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LETTER

Numerical Evaluation on Sub-Nyquist Spectrum Reconstruction Methods

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1 Introduction

As wireless technology continues to expand, there is a growing concern about the efficient use of spectrum resources. Even though a significant portion of the spectrum is allocated to licensed primary users (PUs), studies indicate that their actual utilization is often limited to between 5 to 10 percent [1]. The underutilization of spectrum has given rise to Cognitive radio (CR) technology, which allows secondary users (SUs) to opportunistically access these underused resources [2]. However, wideband spectrum sensing, the key of CR, is limited by the need for high-speed ADCs, which are costly and power-hungry.

Compressed spectrum sensing (CSS) addresses this challenge by employing sub-Nyquist rate sampling. The efficiency of active transmission detection heavily depends on the quality of spectrum reconstruction. There are various reconstruction methods in CSS, each with its merits and drawbacks. Still, existing algorithms have not tapped into the full potential of sub-sampling sequences, and their performance drops in noisy environments [3, 4].

The *GHz Bandwidth Sensing* (GBSense) project [1)], introduces an innovative approach for GHz bandwidth sensing. GBSense incorporates advanced sub-Nyquist sampling methods and is compatible with low-power devices. This project also prompted the GBSense Challenge 2021, which centered on sub-Nyquist reconstruction algorithms, with four leading algorithms to be presented and evaluated in this paper.

2 Signal Models and Techniques

This paper focuses on multiband signals, which comprise multiple distinct frequency bands within a specified baseband range. At the receiver, a combined signal contaminated with noise is observed without knowing the spectrum support.

To efficiently sample these signals below their Nyquist rate, a *multicoset sampling* architecture is employed, whose basic diagram is shown in Figure 1. This method divides the signal into *cosets*, undergoing different and nonuniform time delays before being digitized by ADCs running at a speed far lower than the Nyquist rate. The core challenge is faithfully reconstructing the original signal from the sub-Nyquist samples, formulated as

$$\arg\min \ \|\mathbf{X}\|_{2,0} \ \text{ s.t. } \ \|\mathbf{Y} - \mathbf{A}\mathbf{X}\|_{\mathrm{F}} < \epsilon, \tag{1}$$

where $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{A}$ are the reconstructed spectrum, sub-Nyquist measurements, and the sensing matrix, respectively.

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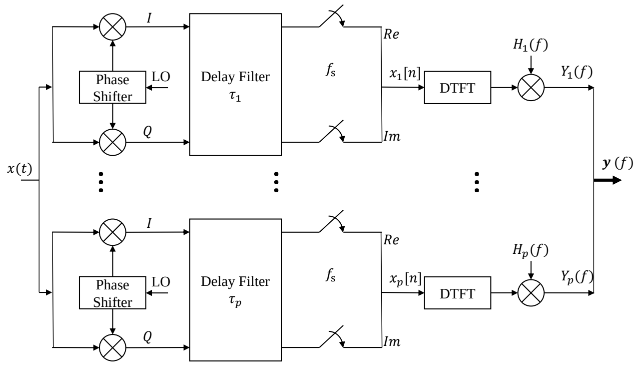

Fig. 1 The basic implementation diagram of the multicoset sampler.

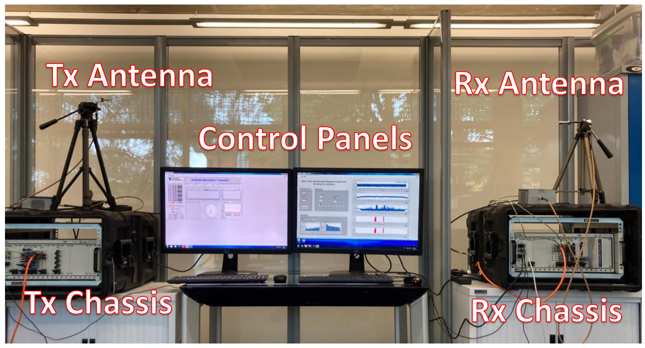

Fig. 2 The GBSense data collection and CSS algorithm testing platform built on the NI millimeter-wave transceiver system.

The notation ϵ stands for a small threshold of error magnitude. Further details are elaborated in the support document, Section 1.

3 Algorithms

When reconstructing the spectrum from sub-Nyquist samples, practitioners typically turn to four principal algorithms, each offering unique advantages and challenges in spectrum sensing applications.

Convex Optimization: This tackles the ℓ_0 optimization challenge by converting it to more manageable problems. The basis pursuit (BP) and basis pursuit denoising (BPDN) methods are notable techniques, with the former optimally decomposing signals and the latter balancing between residuals and ℓ_1-norm simplicity. The focal underdetermined system solver (FOCUSS) algorithm, albeit slower, stands out for accuracy.

Greedy Algorithms: Common in real-time systems for their speed, these algorithms, like matching pursuit (MP), and orthogonal matching pursuit (OMP), iteratively reconstruct signals based on atomic dictionaries. However, they sometimes trade precision for speed. More advanced versions of those matching-pursuit-based algorithms, such as simultaneous orthogonal matching pursuit (SOMP) and joint-block hard thresholding pursuit (JB-HTP), further refine this process for multiple measurement vectors (MMV) problems, but they still often fall short of the reconstruction quality offered by convex optimization methods.

Bayesian Framework: The Sparse Bayesian Learning (SBL) algorithm excels in creating sparse representations, especially beneficial for signals with time correlations. Multi-signal sparse Bayesian Learning (MSBL) improves upon SBL by jointly recovering multiple signals with shared sparsity. In CSS scenarios, MSBL outperforms its counterpart thanks to its collaborative signal recovery.

Non-sparse Approximation: Centered on non-sparse signals, this approach leans on the statistical traits of stationary signals for power spectrum reconstruction. Yen's pioneering research elucidated noise-free reconstruction using the multicoset sampler. An innovative technique, the fast compressed power spectrum estimation (FCPSE), simplifies the estimation process, producing an unbiased representation of the power spectrum while requiring less computational power than its iterative peers.

Four selected algorithms, namely the "slow kill" (SK) method [5], subspace-augmented simultaneous orthogonal matching pursuit (SA-SOMP) [6], MSBL [7], and fast compressed power spectrum estimation (FCPSE) [8]. For detailed mechanisms of these algorithms, refer to the supplementary documentation, Section 2.

4 Testbed Setup

The GBSense Challenge 2021 and the GBSense project provide an experimental platform specifically designed for testing algorithms and producing datasets [9]. This setup includes a transmitter (Tx) and a receiver (Rx), each equipped with a millimeter wave (mmWave) front end and an SDR platform, emphasizing the software-defined radio (SDR) components. A photo of the testbed is shown in Figure 2.

The Tx emulates primary user signals by modulating random sequences. Multiple orthogonal frequency division multiplexing (OFDM) signals are then formed across channels within a 2 GHz baseband and transmitted via the mmWave. The Rx focuses on sub-Nyquist sampling to reconstruct the baseband spectrum. After capturing Nyquist samples via an ADC at 3.072 GSps, multicoset sampling processes these samples through downsampling and parallel fast Fourier transform (FFT). Embedded algorithms tackle the spectrum reconstruction. Given uncertain channel occupation, a method based on Bayesian information criterion estimates the occupation rate of the spectrum.

For performance assessment, the FFT of the Nyquist samples serves as a benchmark, with real transmissions differentiated from noise by energy detection. Recovery results are then compared, and detection probability is calculated.

The system operates on two *National Instruments* PXI setups, each modularly designed with a controller, frequency module, and FPGA. The Tx module produces transmissions,

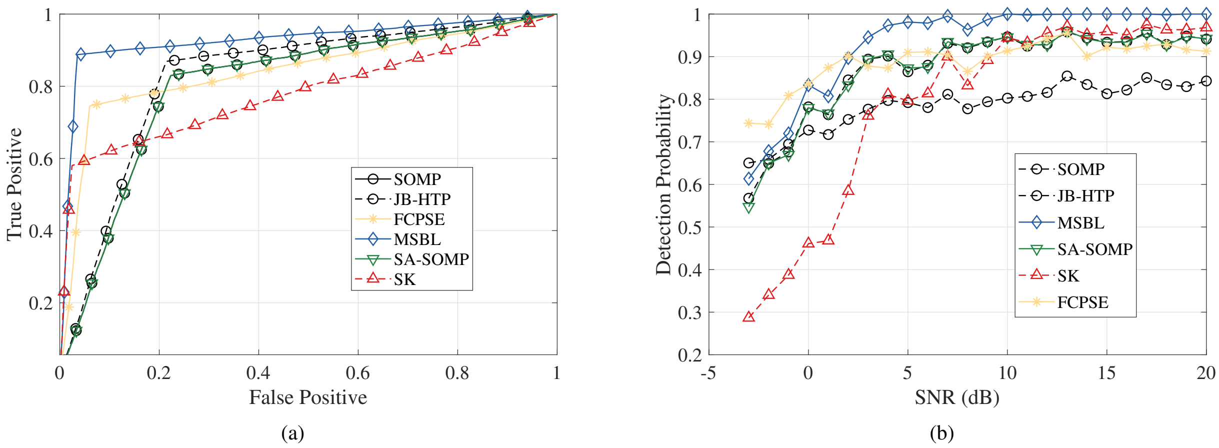

Fig. 3 (a) ROC curves of the associated algorithms at sparsity level $k = 6$. (b) Detection probability as a function of SNR for the associated algorithms.

and the Rx digitizes signals using a dual-channel ADC. Both units connect to 28GHz mmWave radio heads and horn antennas.

5 Numerical Results

Four selected algorithms, together with two benchmark algorithms are tested on the testbed. Experiments were conducted using a 16-channel multicoset sampler, wherein data was sampled at a rate equivalent to 1/40th of the conventional Nyquist rate. The efficacy of the reconstruction methodologies was assessed via the mean area under the curve (AUC) of the receiver operating characteristic (ROC) curve, as visualized in Figure 3(a). It is imperative to note that the SBL technique demonstrated commendable efficacy for a sparsity level $k = 6$.

For a more granulated understanding, detection probabilities at diverse signal-to-noise ratio (SNR) levels were analyzed, as delineated in Figure 3(b). Among the contenders, the MSBL algorithm stood out, delivering unparalleled performance across the evaluated datasets.

Performance evaluations further spanned multiple parameters. While the MSBL algorithm consistently outperformed across diverse SNR conditions and maintained its performance supremacy across varying signal sparsity levels, time complexity emerged as an essential consideration. Detailed insights regarding the average computational times for the algorithms in a MATLAB environment, alongside the implications of modulating the sampling window lengths, can be found in the associated figures and tables within the supplementary documentation, Section 3.

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