Phase Retrieval of Vortices in Bose-Einstein Condensates

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Abstract: We propose a measurement scheme enabling reconstruction of the amplitude and phase of wavefunctions in Bose-Einstein condensates from their momentum power-spectrum. Our method reduces ambiguities and allows the reconstruction of arrays of vortices. © 2021 The Author(s)
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Bose-Einstein condensation (BEC) is a quantum state of matter where particles are trapped and cooled until they form a macroscopic population in a single quantum wavefunction. Measurements of such states are performed by imaging the particles *in-situ* or in the "far-field" by opening the trap and recording time-of-flight (TOF) images. In these measurements, one records the atoms' density (similar to the intensity of optical beams). Alas, as this is a pure quantum state, it also has a well-defined phase structure [1], which cannot be resolved by these measurements alone. Thus, to recover the phase structure of the BEC wavefunction, interference measurements have been suggested, which poses a major experimental challenge. This raises a natural question: Can the phase structure be recovered without atomic interference? As we explain below, this question is related to the well-known phase retrieval problem from optics, but with some important differences.

Traditionally, the phase retrieval problem is defined as the recovery of an object, amplitude and phase, from the magnitude of its Fourier transform [2]. This problem arises naturally in optical imaging as the far-field of an object (or the field at the focal plane of a lens) is proportional to the Fourier transform of the object that one wishes to image. As detectors only measure the field's intensity, the information measured is the magnitude of the Fourier transform, losing the phase information. In recent years, the phase retrieval problem has been studied extensively and several measurement schemes have been proposed that allow recovery of the phase from generalized Fourier measurements [3-6].

Here, we propose and demonstrate in numerical simulations a simple scheme for the complete characterization of a Bose-Einstein condensation wavefunction by applying a phase retrieval algorithm on TOF measurements. Our measurement is based on a simple variation to TOF measurements and does not require any kind of interference. Since the BEC's evolution is nonlinear, described by the Gross-Pitaevskii equation (GPE), our methodology is based on nonlinear dynamical evolution rather than the simple Fourier transform. Our proposed method can resolve ambiguities and facilitates the recovery of single vortices and vortex arrays, including flow directionality, which are highly difficult tasks that have not been demonstrated without atomic interference.

Common phase retrieval algorithms [7] are iterative and work in the following manner. One starts by drawing a random initial wavefunction guess and propagate it by Fourier transform to the far-field, where the magnitude is replaced with the measured magnitude. The field is then propagated back (inverse Fourier transform) to the object plane where constraints are imposed. Most commonly, the constraints are the support (i.e., knowledge about the region within which the wavefunction is contained). The algorithm continues in this fashion until a suitable stopping criterion is met. More recently, optimization approaches have been proposed for recovery from both Fourier and generalized phase recovery problems [8].

Unlike the classical phase retrieval, in the BEC measurement the propagation back and forth is no longer linear and does not correspond to a Fourier transform. Instead, the evolution is governed by the semi-classical approximation of the GPE [9], $i\hbar\partial_t\psi(r,t) = (-\hbar^2/2m\,\nabla^2 + V(r) + U_0|\psi(r,t)|^2)\psi(r,t)$. Here, $V(r)$ is the confining potential in the trap and $U_0 > 0$ is the nonlinear coefficient representing repulsive interactions. Thus, we modify the algorithm by replacing the Fourier propagation with GPE-based propagation. This is done by numerically solving the GPE for each iteration backward and forward, while the rest of the algorithm steps remain unchanged. As in-situ atom-density measurements are available for the BEC (sometimes at low resolution), we impose a magnitude constraint instead of support constraint in the object plane.

While the concept of transferring the phase retrieval problem to nonlinear propagation has been proposed for optics [10] and also for BEC TOF measurements [11], these methods cannot reconstruct complex phase structures such as vortices [12,13]. Vortices imprinted on BEC's quantum wavefunctions are extremely interesting: they inherently possess a quantum number (the topological charge). They are sometimes naturally occurring when they indicate phase-transitions (e.g., the Kosterlitz-Thouless phase transition; Physics Nobel 2016), and they carry quantum information. Hence, the recovery of vortices in BECs is crucial and has many far-reaching implications. However, all methods proposed thus far, with either linear or nonlinear evolution, are not suitable for reconstructing vortex states since the topological charge is conserved in propagation; therefore the charge will manifest in the phase of the "far-field". **Hence, the magnitude measurement does not encode sufficient information on the order and sign of the charge.** In other words, vortices can manifest non-trivial ambiguities in the measurement, and the traditional phase-retrieval methodology is not able to reconstruct them, even when it is adapted to nonlinear evolution. To overcome this problem, we propose a simple technique to augment the conventional TOF measurement and demonstrate (in simulations) the complete recovery of arrays of vortices.

We achieve this feat by a simple and easily implemented adaptation to the TOF measurement. Instead of opening the BEC trap in the x-y axes simultaneously, we open the trap in succession: one axis first and after sufficient evolution, the second axis. In Fig 1. A-D We show a comparison between TOF measurements of a vortex state of order $m=\pm 1$ with ordinary TOF and our augmented approach.

For simplicity, in our simulations we assume that the BEC is generated in a potential well of radius $1[a.u]$, barrier energy $10[a.u]$, and $U_0 \in [0,100][a.u]$. To test our reconstruction, we simulate a 3×3 lattice of vortices, each of order 1, but with random signs and random relative phases. We reconstruct these states for various nonlinear coefficients (determining the strength of the nonlinear evolution) and for choices of initial guesses. We show a typical reconstruction in Fig. 2 for $U_0 = 0$, along with an ambiguous reconstruction given by regular TOF measurements.

An immediate question that emerges from the augmented TOF is what the optimal propagation time under the semi-open trap is? For the case of a single vortex in the linear regime of GPE, we analytically solve and show that the norm between the measured magnitude of a $m=1$ and $m=-1$ vortex is approximately proportional to $\left\| |\psi_1|^2 - |\psi_{-1}|^2 \right\| \propto \frac{1}{\sqrt{T}} \left| \sin\left(\Delta ET / \hbar \right) \right|$ where T is the propagation time under the semi-open trap and ΔE is the energy difference between the first and second mode of the trap. We validate this result numerically, as shown in Fig. 1E. Indeed, when we set $\Delta ET/\hbar = \pi/2$ we can reconstruct the states in Fig. 2.

To conclude, we presented a simple measurement scheme for Bose-Einstein Condensation, allowing complete recovery of the amplitude and phase of the state. Importantly, by breaking the xy symmetry in propagation we are able to reconstruct states containing vortices.

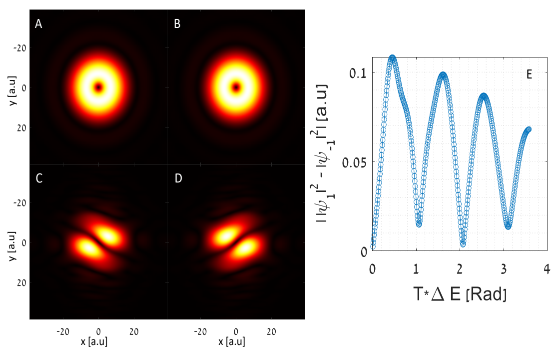

Fig 1. Top row: TOF images of (A) vortex state $m=1$ and (B) $m=-1$. Bottom row, augmented TOF for the same states (C) $m=1$, (D) $m=-1$. (E) Norm difference between augmented TOF measurements of vortex states $m=1$ and $m=-1$ as a function of propagation time T. Time axis is scaled by the energy difference between the second and first modes of the trap.

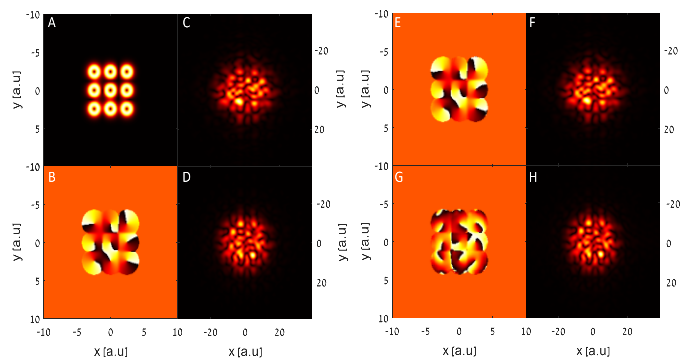

Fig 2. (A) Amplitude of wavefunction for reconstruction and (B) its phase, (C) is the augmented TOF and (D) is the original TOF. In (E) we can see phase reconstruction with the augmented TOF and its corresponding augmented TOF (F). In (G) an erroneous phase reconstruction from the original TOF and its corresponding original TOF (H), giving the same measurement but wrong object phase.

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