Super-resolution and reconstruction of sparse images carried by incoherent light

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We demonstrate theoretically and experimentally the reconstruction of images borne on incoherent light at a resolution greatly exceeding the finest resolution defined by the NA of the system. Our method relies on compressed sensing techniques, which assume that the object is sparse in a known basis, and only that. The approach is robust against noise and can be used for reconstructing subwavelength images through measurements taken in the optical far field. © 2010 Optical Society of America

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A fundamental concept in microscopy is the Abbe diffraction limit, setting the highest resolution of an imaging system to one half of the optical wavelength λ, implying that subwavelength information suffers from inevitable loss due to exponentially decaying evanescent waves [1]. Modern methods overcome this limit by point-by-point scanning with a subwavelength tip [2,3] or a light source [4], or by randomly distributing fluorescent molecules on the sample and averaging over multiple exposures [5,6]. These solutions require scanning or repetitive experiments, thus limiting real-time applications. Other methods rely on negative-index materials [7–12]), but their current technology involves high loss and fabrication at nanometer precision. In addition, several approaches for algorithmic recovery of subwavelength features were proposed [13–15]. However, these algorithms are highly sensitive to noise in the measured data and to the assumptions made on the prior knowledge (see [1], chapter on the super-resolution). Consequently, the diffraction limit is still a practical barrier, especially for real-time imaging [1].

Recently, we proposed a new approach for reconstructing subwavelength images, assuming prior knowledge that the image is sparse (in real space) [16]. This assumption can be utilized for the super-resolution, by using modern information processing techniques known as compressed sensing (CS) [17,18]. These methods assume only that the image is sparse in a known basis, i.e., under a suitable basis transform, the image comprises of only a few nonzero values. Under rather general conditions, these methods can improve the resolution way beyond the numerical aperture of the system (or beyond the diffraction limit, if the aperture is infinite). Ideally, the improvement can be by up to a factor of $1/(2\beta)$, with β being the fraction of basis functions occupied by the original image, i.e., the smallest recoverable feature can be smaller than the diffraction limit by $1/(2\beta)$. We demonstrated this idea theoretically, along with an experimental proof of concept, with coherent light [16]. However, many imaging systems employ incoherent illumination; hence it is important to modify the technique to work with incoherent light.

Here, we demonstrate the super-resolution via CS for images borne on quasi-monochromatic spatially incoherent light, with the only prior knowledge being that the image is sparse (in real space). We discuss the possibility to extend this idea incoherent subwavelength imaging and for white light.

When the light is fully spatially incoherent, the optical intensity in the imaging plane (output plane) is the convolution of the intensity at the object plane (input plane) with the absolute value of the incoherent point spread function (PSF) [1],

$$u_{\mathrm{out}}(x,y) = u_{\mathrm{in}} \otimes h_{\mathrm{ic}}(x,y), \tag{1}$$

where $h_{\mathrm{ic}}(x,y)$ is the PSF of the system, $u_{\mathrm{in}}(x,y)$ is the object intensity, and $u_{\mathrm{out}}(x,y)$ is the image intensity. In the spatial-frequency domain, this relation yields $U_{\mathrm{out}}(\nu_x,\nu_y)=U_{\mathrm{in}}(\nu_x,\nu_y)H_{\mathrm{ic}}(\nu_x,\nu_y)$, where $H_{\mathrm{ic}}(\nu_x,\nu_y)$ is the optical transfer function (OTF). In 2D, the OTF is a cone [Fig. 1(a)], whereas in 1D it is a triangular window [Fig. 1(b)], both acting as low-pass filters with a cutoff frequency ν_c. If the imaging system has an infinite aperture, then $\nu_c=2/\lambda$ [Fig. 1(b)].

In signal processing, the problem is as follows. Given a measured signal smeared by a low-pass filter [say, the triangle in 1D; Fig. 1(b)], we wish to reconstruct the true signal, including its high-frequency information. Since all information carried by frequencies beyond cutoff is lost, there are an infinite number of signals, which—after being smeared by the low-pass filter—will result in the measured image. (E.g., adding any signal containing only frequencies above the cutoff to a candidate signal leaves the measured data unchanged). In order to restore high frequency information, assumptions on the object must be made [15]. A common assumption is that the true signal (object) is limited in space to a known support, in which case iterative methods may be used to restore it. A further assumption that can be made,

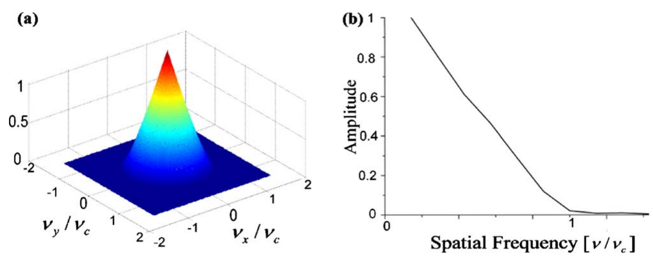

Fig. 1. (Color online) (a) Simulated 2D OTF. (b) Our measured 1D OTF (only positive frequencies shown).

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for images borne upon incoherent illumination, is that the object is nonnegative. These methods offer limited success either for the super-resolution or for subwavelength imaging, mainly due to noise in the measured data. For discussions dealing with the limitations of these techniques, see [1,15,19].

A different approach, which we take here, assumes that the true signal is *sparse in a known basis*—meaning that it comprises of few functions belonging to a known subspace. This assumption is quite general, since many natural images are sparse in some known basis. The simplest examples in optics are images that are sparse in real space, e.g., living cells or bacteria, where the information is only at the contour lines of the various organs, while everywhere else the cells/bacteria are transparent. In the field of signal processing, the sparsity assumption has recently gained widespread use, and many efficient tools for utilizing sparsity have emerged [20–22]. The sparsity assumption can be used to recover signals extending beyond the cutoff of a filter (in our case, the OTF) using the following approach. Among all signals that can be written as a combination of some known basis functions, which yield the measured results after being "smeared" by the known OTF, find the *sparsest one*, i.e., the one comprised of the fewest basis functions. If the true object is sparse enough, then sparsity-based reconstruction methods are bound to find it (exactly—in the noiseless case or up to a bounded error—in the presence of noise) [23,24]. The sparsest solution can be found using simple algorithms [20]. Here, we employ the basis-pursuit denoising method [25], which solves the "sparsest-solution" problem, by relaxing the sparsity constraint. Our consistency requirement is between the low-pass Fourier coefficients of the reconstructed image and the low-pass Fourier coefficients of the measured smeared signal, divided by the OTF (triangular window). The problem we therefore solve is

$$\hat{x} = \arg\min_{x} \|x\|_1 \ \text{subject to} \begin{cases} (\Sigma^T W \Sigma)^{1/2} < \varepsilon, \\ x \geq 0, \end{cases} \quad (2)$$

with $\Sigma = FAx - b$, where x is the sparse vector, F is the partial Fourier matrix (up to the cutoff frequency ν_c), A is a matrix representing the basis in which the object is sparse, b's are the measurements in Fourier domain, after normalization-division by the OTF, and ε is a parameter determined by the noise value. W is a diagonal matrix containing the squared magnitude of the OTF, for increasing frequency values on its diagonal. Its purpose is as follows: When the low-pass filter is a sharp window (as in [16]), the measured data at frequencies beyond ν_c contain only noise and can be set to zero. However, the OTF is a gradual filter, yielding small signal values at frequencies slightly below ν_c, where the Fourier coefficients are divided by small values (hence noise is amplified greatly) near ν_c. The weight matrix W allows greater errors near ν_c, through a weighted basis-pursuit method. The reconstructed object is then $y_{\text{rec}} = A\hat{x}$.

We first demonstrate the idea theoretically, on a 1D optical image containing delicate features borne on incoherent light. Figure 2 shows an example: the input image [Fig. 2(a)] is a set of shifted rectangles, containing spatial frequencies extending way beyond ν_c of the 1D triangular filter [Fig. 1(b)]. The input image is sparse in the shifted-rectangle space ($\beta = 4/100$ in this example). The triangular OTF multiplies the power spectrum and cuts it at ν_c [Fig. 2(d)], which is translated into a smeared image [Fig. 2(c)] where all the delicate features are lost. In addition, noise with energy at 1% of the measured energy is added. Our method yields the reconstruction of the image [Fig. 2(e)] and its power spectrum [Fig. 2(f)], both displaying excellent correspondence to the input image and its spectrum [Figs. 2(a) and 2(b)].

Next, we provide an experimental proof of concept, demonstrating image recovery at a resolution greatly exceeding the finest resolution defined by the OTF of a spatial filter. Our system (Fig. 3) is a 4-F imaging system, with a tunable low-pass spatial filter positioned at the Fourier plane. The light source is a 532 nm laser beam, passed through a rotating diffuser, making the light partially spatially incoherent with a speckle size of $\sim 2-3\ \mu$m, while the feature size in our image is $>100\ \mu$m. Our reconstruction method is designed for taking the data in the Fourier plane, where the data are the least sparse [16]. However, when the low-pass filter is sufficiently narrow such that the smeared data are broad enough, one can

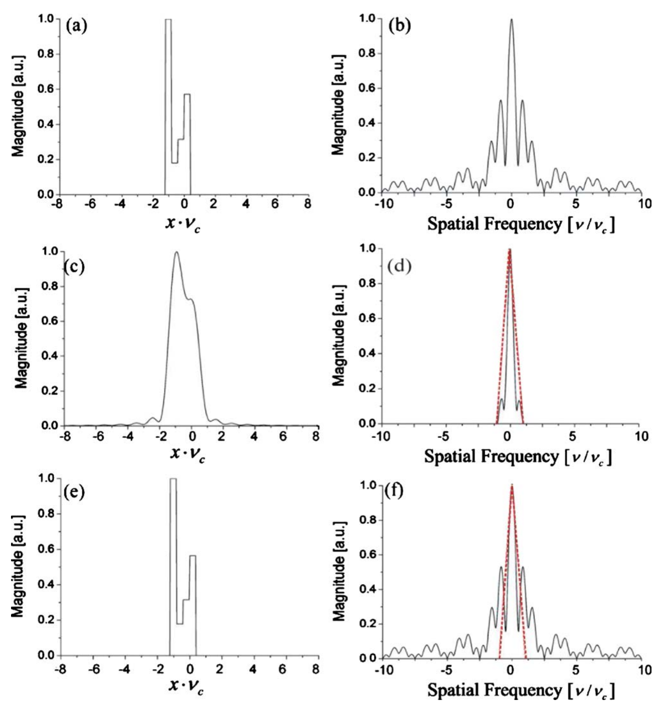

Fig. 2. (Color online) Theoretical example demonstrating reconstruction of a 1D image carried by incoherent light. (a) 1D image comprising of rectangles $2/(5\nu_c)$ wide ($\lambda/5$, if $\nu_c = 2/\lambda$). (b) Spatial power spectrum of (a). (c) Smeared (filtered) image. (d) Spatial power spectrum of the filtered image of (c), with the triangle marking the OTF. (e) CS-reconstructed image. (f) Spatial power spectrum of the reconstructed image. Note the reconstructed frequencies in (f), residing far beyond ν_c. Although showing only the magnitude of the spectrum, the phase of the Fourier transform was also reconstructed well.

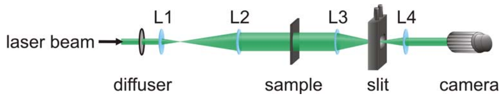

Fig. 3. (Color online) Experimental setup.

take the measurements in the plane of the smeared image [16], which is experimentally more convenient. Accordingly, we measure the smeared image with a CCD camera and utilize the CS technique to recover the original image. For simplicity, we demonstrate here the reconstruction of 1D signals, achieved by averaging the measured data over the vertical axis. The extension to 2D is straightforward—as it simply requires a 2D model for the signal, and F from Eq. (2) to be the 2D partial Fourier matrix. Figure 4 shows the true image [Fig. 4(a)], the measured image after the low-pass filter [Fig. 4(b)], and the CS-reconstructed image [Fig. 4(c)]. The recovered image has excellent correspondence to the original image. This experiment shows a resolution improvement by a factor of ~5, over the resolution limit imposed by the low-pass filter. The only prior information used to reconstruct Fig. 4(c) from Fig. 4(b) is that the object is nonnegative, comprising of few rectangles—i.e., it is sparse in the basis of shifted rectangles.

The ideas demonstrated here call for the discussion of the possibility of reconstructing subwavelength images. Our previous work with coherent light [16] works well in the subwavelength regime (as indeed current experiments in our laboratory prove), because the transfer function for coherent light is exact [1]. However, for spatially incoherent light, Eq. (1) is approximate—assuming that the light in a completely uncorrelated, i.e., the transverse correlation, distance (~speckle size) is much smaller than the feature size in the input image. In the above-wavelength domain, this assumption is easily applicable, because the speckle size can be as small as the Abbe diffraction limit since it arises from interference among radiation (nonevanescent) waves. For sub-wavelength images, the speckle size at the plane of the input image depends on the distance of that plane from the light source. If the subwavelength image is placed within the near-field range of an incoherent light source (e.g., the fluorescent molecules of [5,6]), the evanescent waves make the speckles smaller than λ. In this case, the relation between the width of each stripe in Fig. 4 and the cutoff frequency of the low-pass filter is equivalent to image reconstruction in a system with subwavelength stripes

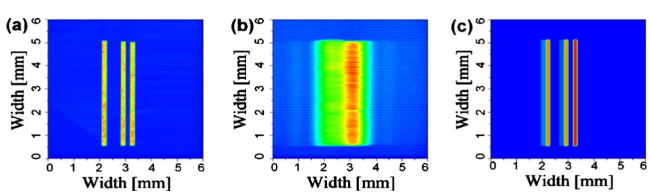

Fig. 4. (Color online) Experiments. (a) Original 2D image of three stripes. (b) Smeared (filtered) image. (c) CS-reconstructed image. The weak "ghost" rectangles in (c) arise from slight mismatch between the locations of the basis functions in the model and the true locations of the stripes; this happens because the blurred image is sampled by the pixels of the camera, leading to quantization error.

with a width of λ/9.5, *way beyond the Abbe diffraction limit*. However, if that distance between the light source and the input subwavelength image is beyond near-field range, the speckle size is diffraction limited, implying that Eq. (1) is no longer valid. In this case, the OTF should be replaced with the expression for the intermediate case of partially spatially incoherent light, accounting for partial correlations in the image. The analysis is more complicated, but doable. Clearly, CS techniques can be used for reconstructing subwavelength features also under incoherent illumination. Finally, these ideas can be extended to white light if the spectrum of the light is known. This could facilitate the super-resolution and recovery of subwavelength features in every microscope.

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