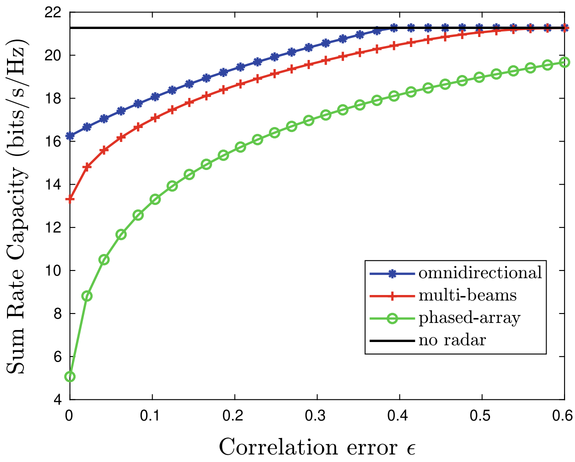


Fig. 8 Ergodic sum-rate capacity versus ϵ for different $\mathbf{R}_o$, where $P/\sigma^2 = 13$dB



the emission power towards the side-lobe region can be quite low. Therefore, if the communication receiver is located in the side-lobe region, the performance degradation of communication can be serious because of the limited signal power. When radar forms a omni-directional transmit beam, the communication receivers can always be covered so the highest sum-rate capacity is achieved.

The trade-off curve between radar and communication is plotted in Fig. 8, where the transmit SNR is 13dB. Similarly, the sum-rate capacity is the highest for omni-directional radar transmit beam, and is the lowest for phased-array single transmit beam. The performance gap between omni-directional beam and phase-array beam can be significant, especially when ϵ is small, while the performance gap betweem omni-directional beam and multiple main beams is not that significant. When ϵ is large enough, the capacity in the joint system approaches the capacity with only power constraint and without the radar constraint in (23).

6 Conclusions and Future Directions

In this chapter, we considered the joint waveform design for MIMO radar and multiuser communications. First, we reviewed a new signaling scheme which jointly precodes the radar waveform and communication symbols. Compared with previous methods which only transmit communication symbols, the proposed scheme provides more DoF for MIMO radar and achieves better performance. Under the joint precoding scheme, we discussed the design of the precoding matrices for radar and communications. We showed the expressions of receive SINRs for multiuser communications and demonstrated that the performance of MIMO radar is determined by the transmit covariance. The solution of two kinds of joint optimizations were introduced in detail, where the first optimizes the radar performance under individual SINR constraints at the communication receivers and the second performs SINR balancing for multiuser communications with a given optimal transmit covariance for radar. Both of the optimizations can be effectively reformulated and solved by convex optimizations. Next, the computation of the sum-rate capacity for multiuser communications under certain constraints on the transmit covariance of radar was discussed, including a toy example study for the single-user and single-target scene. Finally, numerical results were provided demonstrating the performance of joint radar and communications.

The works reviewed here rely on some assumptions on the channel. First, the channel is assumed perfectly known at the transmitter. However, in practical applications this assumption may not hold, e.g. when sthere exists channel estimation error. In the future, it is meaningful to consider the precoder design in the case that the channel is only partially known at the transmitter, for example under some prior knowledge on the difference between the actual channel and known channel at transmitter. Second, the channel is assumed to be non-frequency selective. However, in wide-band communications, the channel may be frequency selective and MIMO communication transmitters generally transmit OFDM waveforms to eliminate the effects of multipath. OFDM waveform design for joint radar and communications under frequency selective channels is also an interesting avenue for future research.

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