ANCIENT AND CLASSIC WORLS

Until the 5-th Century

MECHANICS

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MUSIC

Typical music instruments in the ancient world:

Harp, Lyre, Violin, drum, flute, horn, shofar (horn made of ram's horn).

Sounds produced by musical instruments are created by periodic vibrations in the air. It may be produced by a vibrating stretched chord (string instruments), played by plucking the strings (harp) of with a bow (violin). Membrane (drums) vibrates in response to hitting by fingers or sticks.
Wind instruments induce vibrations of air in a tube by blowing in through a narrow orifice or through reeds. The tube may be straight (flute) or bent (Cornu, Roman military horn)

Harp in Babylon

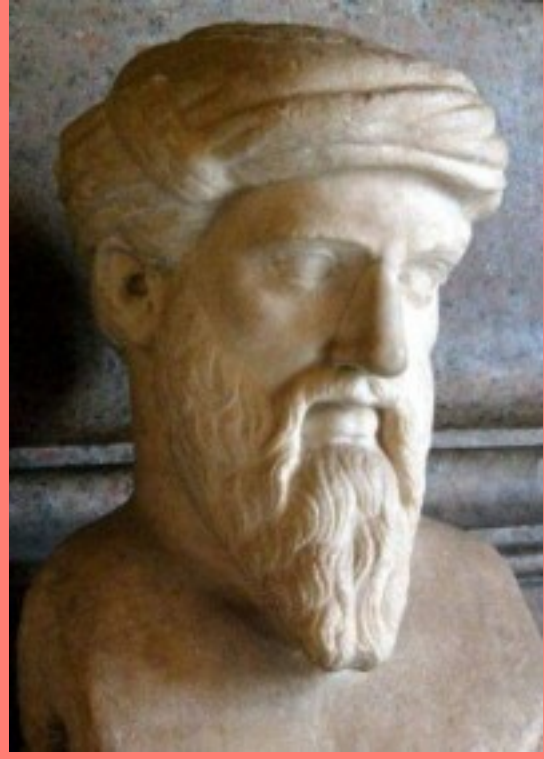
African bow-harp

Roman Cornu

Jewish Shofar

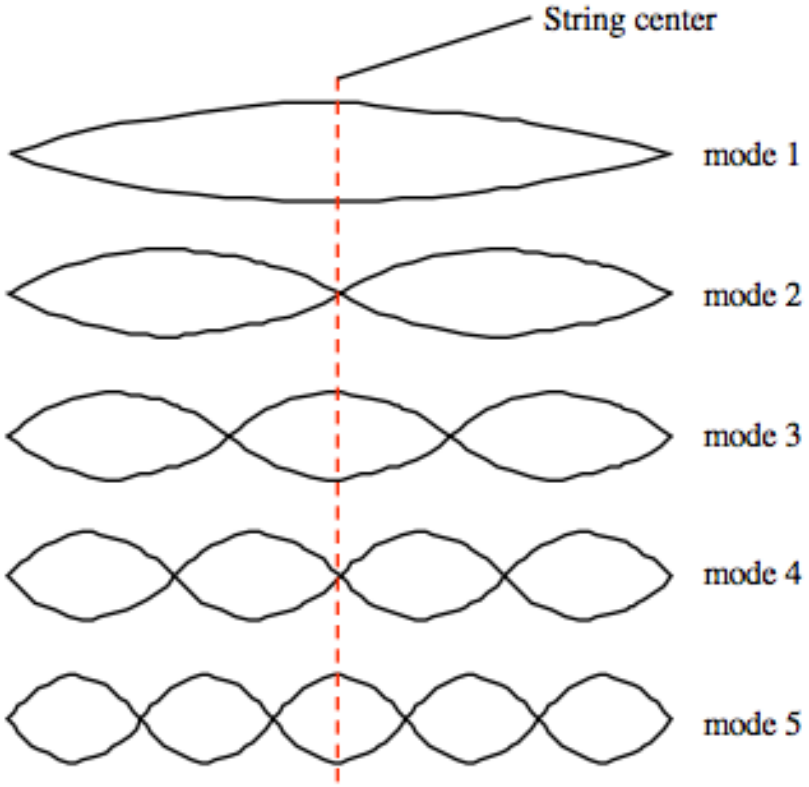
Pythagoras (570-495 לפה"ס)

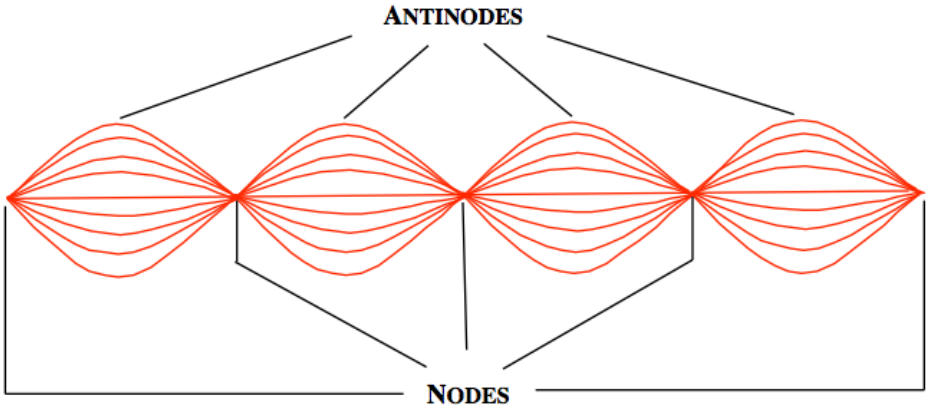
Contributions in mathematics, chemistry and music



String vibration

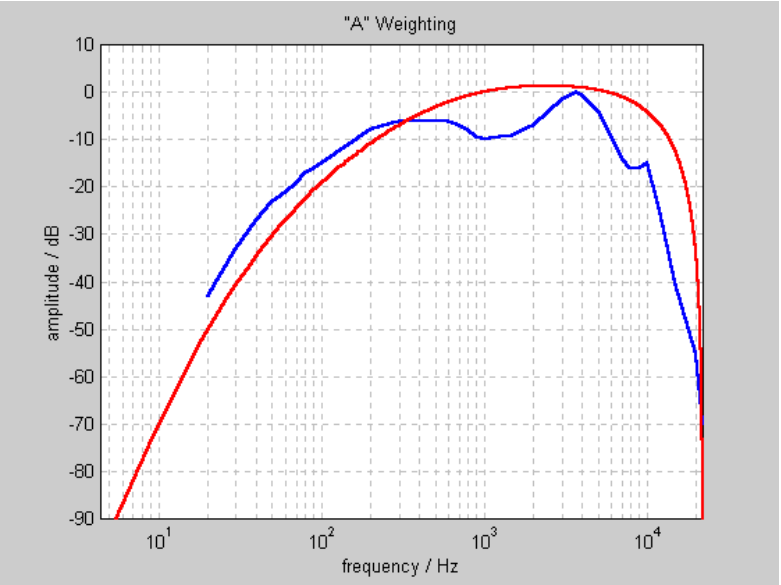
Pythagoras believed in explanation of natural laws by numbers.
A string stretched between two anchoring ends can vibrate at different frequencies, by forming an integer number of equally-spaced “nodes” (wavelength). The vibration frequencies are integer multiples of the basic frequency of one-mode vibration, and are called “harmonies”. All integer multiples of a given frequency are perceived as the same musical note by our ears.
Harmonic notes were for Pythagoras a confirmation to his theory about the role of natural numbers in nature.





INSIGHT: our senses are LOGARITHMIC. We feel similar changes when the physical quantity we feel (sound, pain, light brightness) is doubled or halved, and not when the change is due to addition or subtraction of equal differences.
For this reason sound levels are measured in decibels, the logarithm of the ratio of the sound energy (or amplitude).
Below is a table of the levels of various common sounds.

In addition, the threshold level of sounds we hear depend on the sound frequency: We do not hear very low frequencies, as well as high frequencies. Dogs hear well higher notes than us (dogs whistle).

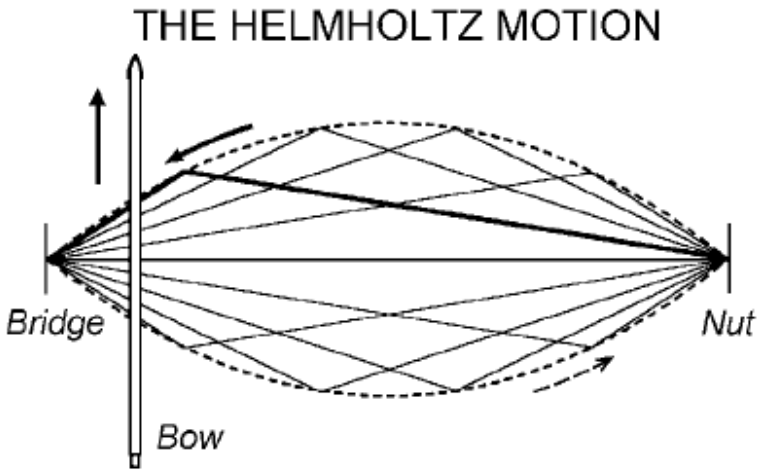


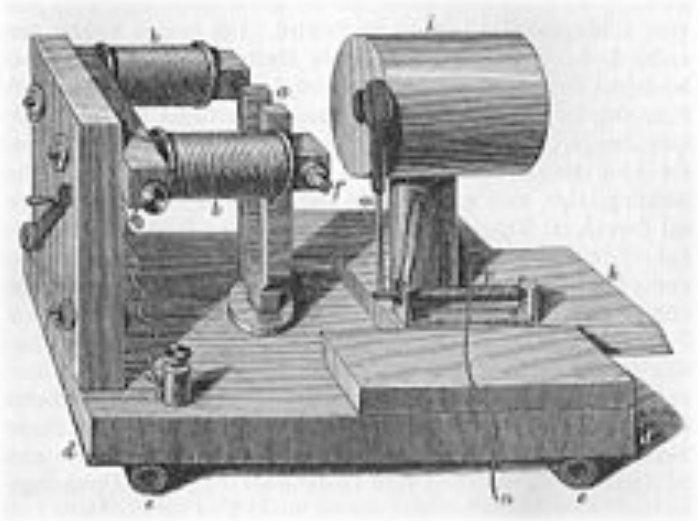
Source of sound	Sound Intensity Level (dB)	Sound Intensity $\left(\frac{W}{m^2}\right)$
Threshold of hearing	0	1×10^{-12}
Breathing	20	1×10^{-10}
Whispering	40	1×10^{-8}
Talking softly	60	1×10^{-6}
Loud conversation	80	1×10^{-4}
Yelling	100	1×10^{-2}
Loud Concert	120	1
Jet takeoff	140	100

Table 1.1 Decibel levels for typical sounds

Ptolemy - following Pythagoras, defined pleasant (harmonious) tone intervals based on the ratio between their frequencies – e.g. Tetrachord: four notes with 1/3 equal interval ratios in between, Octave: two notes one has double frequency of the other.
Kepler – also claimed that the planet radii of cycling around the sun followed integer ratios.
Helmholtz 1821-1894 - was the first modern scientist who experimentally studies musical tones created by strings. He induced vibrations by an electromagnet, and devised a circular bow to continuously set vibrations in strings of different length and tension. He showed that the spectrum of frequencies created were altered by the distance of the bow touching-point from the string anchor, and determined the physical basis for different textures of the same tone (overtones spectrum and tones attack and decay dynamics, see following chapters).

NOTE: **Graham Bell** saw Helmholtz drawing below, but since he did not know German he misinterpreted its application, and used it to construct the telephone hearing speaker.





There is an inverse relation between the string length, L, and its vibrating frequency, F

$$F = c/T$$

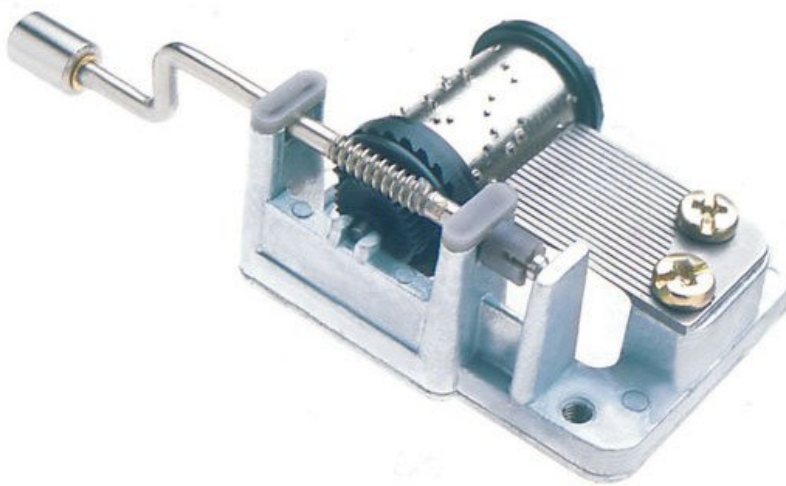
The constant c is proportional to the string tension, or the force of pulling on its ends. The higher the tension is, the higher the vibrating frequency.

WHAT IS THE PURPOSE OF THE SOUND BOX IN A VIOLIN?

The sound box resonates with the string frequency and enhances the sound. The bridge and sound post couple the strings to the front and back of the sound box.

Ancient harps and lyres were often built with a sound box to amplify the musical sounds. Can you explain the shape of the violin sound box ? Compare to the sound box of Mandolins, and the different sound texture.

Hint: violin sound box has different length at various directions, therefore can resonate sounds of different frequencies (=wavelengths).

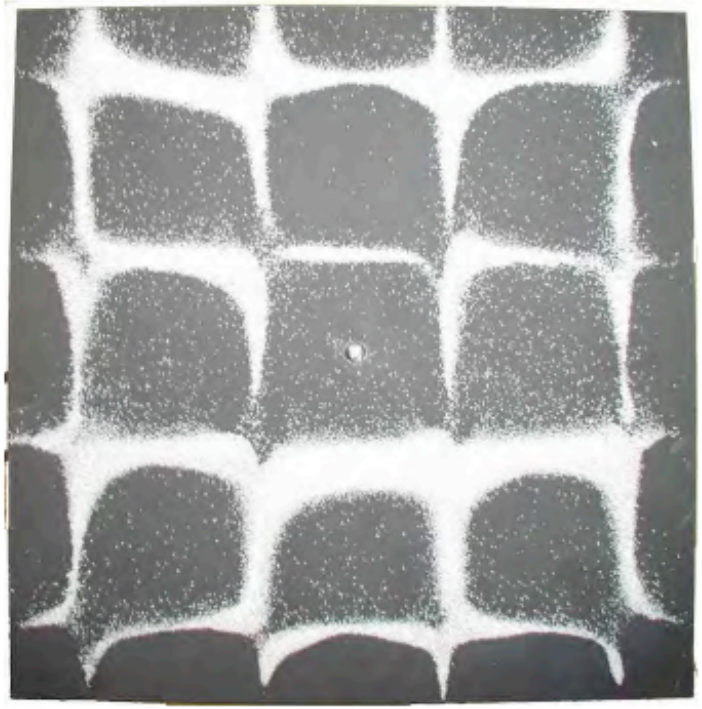


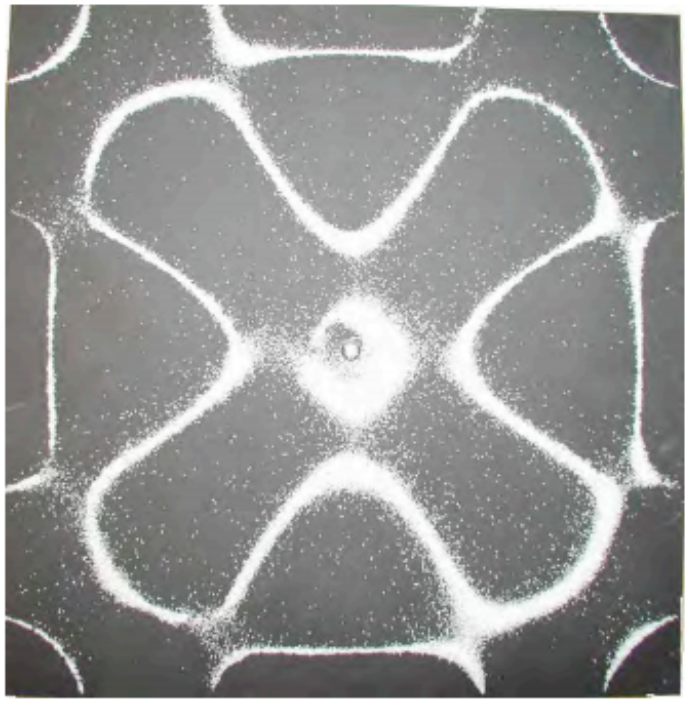
DEMONSTRATION: Take a music box mechanism and wind it up. The music, created by the pricking of a metal comb by pins, hardly sounds. But if the Mechanism touches the sound box of a violin, (or even a table) the sound is loud and clear.

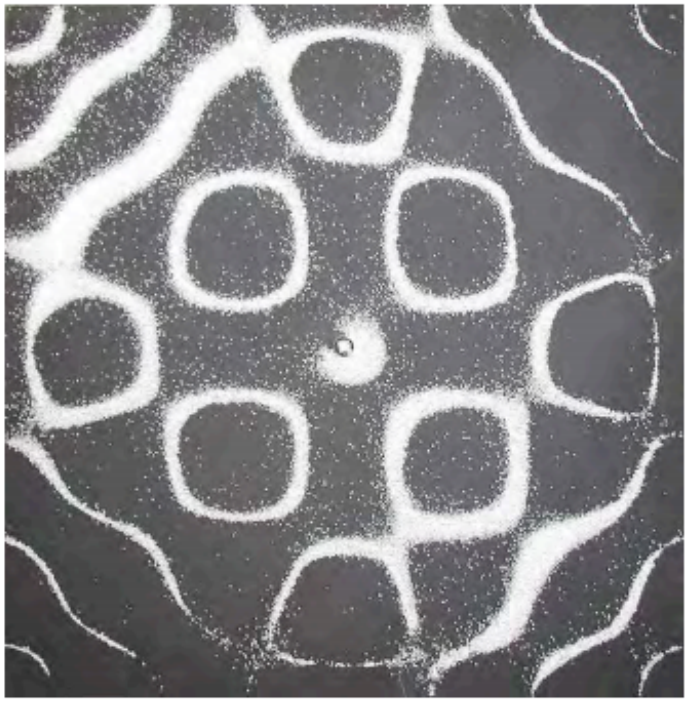
VIBRATING MEMBRANES:

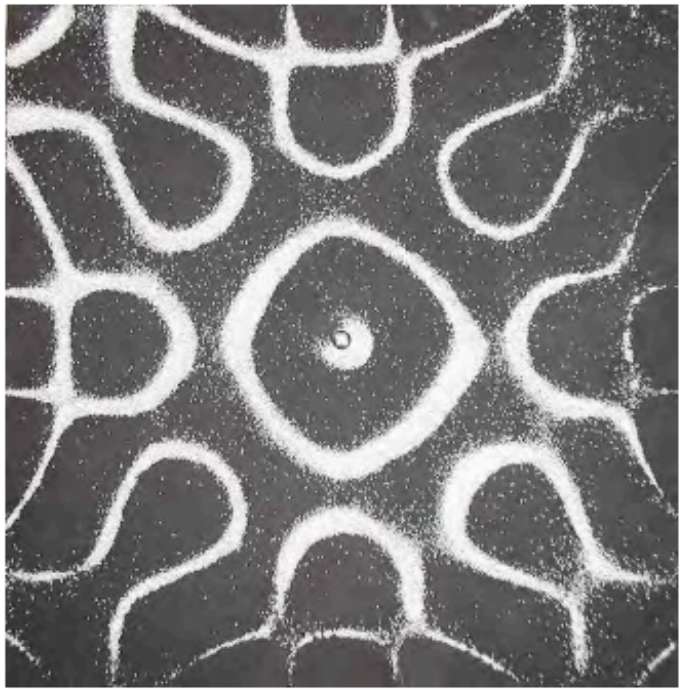
DEMONSTRATION:
Spread fine sand on the drum membrane.
Hit the drum with a stick at the middle, and close to the edges.
Record the sand patterns.
Correlate the patterns to the texture of the sound.

Hitting the center excites low harmonies. Hitting close to the edges excites high harmonies.
Describe these textures.









Air vibrations in a flute

Vibrations in air are periodic changes in air pressure. Similar to the nodes in a vibrating string, the position of the nodes in a pipe are also influenced by the pipe length, as well as by holes along the pipe. Similar to the stress in a string, higher air pressure increase the sound frequency, although we do not regularly play concerts in pressured tanks…
However, unlike strings, where the first and last nodes are strictly restricted to its ends, it is the antinode that tend to be positioned near the pipe opening, but not strictly at the end. Depending for example on the shape of the wider end of the pipe, the antinodes of higher frequencies tend to be positioned closer of farther from the end of the pipe. This feature modulated the harmonic texture of the instrument, making the same note played By a flute and a clarinet sound so differently.

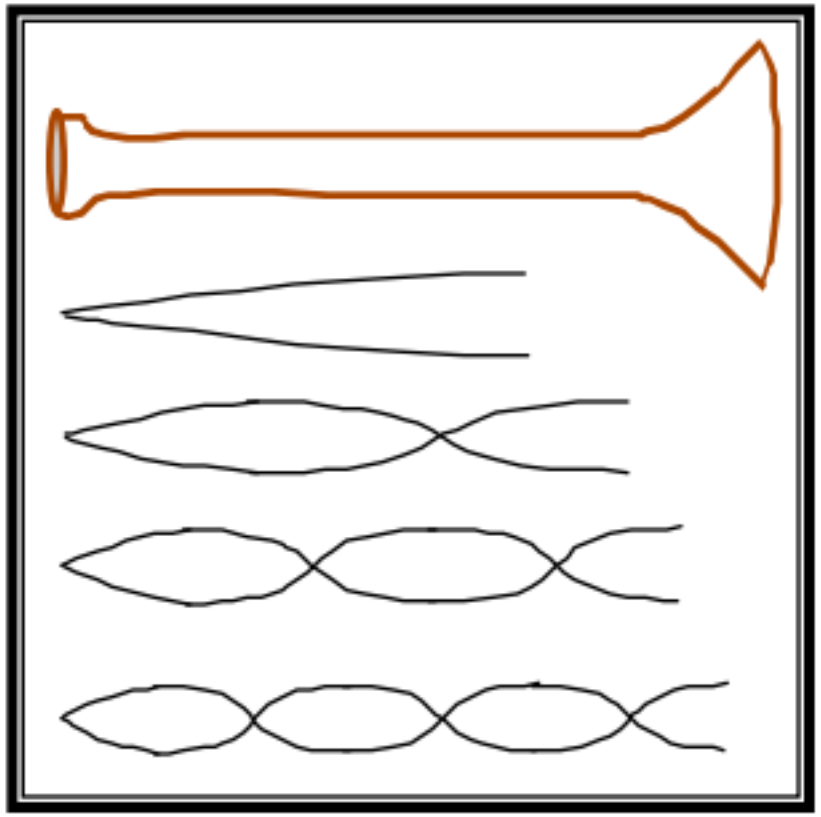


Figure 6.11: The bell of a brass instrument causes lower modes to reflect prior to reaching the end of the instrument. This smaller wavelength for the lower modes increases their frequencies, forcing them to approach the harmonicity of an open pipe.

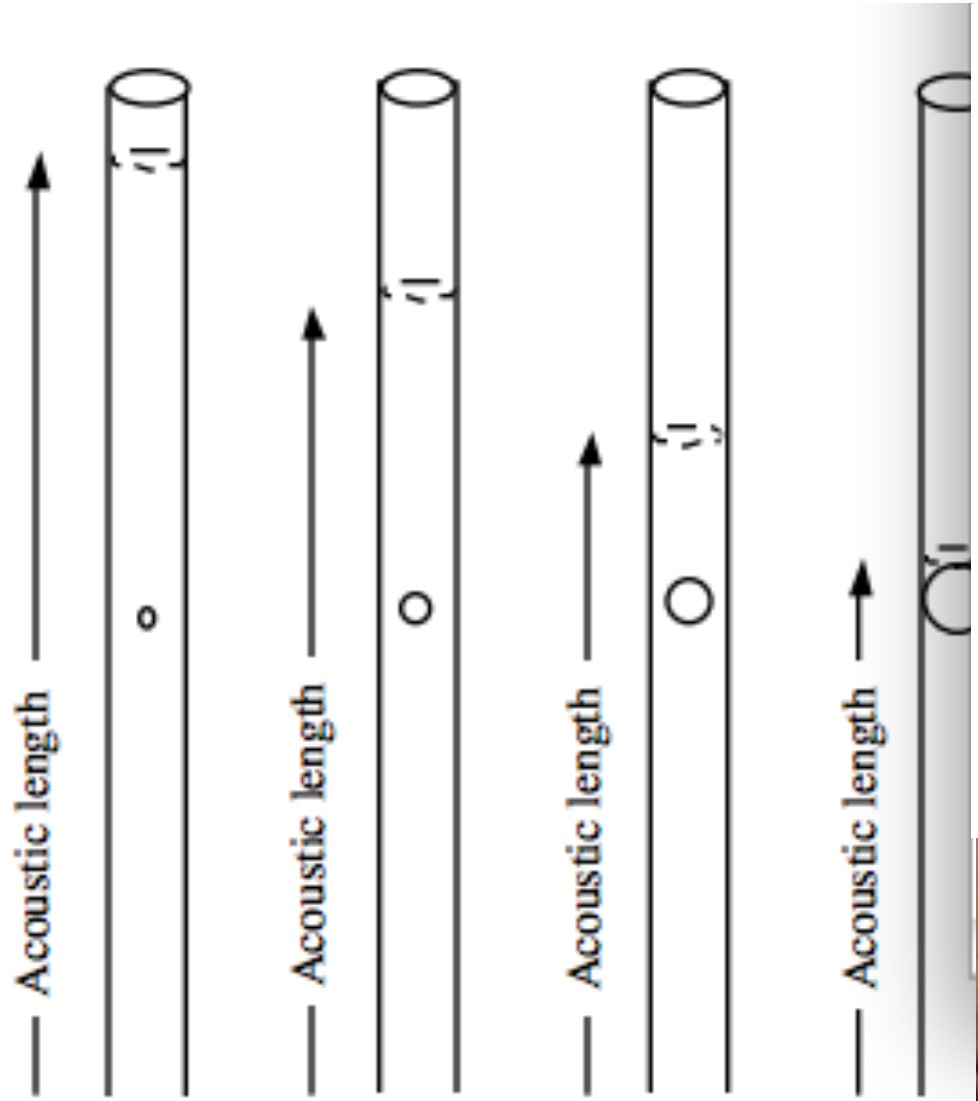


Figure 6.14: A hole drilled on the side of a pipe changes the acoustic length of the pipe. The larger the hole, the closer the acoustic length will be to the hole position.

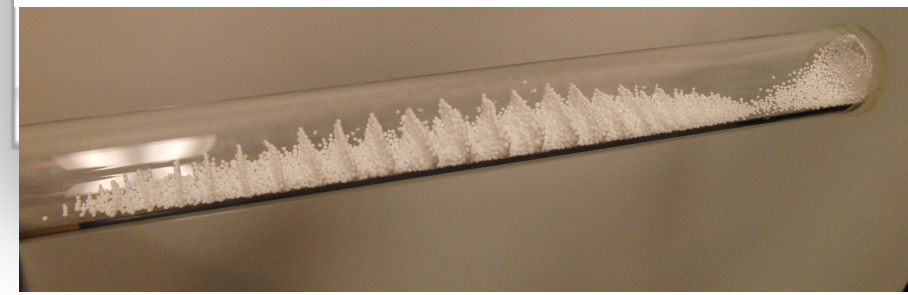
The hole along the flute tube modulate the “effective” length of the pipe, and change the produced tone.

DEMONSTRATION:

Kundt’s tube. Filled with fine powder. A vibrating piston at one end, and a stable piston at the other hand. When the distance between the two pistons equals

$$(n+1/2)\,\lambda$$

The powder is pushed away from the antinodes, and concentrates at the nodes of the standing wave (see picture below).



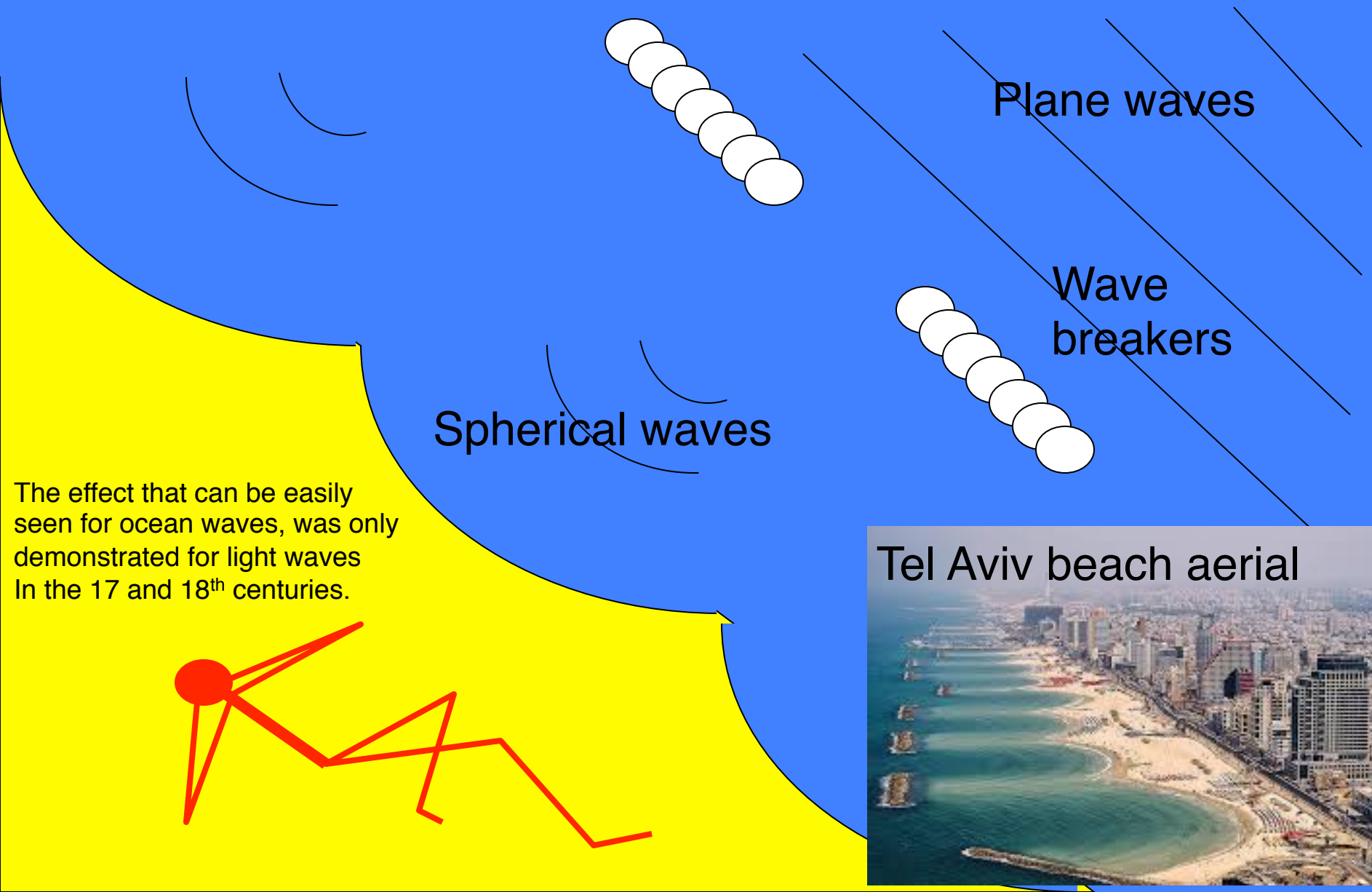
RESONANCE

We saw that motion of mechanical structures “prefer” distinct frequencies. When they are vibrated in these frequencies their vibration amplitude grow drastically. For example, poles stuck in the ground can be made to vibrate vigorously if one “tunes” the vibration frequency to the natural frequency that the pole moves when you push and let it go. Tuning to a mechanical structure natural frequencies creates vibrations in resonance. Soldiers are ordered not to march to rhythm when walking on a bridge, to prevent resonant vibration and collapse of the bridge.

During the 50th anniversary of the Golden Gate Bridge in San Francisco (built 1937) thousand of people paraded on the bridge, and caused it to vibrate in resonance with their walking.
A catastrophic event was prevented only when everyone was ordered to lie down, arresting peoples motion with the bridge.

PROPAGATION OF WAVES

A lesson of the beach.



The effect that can be easily seen for ocean waves, was only demonstrated for light waves In the 17 and 18th centuries.

DYNAMICS

LAWS OF MOTION

350 BC Aristo against **Leucippus -**
"Vacuum cannot exist, since if it did, object could move infinitely fast (no air friction)".

Aristo's laws of motion:

1. Heavier object fall faster, speed is proportional to weight. (proof: feather vs. stone).
2. Falling speed is inversely proportional to the density of the medium in which the object falls (fall in air vs. water vs. oil).
3. The planets are made of aether and their natural motion is in circles (as opposed to straight motion of earthy bodies).
4. Earth is a sphere, and will arrest without force acting upon it (implied: speed is proportional to force).

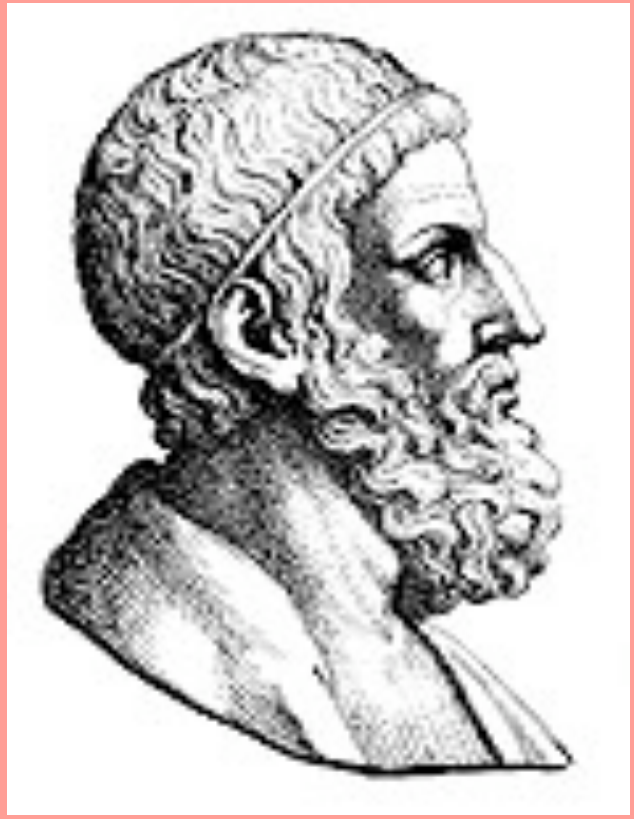
If you have lived in Aristo's times, could you provide experimental proofs to support or disprove his laws? Why didn't they use slower balls rolling down tilted slopes? Measurements of free falling speeds is difficult, therefore Galileo compared the time of reaching ground for large and small stones released from the Pisa tower. He understood that feathers fall slowly because of air friction, and minimized this effect using stones.

Note that Aristo did not distinguish between speed and acceleration, and attributed the falling of bodies to the medium. Other Greek scientists attributed falling to bodies "inherent" character.

268 BC Strato died. He was the first scientist who noticed that bodies accelerate while falling. His observation was based on water swarm flowing from the roof. He noticed that water stream started flowing slowly, and speeded up till the stream broke into droplets.

287–211 BC Archimedes

Born in Syracuse, an independent Greek city in Sicily, with 500 years long history. His father, Phidias, was an astronomer probably connected to King Hero II. He studies in Alexandria, and was one of Euclid's students. Was admired in his times by all the ancient world. During the 2nd Punic war he invented devices to defend Syracuse against the Romans. The Roman commander, by respect and desire to take advantage of his military skill, issued special order while breaking into the city to bring Archimedes alive. The story tells that Archimedes scolded a Roman soldier who stood against the sun when he was drawing a geometrical problem on the sand, and the soldier killed him.

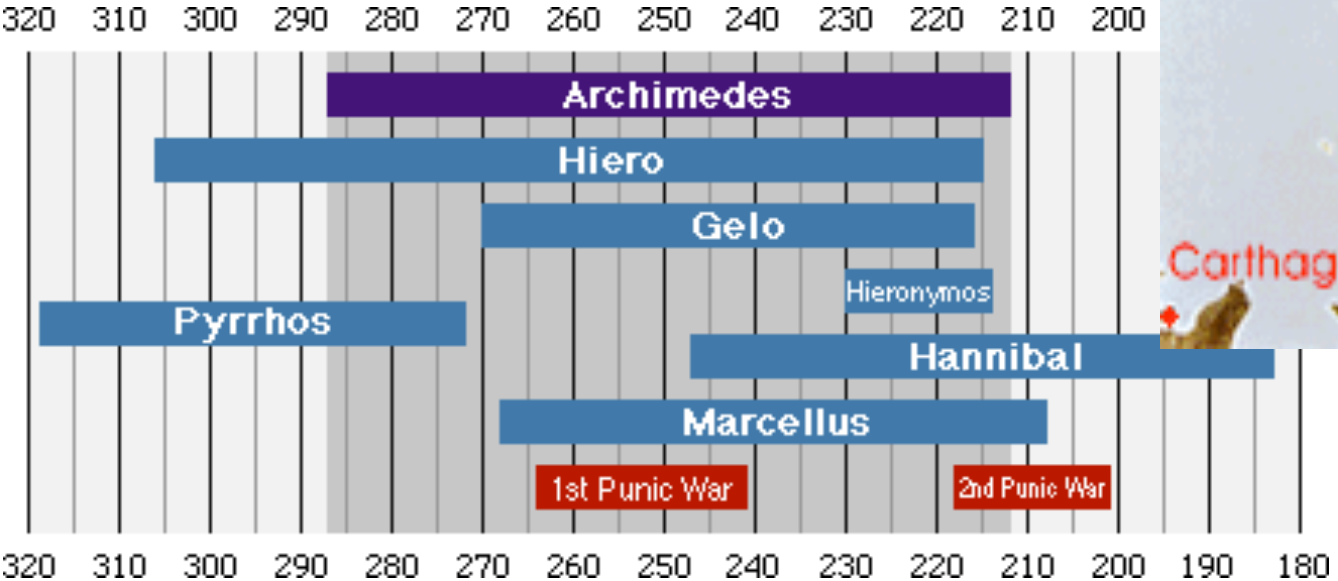


The Punic wars

The 1st Punic war: Syracuse, a colony of Corinthian Greeks in Sicily, allied with Carthage, a Phoenician colony in North Africa, against the Latin tribes of Rome, to prevent Romans expansion to Sicily.

The 2nd Punic war: Hannibal, the Carthaginian general, conquers Cannae, crosses the Alps with elephants and reaches Italy, but the Romans win over his troops.

The 3rd Punic war: ends in demolishing of Carthage 146 BC..



3rd Punic war

149-146

Archimedes heritage:

”On the method of Mechanical Problems” a book preserved through Arabic and Latin translations of the original in Greek, with the following chapters:

*** Statistics – The law of the lever
*** Hydrostatics - Law of floatation
Picnometry - The measurement of volumes and densities
* Equilibrium - The centers of mass of planar geometrical shapes
* The Sphere and the Cylinder.
* Measurements of Circles
* Spirals, spheroids and cones.
Quadrature of the Parabola.

Archimedes is the father of calculus (infinitesimal mathematics) and of mathematical physics.

Archimedes heritage:

Recently his compositions were recovered through **“Archimedes Palimpsest”** a parchment (animal skin that preceded paper) codex palimpsest (pages that the original writings were washed or scraped, and were reused, in this case for prayer book). The reconstruction of the original Archimedes text used Infra-red, Ultra-violet and Synchrotron x-ray imaging.

The contents:

"On the Equilibrium of Planes”; **Eudoxus'** method of exhaustion and indivisibles.

"Spiral Lines”; Considered the paths of planets around earth.

"Measurement of a Circle”; Lower and Upper bounds to π

"On the Sphere and Cylinder”; Includes cylinder and enclosed sphere volumes

*** "On Floating Bodies”; Include the law of floatation.

*** "The Method of Mechanical Theorems”; Include the law of moments.

*** "Stomachion” the puzzle game.

Speeches by the 4th century BC politician **Hypereides**

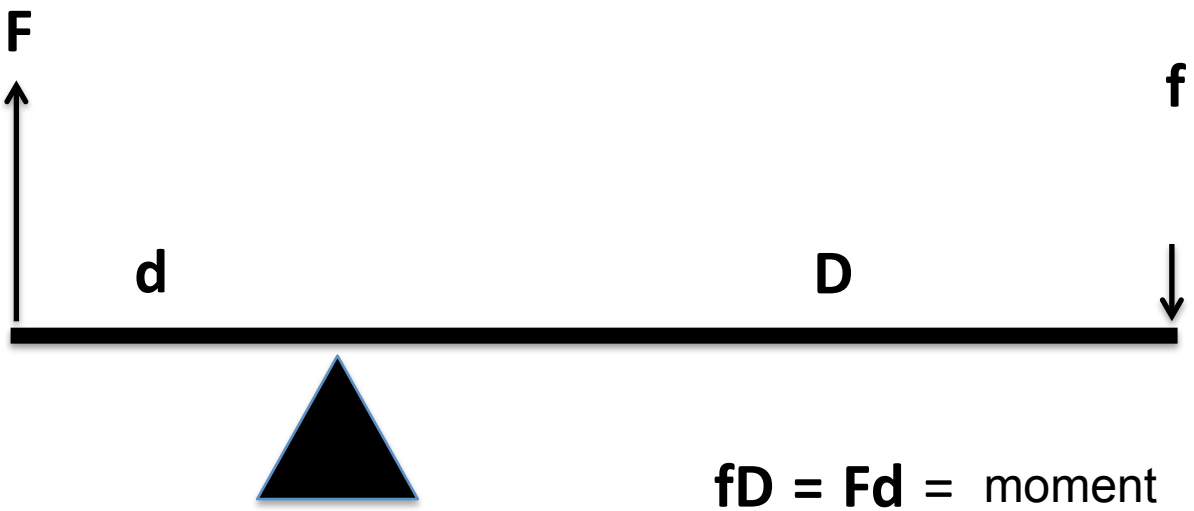
A commentary on Aristotle's Categories by **Porphyry** (or by **Alexander of Aphrodisias**)

Other works

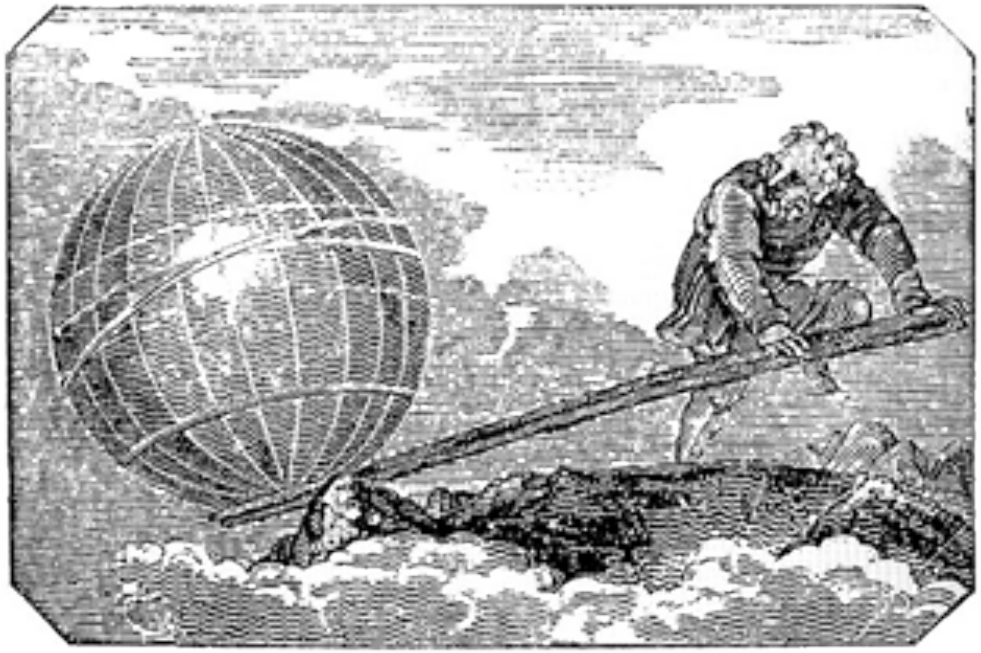
The original Archimedes scripts were probably kept in the Alexandria library, and maybe copies existed in Byzantium. The first burnt down completely ~600AC after a long decline in imperial support, and the second ~12000 in a crusades capture of Byzantium. The three chapters marked by stars are the only copy of Archimedes texts.

LEVERS

Archimedes said: “Give me a place to stand on, and I can move the earth.”



fD = Fd = moment

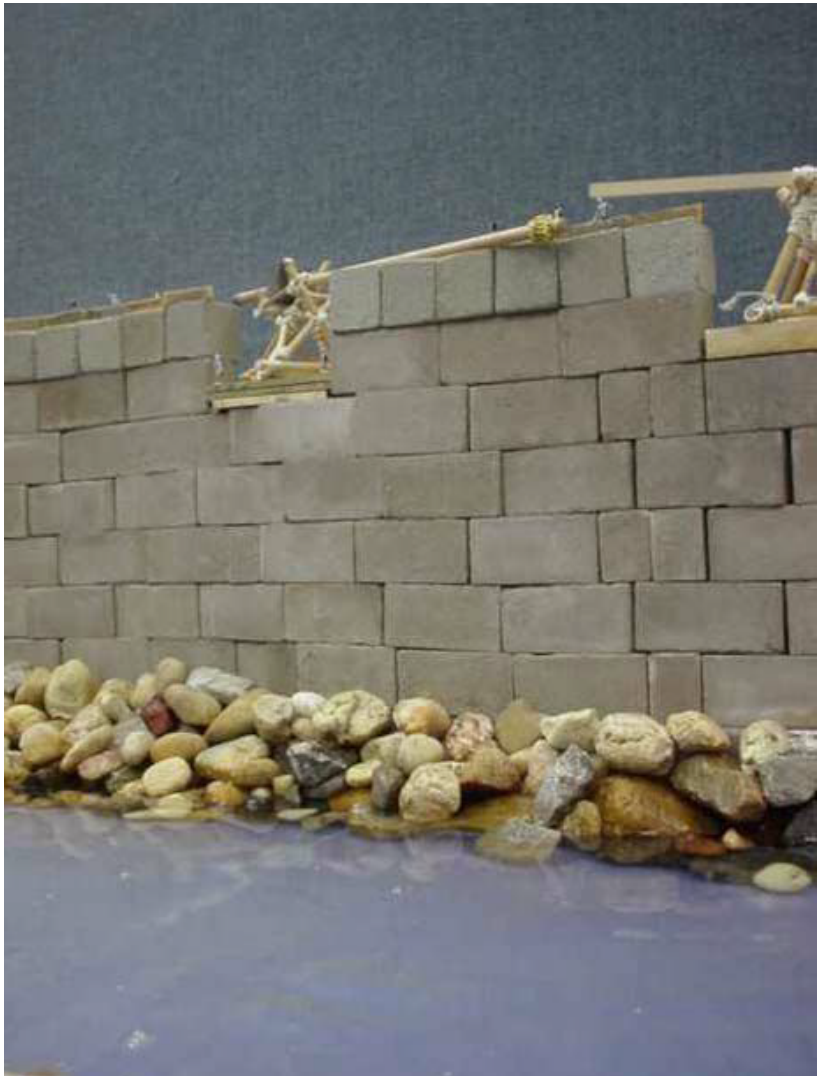


DEMONSTRATION:

Level arms, forces, balances

ARCHIMEDES JAWS

Using a long arm he grabbed Roman ships and turned them over



260 BC Archimedes describes the **law of floatation**

King Hero had his goldsmith prepare a crown. The crown was beautifully made, but he suspected that it was not made of pure gold. Hero asked Archimedes if he could determine if the crown contains only pure gold without melting it.
Vitruvius tells that the idea came to Archimedes when he was bathing, and felt his reduced weight inside the water. He jumped out naked and ran outside screaming "**EUREKA**", which became the headline of scientific inventions.

Archimedes law of floatation:

A body immersed in liquid loses in weight the weight of water it repels.

Pliny describe Vitruvius version of Archimedes solution to Hero's crown purity as follows:

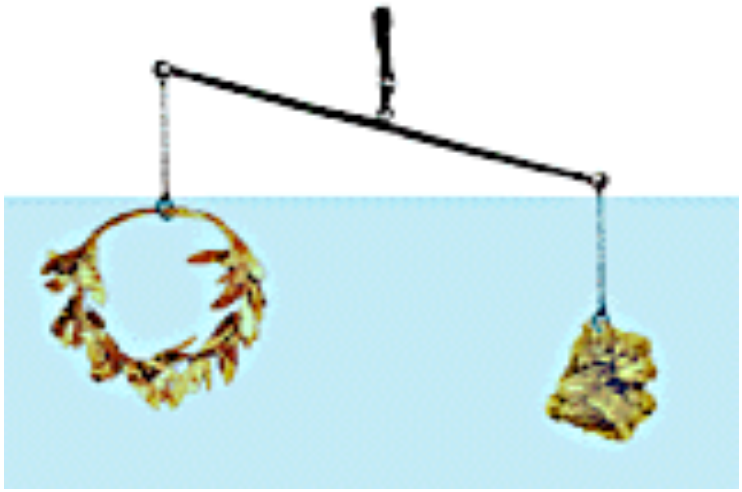
Archimedes immersed a piece of pure gold with weight equals to that of Hero's crown in a container filled with water to its edge, and let the water flow out. He then took out the pure gold and immersed the crown, finding more water flowing out. This confirmed that the volume of the crown was larger than pure gold of same weight, implying it contained other metals with lower density than gold.

Would you believe this version?

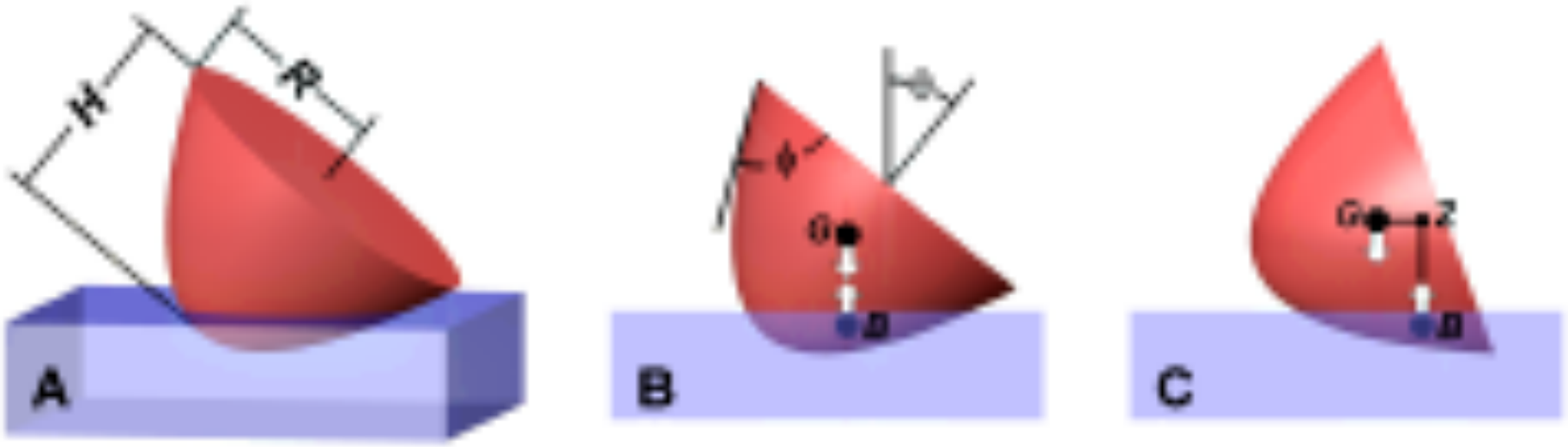
Lets assume Hero's crown weight was 1 kg, and it was immersed in a cylindrical container with 20cm diameter (area 314 cm^2). Density of gold is 19.3 gr/cc, therefore its volume is 51.8 cc

If the goldsmith changed 300gr of gold by silver, with density of 10.6 gr/cc the volume of the faked crown would be 700/19.3 + 300/10.6 = 64.6 cc , 12.8 cc more than the gold, causing water to rise 0.41 mm more than the pure gold. Considering water and also air bubbles adhering to the objects and the container edges (water surface tension) this would be too small a difference to measure.

What Archimedes possibly did is to balance the crown with equal weight of gold in air, then immerse both in His bath, easily finding the tilted balance due to 12.8 gr different weights inside the water.



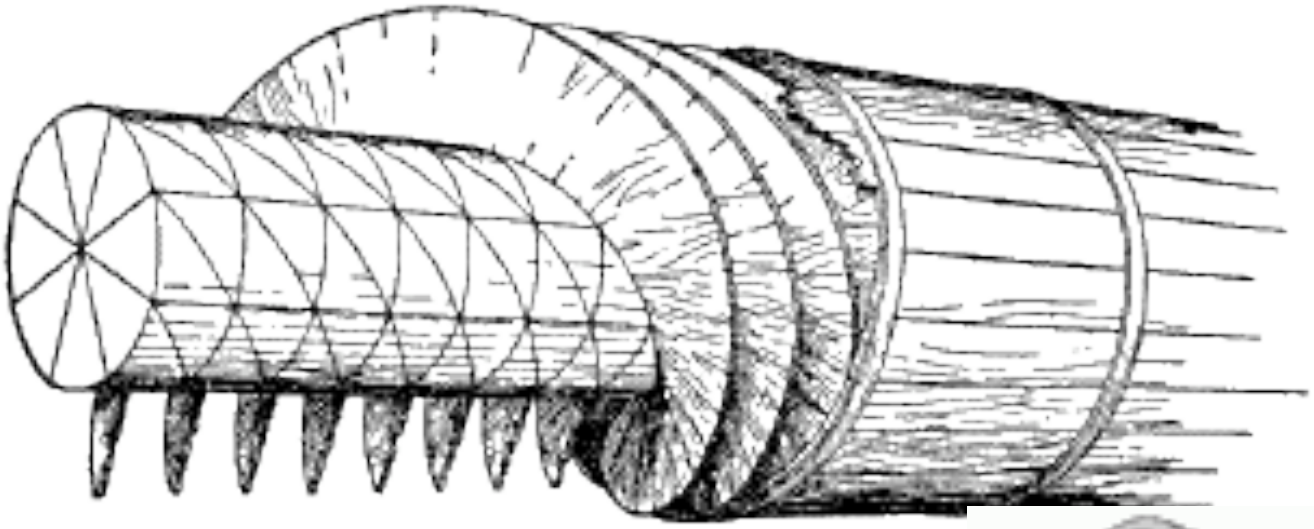
Archimedes not only found the weight loss of immersed bodies, but also described that the floatation force acts at the center of mass of the repelled water. This is the force that stabilizes a boat with weights at its bottom. The center of mass of the added weights is below this of the floatation force, turning the boat up.

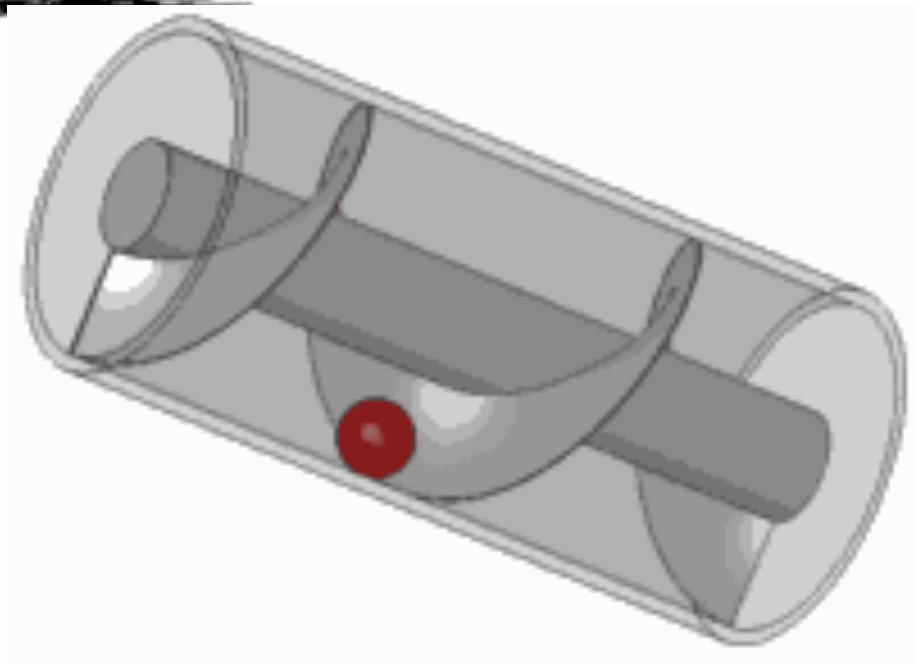


We are reminded in this respect about the sinking of the Swedish **Vasa boat** at 1676. The boat was designed to be high above water, to extend the range of its cannons. It is not clear if the design was mistaken, or the loading of weights at its bottom was not as planned. In any case, upon its first exit from Stockholm harbor to the Baltic see the boat turned over and sunk into the water. It was recovered recently and is displayed with all its glory in a museum in Stockholm.

Archimedes screw

An ingenious invention to continuously pump up water, possibly designed by Archimedes in Egypt to irrigate fields from the Nile water.

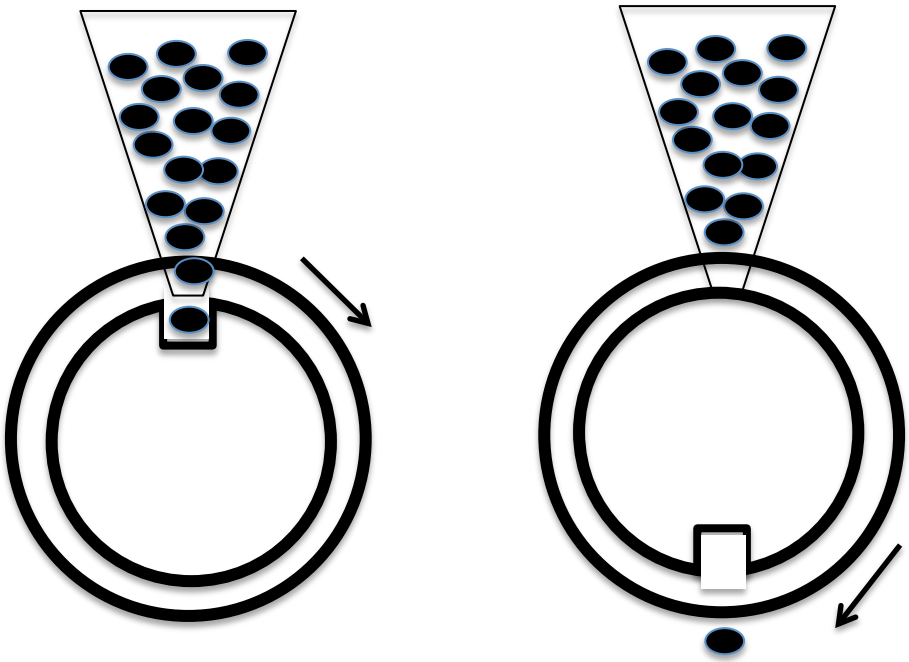




ODOMETER: measures distance of travel

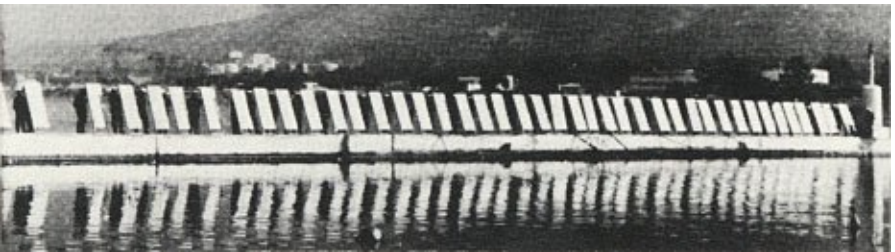
This arrangement automatically counts the number of turns that a cart performed on its journey. The container is filled with a known number of pebbles. The journey length is the number of lost pebbles times the perimeter of the wheel.

It is possible that **Eratosthenes** measured the distance between Alexandria and Syene using this method, when he calculated the perimeter of earth (see chapter below).



Even if the story is not true, Archimedes must have understood optics to propose reflecting sunlight from many polished shields onto the sails of Roman ships in order to set them in fire.

Modern astronomers re-invented this trick in “Adaptive optics”: Large telescope reflectors built from multiple small segments designed each to shift a bit in order to compensate atmospheric aberration (see 20th century optics).



Archimedes used **infinitesimal calculus methods** and the converging sums of infinite series in his calculations, e.g. centers of mass of bodies that he segmented to many small triangles, perimeter of a circle approximated by polygons, and the volume of bodies created by turning two-dimensional shapes around an axis.
We shall come back to these fascinating subject in the chapter on "Numbers"

In the following slide we shortly describe practical applications of science in antiquity as they anticipating later inventions:
Aviation, Clocks, Electricity and Magnetism and application of mechanics and statics to Buildings Construction,

AVIATION

FLIGHT

Although human succeeded in flying only at the 20th century, the thrive to fly like birds paves historical and legendary records from antiquity.

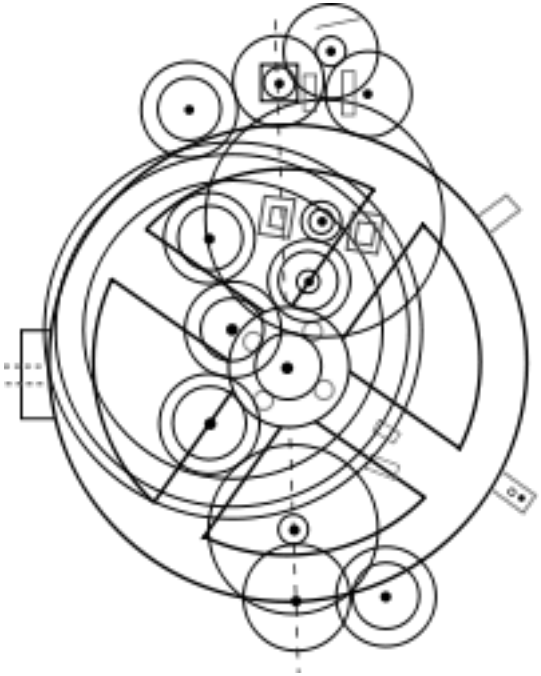
Icarus and Daedalus prepared wings from feathers stuck with wax, and only failed since they reached too close to the sun that melted the wax.
1000 BC a flying device called **Vimanas** is described in Indian Vedas. Probably a kite.
800 BC King Bladud floats above new Troy and falls to his death.
500 BC Chinese in Shan-Dung use kites for games, human flying (Mu Zi, Lu Ban black wood kite) for mapping in Qufu region (the city of birth of Confucius, 551-479 BC) , for communication and navigation, and at 200 BC to calculate heights and distances from vision angles.
400 BC Archytas of Tarentum describe his “dove”, probably a kite.

200 BC Chinese invent paper-made hot air balloons: **Kongming Lantern**, used in celebrations, as well as for sending messages from city under siege.

CLOCKS

150-100 BC The **Antikythera** mechanism, discovered in sunken Greek ship, is possibly an astronomical clock, or Planetarium, containing 30-70 wheels and gears to time position of the sun, moon and five, then known, planets.
Pliny writes that Archimedes built a planetarium, and the mechanism found may have been a copy of it.
Planetarium was a common instrument displayed at wealthy people's homes during the middle ages.

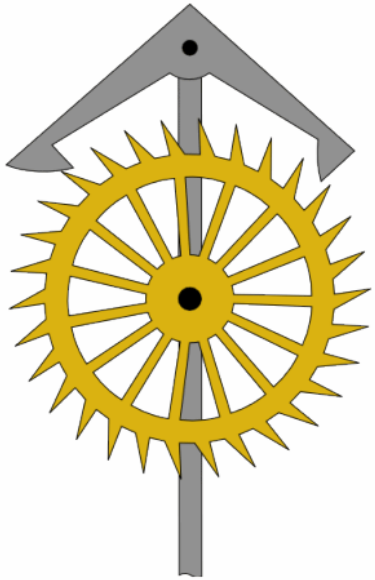
270BC Ctesibius - builds the **Clepsydra**, a water clock that became popular.



Sun clocks, or **Sundials**, were found all over the ancient world. **Obelisks** may have been used as sundials. The projected shadow speed depends on the sun's path angle that changes with the season. To get shadow movement at constant speed sundials base is tilted. The angle is the geographical latitude and the pole directs to the North Star.
Some propose that **Stonehenge** is a symbolic astronomical clock.

Sand clocks, or **hourglass**, spread after glass technology developed. Its advantage is the constant flow of sand controlled by the friction between the grains, unlike the flow of water that changes with the amount of water left in the upper container. These clocks were useful to set short time length, but not to indicate long times since they needed to be turned over exactly every hour.

Pendulum clocks were developed as tower clocks in city centers During the medieval ages. The gears and wheels were first made of wood, and later from bronze and steel. Pendulum period changes with its length, therefore it is sensitive to temperature changes. They also require stability and thus are not useful on boats, where clocks Were needed for navigation. Such clocks were only developed at the Renaissance.



ELECTRICITY AND MAGNETS

Magnets and Electric charges induce forces acting at a distance

Magnets can be found in nature – **Lodestones**. It contains minerals with high content of Iron Oxide, "Ferromagnetic" compound that was probably magnetized by lightening. When metal iron was purified, people learned to induce magnetism by heat and hammering.

Egyptians called magnetic stones **Haroeri**, the grandson of Earth.

Greeks, so Pliny tells, called **Magnesia** the place where **Magnes the Sheppard** found out that his sandal iron nails were pulled out by the ground.

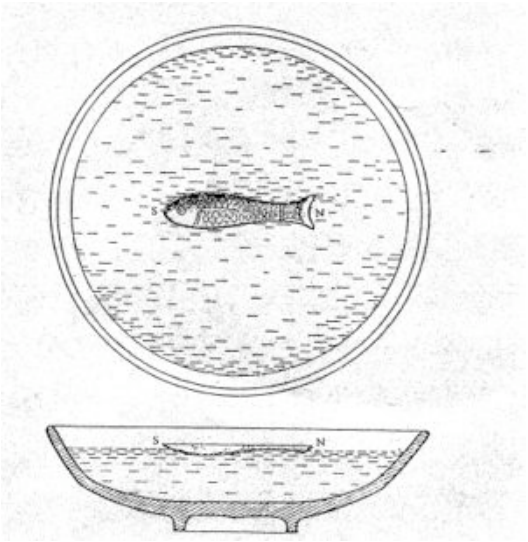
600 BC Thales - rubs Amber (**Electron** in Greek) with cats fur and pulls feather flakes. He does not discriminate between Electrical and Magnetic forces.

384-322 BC Aristo – in his laws of mechanic unify the force and the body moving in the aether – to explain forces acting from far.

Maya culture - pre-Columbian legends about attracting stones.

China – discovery that freely floating magnets are aligned North-South – invention of the **Compass**

1040 BC Wu Ching Tsung Yao - describes production of magnets by cyclic heating cooling of iron for his compass, made of a magnet attached to a fish-shaped wood and floating on water.



Do you know another force acting from distance?

Why Gravitational force was not described in antiquity?

Compass was essential for navigation in open sees.
The Chinese admiral **Shen He** sailed from China to Cambodia, Ceylon and Africa.
Following the loss of the Chinese fleet in a storm, the **Ming dynasty emperors** ban all shipping activities (against the will of the gods).
The compass was brought to Europe, possibly by **Marco Polo**, and was seminal in the Venetian and later Spaniards and Portuguese naval explorations to India, the far east and the Americas.

1269 AC Petrus Perigrinus - studies magnetic poles.
1600 AC William Gilbert – studies the magnetic field of earth: Earth is a magnetic dipole.
17th century René Descartes – describes the magnetic field as pipes leading particles.

DEMONSTRATIONS:

Pulling paper flakes by plastic comb after rubbing with silk or cloth.

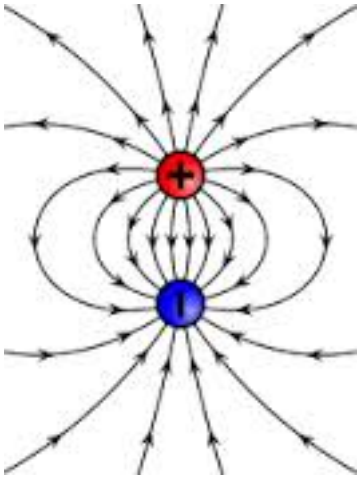
Iron sawdust on a glass plate ordered by a magnet below the glass – magnetic force lines.

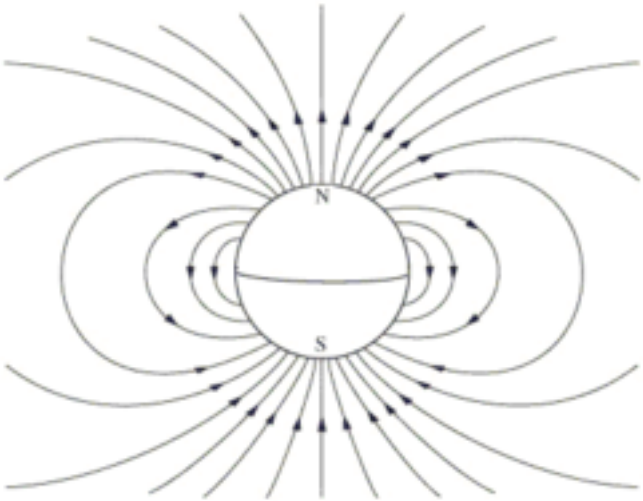
Floating magnetic needle on water (surface tension) – a compass.

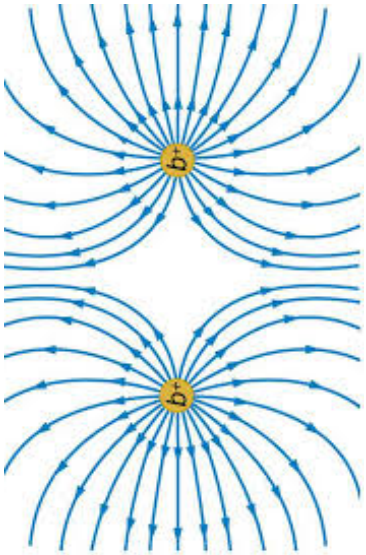
How can you discriminate between electric and magnetic forces?

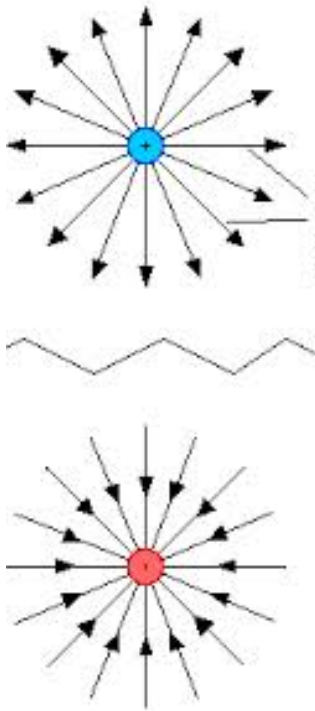
There is no magnetic monopole. North and South poles always come together. Cutting a magnet will produce two magnets each with two poles. Electrically charged body can be charges positively or negatively (e.g. by rubbing with different materials). Electric charge does not turn in earth magnetic field. Two magnets pull or push each other depending on their orientation. Two electric charges pull or push only depending on their charge, but independent on their orientation.

Can you assign the figures below to magnetic or electric field lines?









CONSTRUCTIONS

(and Static Forces)

Common to large building structures in ancient times:
Dependence on rivers for drinking, irrigation, transportation and hygiene.
This is true for Mesopotamia, Egypt and the Indus.
However, some Greek and Maya cities were far from rivers: Their strong social organization provided water reservoirs and food imports.
Earliest remaining architectural structures were temples and palaces, which building demanded abundant resources for stone quarries, transportation and construction.

Interesting: far away cultures in Egypt and Central America (Maya in Yucatan) built pyramids, since this was the most stable form of large and impressive structure.

Gate for a temple in Malta: Large stones are places on top of the entrance.

England 4-5000 years ago: Stonehedge
Circular construction of huge stones, brought from far. How did they position the tops?

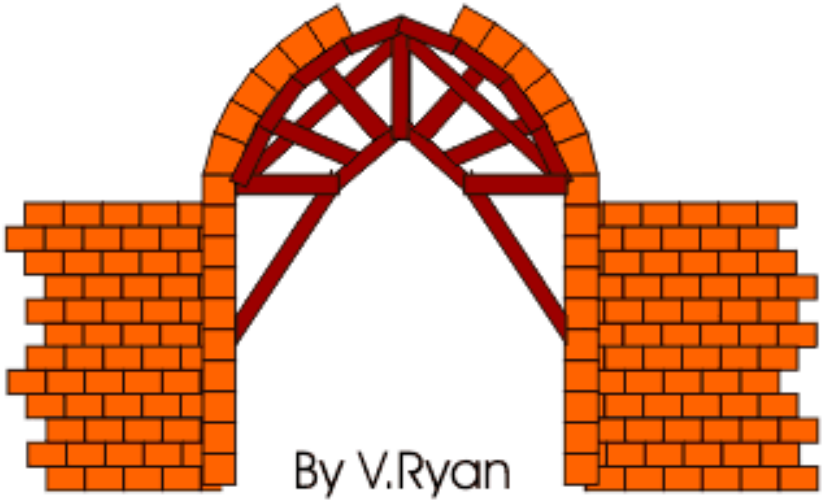
The innovative construction of large buildings using small stones:

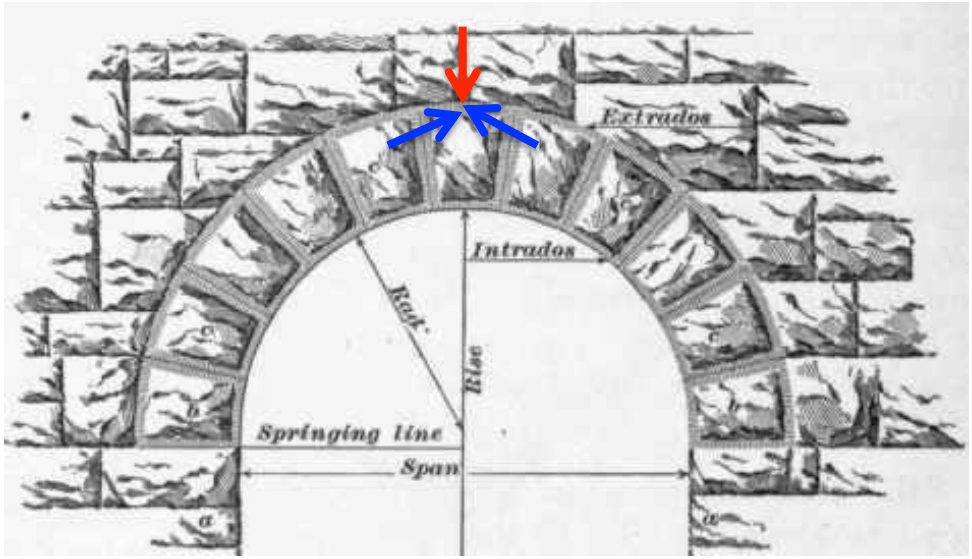
Enables individuals to build their homes at moderate work efforts:

Walls from piled stones, if not cut square, walls are thicker, and enforced by mud or later plaster.

Ceilings of wooden beams were supported on the stone walls and supplemented by straw or thinner wooden branches.

Later, construction of arches and domes from shaped stones that fit well each other, piled on a supporting temporary scaffold, and finally stabilized by a **<u>headstone</u>** locking and fixing the structure. The architects intuitively understood the static forces acting to stabilize the arch.





In the Islamic world, e.g. for Sheik graves, domes were built on sand-filled cavities..
After positioning the head stones the sand was removed.

BUILDING IN ROME

SCIENTISTS IN ROME

As we already noted, Roman natural sciences were highly considered for their practical, engineering applications. The Romans therefore nurtured Greek scientists, and Greek was the language of scientific texts. The Roman unique contribution was political philosophy, and encyclopedia editions to avail the scientific knowhow to present and future generations.

Mathematicians (studying the Quadrium- Arithmetic, Music, Geometry and Astronomy):
Eratosthenes, Sosigenes, Manaechmus, Callipedes, Isidore, Boethius, Rabanus Maurus, Macrobius

Philosophers:

Cicero, Seneca, Lucretius, Lucullus

Science Philosophy:

Martianus, Plotinus, Tertullian, Cassiodorus

Science history:

Pliny the Elder, Pliny the Younger, Agricola, Livy

Social sciences:

Varro, Hero, Strato, Callidus

Geographers:

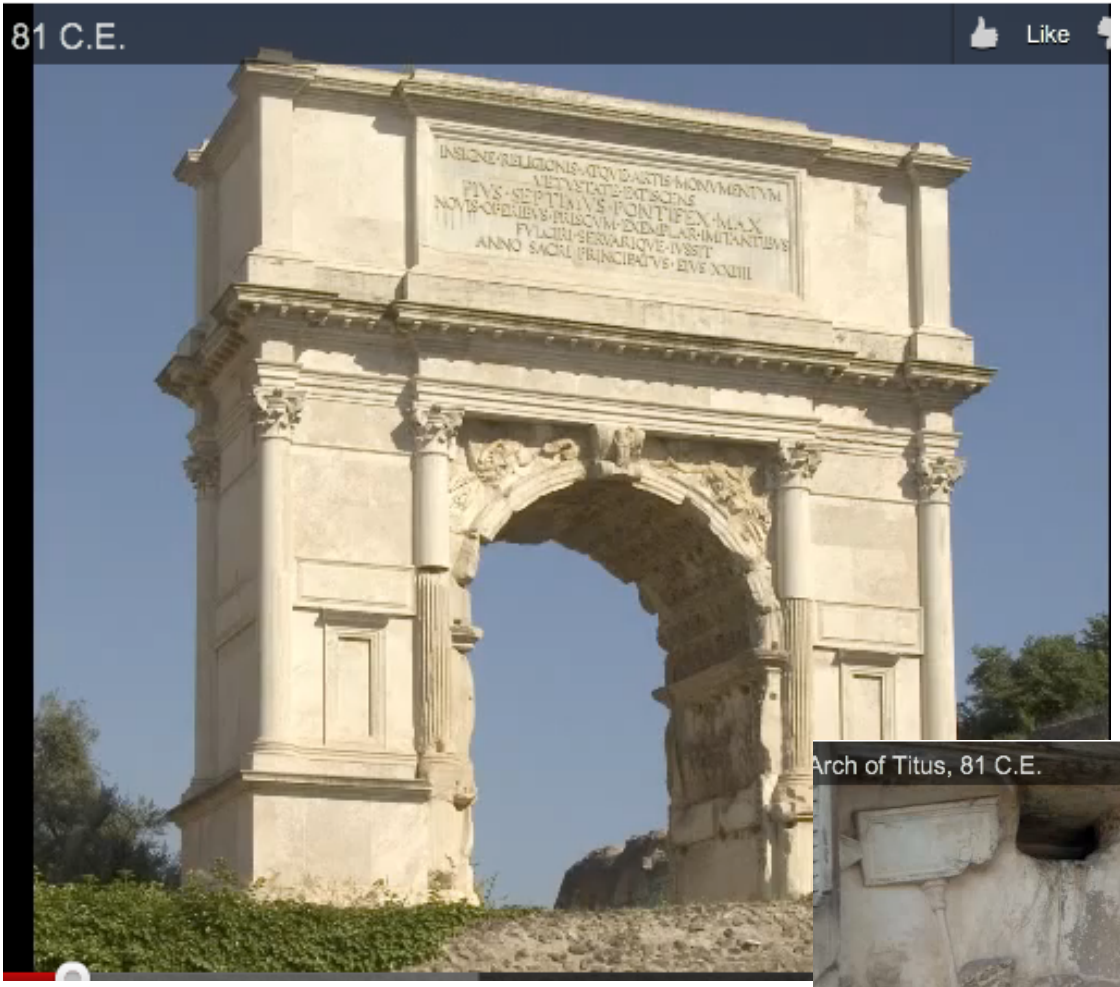
Phidias, Eudoxus, Polybius, Seutonius, Pythias, Hecateus, Eudoxus, Strabo

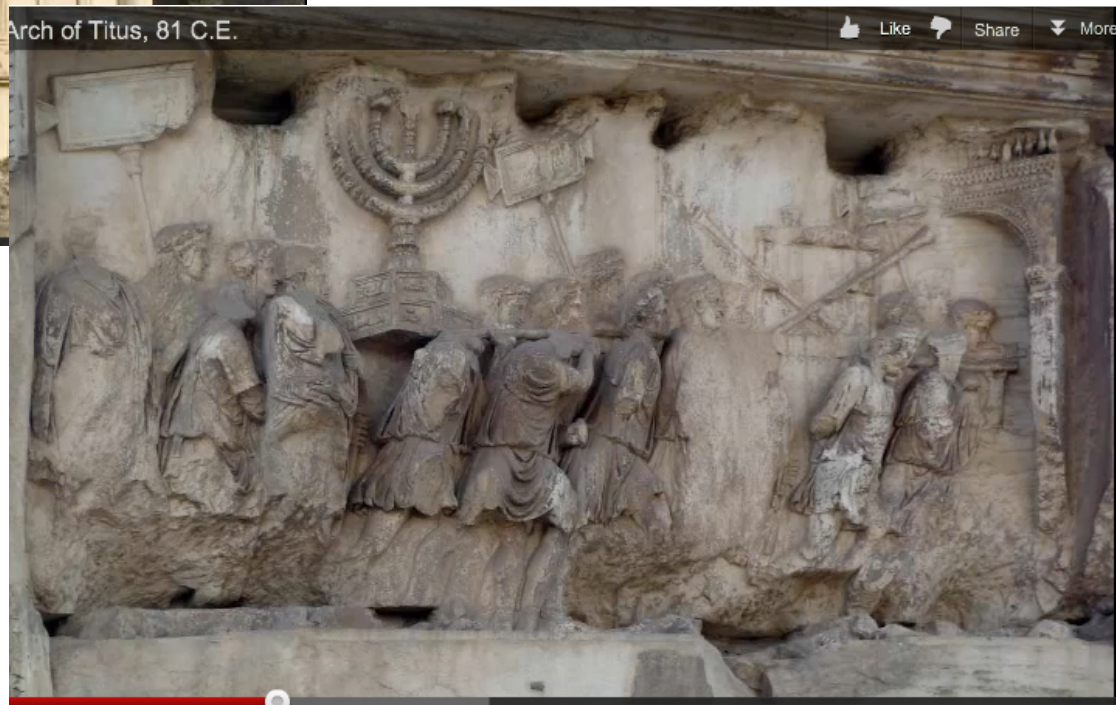
Physicians:

Varrus, Celsus, Junius, Livius

Architects:

Collidius, Vitruvius, Alberti, Palladio





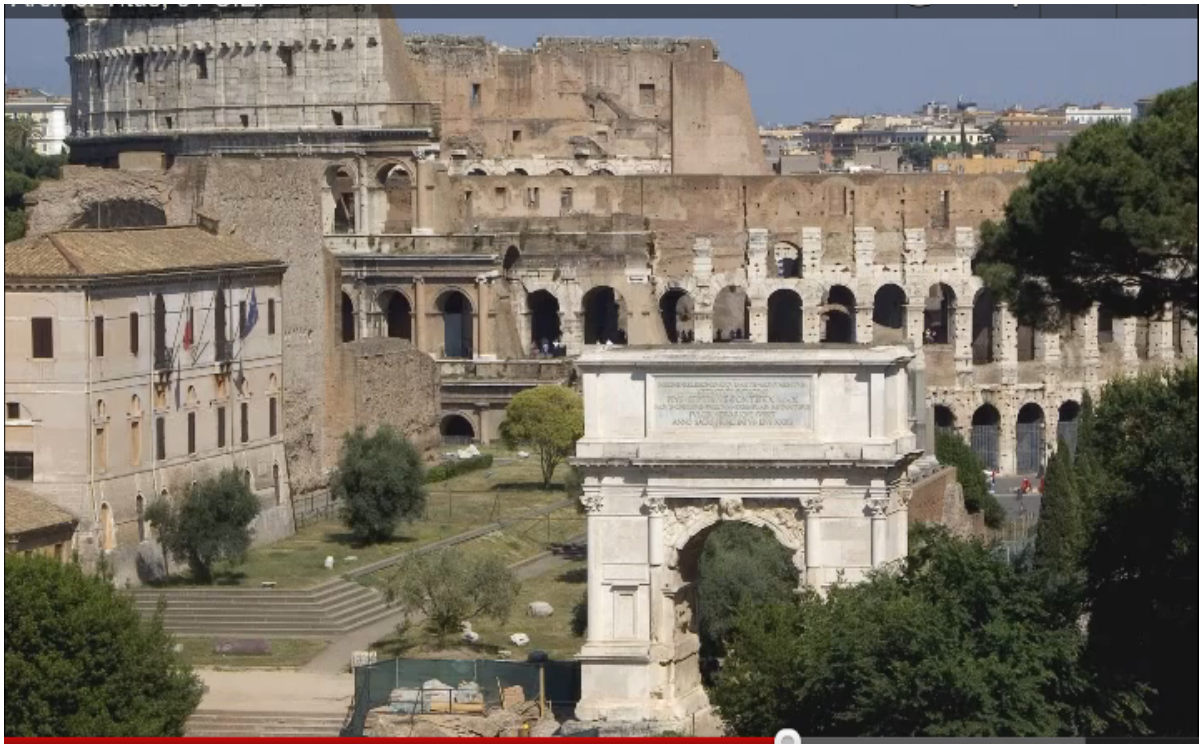
Titus arc in the Forum Romano with the relief describing the capture of the Jewish Temple artifacts with the MENORAH.

Practical achievements of the Romans:

Architecture: Arches, gates, aqueducts, Amphitheaters, Public baths, Sewage system.
Communication: Paved roads (via Apia from Rome to the port), commerce and monetary systems across the Roman Empire.
War technology: Catapults, two and three-story row boats with iron heads (sailing independent on winds), Burning arrowheads, ramps and battering ram for breaking city walls under siege.
Also military medicine: wound bondages, fixation of bone fractures, surgery e.g. for arrow extraction.

Via Apia

The Coliseum in Rome



Aqueducts

Accurate leveling
of the water channel
Along kilometers distance

SUMMARY:

Statics mechanical theories were relatively easy to test. Archimedes law of moments was a mathematical formulation of balances, commonly found in every ancient market place. Static forces in buildings, although may not have been calculated by civil engineers as they are today, were intuitively understood, and helped architecture evolve from bulky pyramids to Roman styled arches and aqueducts.

Dynamics was harder to examine experimentally, with the lack of short times measuring devices. Velocity and acceleration were confused. Since continuously moving bodies did not exist in the real world (due to friction), force was assumed proportional to velocity. Falling down was considered a “natural character” of bodies. The circular motion of stars thus was a special character of celestial bodies.

Hydrodynamics was another field that was intuitively embedded in boat building. Yet, it required the ingenuity of Archimedes to formulate its rules.

The mechanics of waves was amazingly quite developed, maybe because waves on water and in strings formed a good study model for musical instruments.