

MIDDLE AGES
6 - 15th Centuries

SCIENCES
IN THE ISLAMIC WORLD
EUROPE, ASIA
& AMERICAS

Collected and edited by Prof. Zvi Kam,
Weizmann Institute, Israel

Middle ages - content

Byzantium

China

Islam golden age

Architecture and art in Europe

Biology and Medicine (mainly Arabic)

Chemistry (mainly Islamic world)

Physics

Astronomy and earth sciences

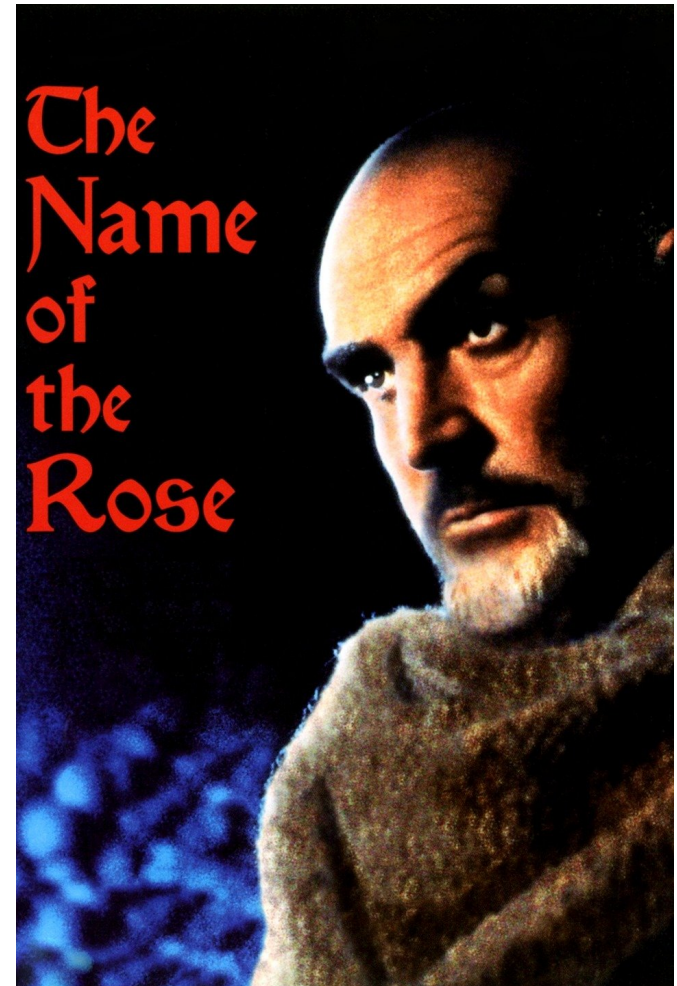
Technology

Mathematics

Late middle ages

Appendices:

- * List of middle age scientists
- * Movie: "The name of the Rose",
based on Umberto Eco's book
- * Plagues in history



SCIENCES IN THE MIDDLE AGES

Golden age of the Islamic world

The middle ages are considered a **dark** chapter in human history. Dark compared to the glorious Classical and Roman period, and dark because of poor life of farmers, high mortality due to medical ignorance and plagues, and dark because of the secluded monasteries where some sort of learning persisted in secret dark libraries. Yet below the surface the middle ages prepared the grounds for the Renaissance explosion.

We tend to mark revolutions in history, and link them to famous figures. But the understanding of the background that nourished these revolutions is no less important. On such backgrounds revolutions are inevitable and if not these “famous” revolutionaries, others would almost certainly emerge.

Scientific discoveries are based on known **existing knowledge** and often discovered simultaneously by different scientists. We attribute findings to the scientist who best described his discovery with clarity, and convinced his contemporaries of the truth of his theories and laws of nature. As scientific knowledge grows, every discovery is adding a block to the existing structure. In most cases it is impossible to define what are the independent discoveries.

During the middle ages the classical sciences were preserved through scripts from Alexandria, Rome, Byzantium and mostly Arabic translations in Baghdad, Damascus, North Africa and Maori Spain. The Islamic science benefitted from a common language, much like the classical world used Greek. But Arabic, unlike Greek, was the common spoken language and not the intellectuals language, which was different than the many spoken languages in the Roman empire.

The Islamic scientists had consistent methods to develop and prove scientific theories experimentally. They developed measurement tools and applied them for sailing in open seas, develop commerce with far away countries, and defend cities. Reading Arabic publications influenced Francis Bacon and Isaac Newton in their experimental approach, that was revolutionary following the classical science based on theoretical logical argumentations.

The Caliphates of Baghdad encouraged research, and scientists flourished in “schools” or universities from the 7th till the 15th centuries. They based their studies on translations from both the classical world and India, and encouraged independent developments. They took advantage of the yearly Hajj to Mecca and Medina to organize Islamic-world “conference” of scientists. The Caliph Harun al-Rashid established in Baghdad “the house of wisdom” at 810, the first university devoted to translation and studies of scientific scripts. Under the caliph support experiments were carried to establish or deny theories in physics (optics), mathematics, astronomy, medicine and alchemistry. Although the Arabic scientists are not associated with major breakthroughs, they were critical and open minded much more than their contemporary Europeans.

The crusades from the 11th century, turned Jerusalem and Maori Spain to war zones, that also created tight contacts between east and west. The Europeans learnt to build fortresses and produce steel swords, as well as to navigate and do mathematics (including the Arabic number system). Florence and Venice were trading with the east via land and sea. The Arabs also help Europeans recover the lost scientific literature of the classicists. For example, Gerard of Cremona traveled to Spain in order to translate one volume, and stayed till he finished translating 7 large volumes from Arabic to Latin.

Maori Spain enjoyed cultural and scientific prosperity, contributed by all three coexisting monotheistic religions. They developed ocean sailing technologies, including triangular sails for navigation against winds and in storms, and localization tools (astrolabes). These were enabling technologies for discovering the way around the Cape of Good Hope to India by Vasco da Gama and of America by Columbus. The inquisition and the Reconquista of the whole of Spain by the Catholics put an end to this golden age. Even the discovery of America and its rich gold storage did not help Spain to end the dark middle ages and enter the renaissance.

Other than the large kingdoms of Spain, France and England, most of Europe was divided between small city-states. Yet art, science and technology was considered the power and pride of the ruling princes and kings.

Universities developed from research centers established by Charlemagne, and from monasteries under the pope (e.g. Salerno, Bologna, where classical Greek scripts and Latin translations were kept). These early signs of cultural renaissance were cut by the **black plague**, that spread fast all over Europe by rats in grain transportations, and killed third to half its population. It mostly affected farmers who lived in poverty and terrible sanitary conditions. Interestingly, the severe shortage in working power advanced **rights to farmers and craftsman**, and freedom from feudalism.

With the fall of Byzantium at 1453 to the Ottoman Turks a flux of intellectuals ran away to Italy, brought with them Classical Science heritage with Arabic Science, and the Renaissance.



The monastery in Pavia, the first University in Italy

Important middle ages dates:

598-670 Brahmagupta the ZERO

750 Abbasid Caliphs succeeded the Umayyad Caliphs at Baghdad

820 al-Kwarizmi Algebra: Al-Kitab al-mukhtasar fi hisab **al-gabr** wa'l muqabala

830-870 Geber and Rhazes fathers of modern chemistry

~1000 Alhazen publishes "Optica"

980-1037 Avicenna medical canon.

1170-1250 Fibonacci brings Arabic number system & zero to Europe.

1282 Marco Polo voyage to the far east, import of Chinese technologies.

1347-1351 The Black Plague

1453 Constantinople is taken by the Ottoman Turks

1485 Tudors in England.

1492 Columbus discover America,

1492 the Spanish **Reconquista**.

1497 Vasco de Gamma surrounded Africa via the Cape of Good Hope

14th century Bacon, Spina, Cusa Eyeglasses, lenses for telescopes and microscopes

1500 Paracelsus father of modern pharmacology.

CHINA

The Chinese culture developed independently, with minimal contacts with the Europeans or the Arabs. It is therefore interesting to compare achievements in science and technology. Grossly, classical scientific philosophy was more advanced, but the Chinese contributed many technologies unknown in the western world, both for the benefit of mankind (print, compass) as well as for turning military confrontations more lethal (explosives)..

Technologies associated with Chinese inventions are:

- * Kites, hot balloons, human gliders
- * Water clocks
- * Controlled irrigation channels, pumps and pistons
- * Inspection tubes to precisely localize the position of stars, accurate sky maps
- * Explanation to the phases of the moon
- * Fossils and Geology – flooding and retreat of sea water and land erosion.
- * Seismograph to measure earthquakes
- * Print blocks and paper
- * Bronze and Iron casting in furnaces aerated by water-driven blowers. Steel work.
- * Fine porcelain and painted glazing
- * Fireworks, gunpowder, rockets
- * Eye glasses
- * Color dispersion in the rainbow

Let us credit some of the Chinese inventors:

559 Yuan Huangtou, Ye, glided from a tower

Li Chunfeng (602–670) Mathematician and astronomer

Shen Kuo (1031–1095) A floating magnetic needle as a compass is applied for navigation.

Light refraction in the atmosphere, astronomical gnomon (clock, see pic1),

clepsydra (water clock)

armillary sphere (spherical astrolabe, see pic2), astronomical tube for precise localization of star latitude and longitude angles.

Geological explanation to buried marine fossils on hills due to erosion by rains and rivers

Yang Hui (1238–1298), Mathematician

Bi Sheng (990-1051) pic4. Inventor of printing tabled with exchangeable letters, pic3.

1450: Johann Gutenberg Implemented this invention in Mainz, Germany.

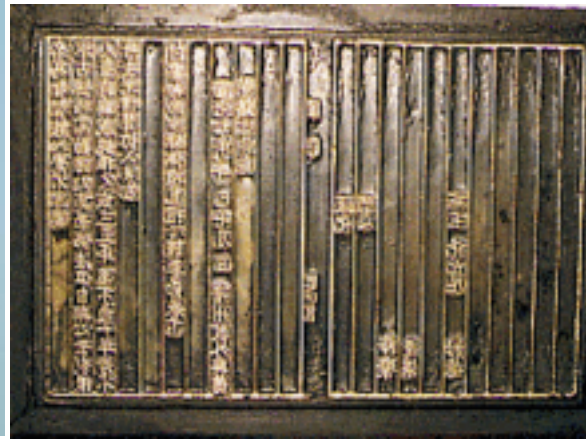
pic1



pic2



pic3



pic4





Su Song (1020–1101)

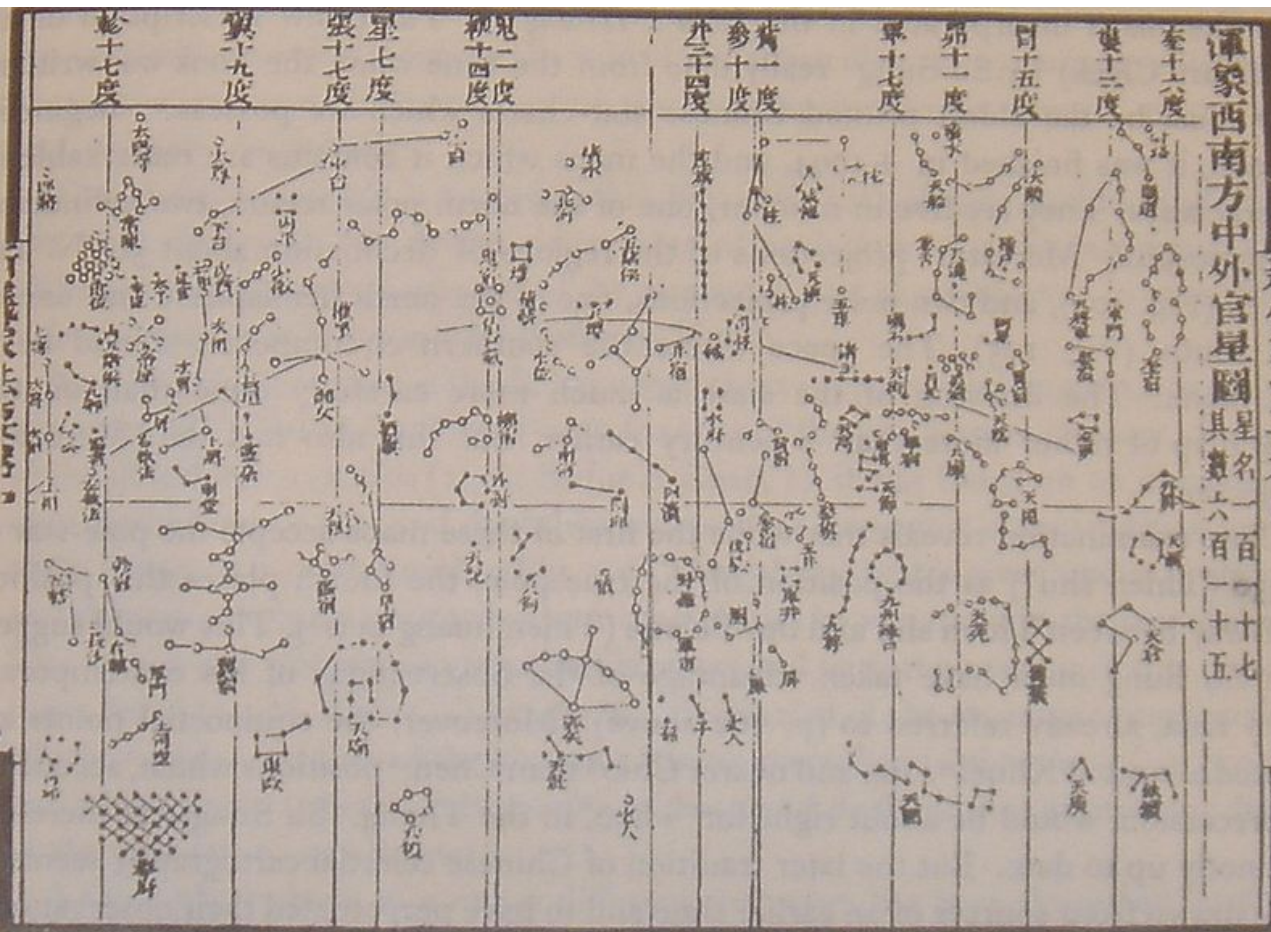
Star map- cylindrical (Mercator) projection, pic1.

Pharmacology, Botany, Zoology, Mineralogy and Metallurgy.

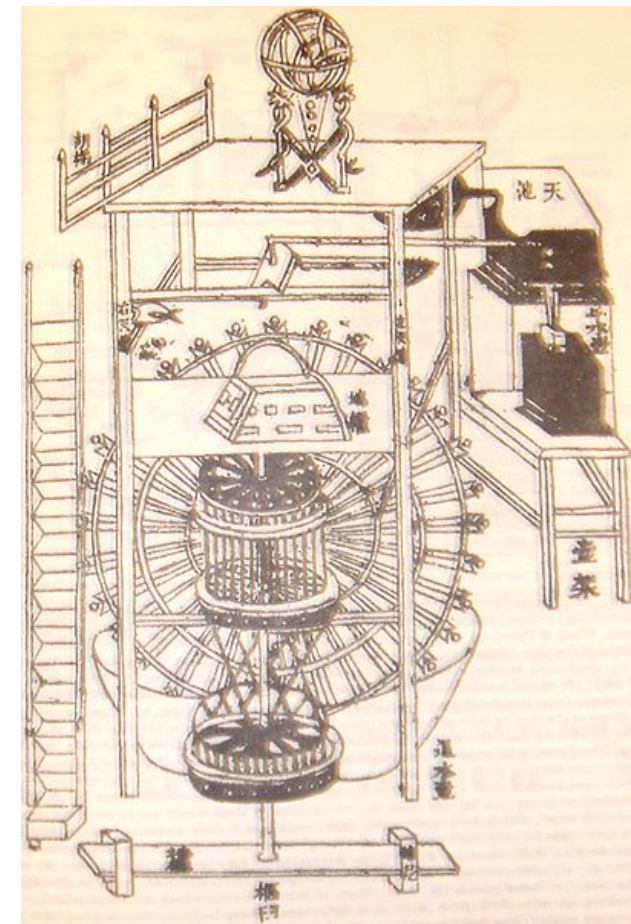
880 AC built a tower clock in Kafieng, driven by a chain, pic2.

Built gates to regulate water flow in irrigation channels: what turned Egypt and Babylon wealthy 3000 years before.

pic1



pic2



Zhang Heng (2nd century) Seismograph to sense earthquakes.

Also, possibly independent from the Arabs, invented decimal number system

Joseph Needham the Jesuit describes **Chinese inventions** that were transferred to Europe: Shipyard docks for building and repairing ships, Calipers, Piston pump, Iron casting, Furnaces with water driven air blowers, Iron ploughs, seeding machine, wheelbarrow, Sieving machine, Air fan, Propeller, Parachutes, Hanging bridges, Natural gas for warming, Matches, Arches, rackets (including multi-stage), Bridle and Horseshoes, Embossed maps, Sky Globe turned by water.

Marco Polo (1282) Reports about man flying by kites.

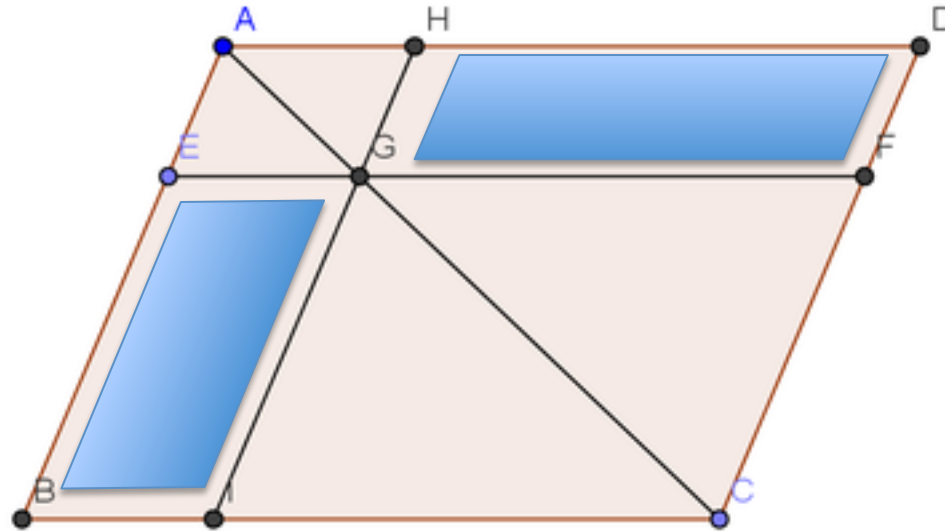
From ~1000 development of explosives from tar, sulfur and coal.

Guns and rifles evolved from Bamboo shooters 1044, then iron pipes shooting arrows. After 1280 shooting balls, From the 13th century – Iron projectiles, rifles, mines and rackets. 14th century – Exploding cannon projectiles, two stage rackets, marine mines.

From the **10th century** – eyeglasses

1279 Yuan dynasty – baked hard porcelain. Produced from white fine-grain clays precipitated in riverbeds and lakes. Imported to Europe by Marco Polo, and cost fortune, The secret of its production was broken to Europeans only at 1707.

Yang Hui (1238–1298) Writes a proof to the theorem: Complementing Parallelograms BEGI & DFGH. (blue) to Parallelograms EGHA & FGIC lying on the diagonal AC of a parallelogram ABCD have equal area:



Two parallel lines crossing other two parallel lines. Therefore equal triangles area:

$$ABC = ACD \quad AEG = AHG \quad GIC = GFC$$

Therefore: $AEG + GIC = AHG + GFC$ thus: $ABC - AEG - GIC = ACD - AHG - GFC$ or: $BEGI = DFGH$ Q.E.D

Typically, Chinese mathematicians wrote theorems without proofs. It is hard to know if they knew to prove them, or intuitively believed that they are true.

This is in contrast to the Euclidian heritage of European mathematicians, who defined a set of axioms and proved all theorems derived from them.

Sun Sikong (1015–1076) Rainbow is formed by reflection and refraction of sunlight by water tiny droplets in misty fogs.

Han Dynasty (202 BCE–202 CE) : Jing Fang (78–37 BCE) Zhang Heng (78–139)

Sun eclipse is due to moon shadow. Moon eclipse is the earth shadow.

The moon is a sphere, not a disk, and reflects sunlight. According to direction of the observer moon phase is full or sickle.

Until the 17th century, earth was believed to be flat.

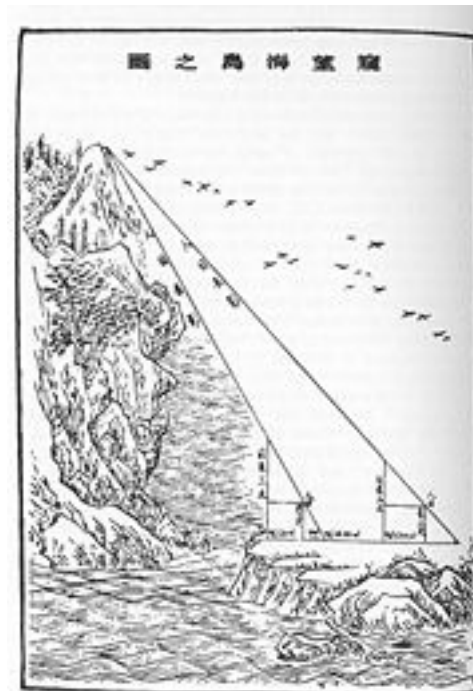
Matteo Ricci (1552–1610) the Jesuit and **Xu Guangqi (1562–1633)** the Chinese astronomer spread the notion that earth is a sphere.

1054 Supernova explosion in Cancer constellation was observed at the **Sung national observatory at K'ai-feng** for 23 days. It was so bright that could be seen by eye in full daylight.

Liu Hui (3rd century) “The Sea Island” manual

Is an extensive mathematical document.

For example: a diagram showing how to measure the height of a mountain.



Guo Shoujing (1231–1316) Spherical trigonometry. Deviation of the compass from true north.

Qin Jiushao (c. 1202–1261) Zero as a space in numbers

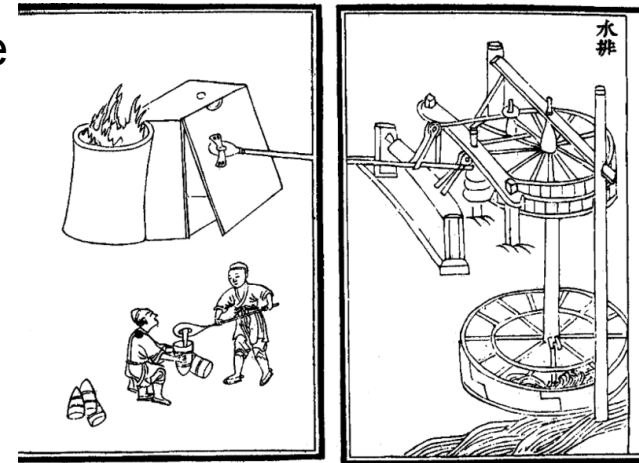
Jia Xian & Yang Hui Pascal triangle

Zhu Shijie (fl. 13th century) Calculation algorithms

11th century – massive amputation of forests for fueling the iron industry before coal mines were discovered. May have been the reason for drought causing millions to die of hunger.

Wang Zhen (fl. 1290–1333) quotes the invention of water-driven blowers in blast furnaces by

Du Shi (1st century). Cast iron was produced in China Since the 5th century BC.

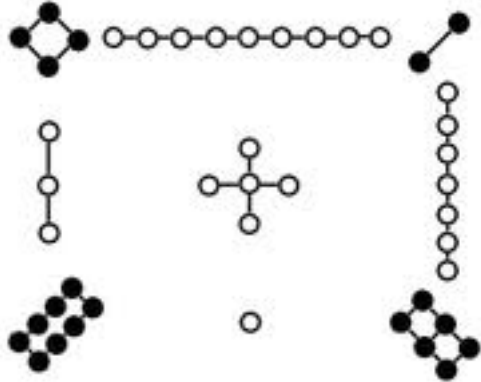


Invention of lacquer to impregnate silk costumes for divers and other clothes, leather and iron weapons from rust. Invention of mirror polishing pastes.

Marco Polo (c. 1254 – January 9, 1324) The Venetian who travel by land through the “silk road” in central Asia to China under the rule of Kublai Khan. Although he was not the first westerner to visit China, he left detailed account of his trips.

Vasco de Gamma (1460 or 1469 – 24 December 1524) The Portuguese sailor who surrounded Africa and the Cape of Good Hope to reach India.





4	9	2
3	5	7
8	1	6

A mathematical game favored by Chinese:
Magic squares
Left: Chinese number script, right: today's numbers

2	16	13	3
11	5	8	10
7	9	12	6
14	4	1	15

12	27	33	23	10
28	18	13	26	20
11	25	21	17	31
22	16	29	24	14
32	19	9	15	30

13	22	18	27	11	20
31	4	36	9	29	2
12	21	14	23	16	25
30	3	5	32	34	7
17	26	10	19	15	24
8	35	28	1	6	33

46	8	16	20	29	7	49
3	40	35	36	18	41	2
44	12	33	23	19	38	6
28	26	11	25	39	24	22
5	37	31	27	17	13	45
48	9	15	14	32	10	47
1	43	34	30	21	42	4

61	3	2	64	57	7	6	60
12	54	55	9	16	50	51	13
20	46	47	17	24	42	43	21
37	27	26	40	33	31	30	36
29	35	34	32	25	39	38	28
44	22	23	41	48	18	19	45
52	14	15	49	56	10	11	53
5	59	58	8	1	63	62	4

31	76	13	36	81	18	29	74	11
22	40	58	27	45	63	20	38	56
67	4	49	72	9	54	65	2	47
30	75	12	32	77	14	34	79	16
21	39	57	23	41	59	25	43	61
66	3	48	68	5	50	70	7	52
35	80	17	28	73	10	33	78	15
26	44	62	19	37	55	24	42	60
71	8	53	64	1	46	69	6	51

1	20	21	40	41	60	61	80	81	100
99	82	79	62	59	42	39	22	19	2
3	18	23	38	43	58	63	78	83	98
97	84	77	64	57	44	37	24	17	4
5	16	25	36	45	56	65	76	85	96
95	86	75	66	55	46	35	26	15	6
14	7	34	27	54	47	74	67	94	87
88	93	68	73	48	53	28	33	8	13
12	9	32	29	52	49	72	69	92	89
91	90	71	70	51	50	31	30	11	10

Byzantium

Byzantium (Constantinople)

At the collapse of the Roman empire in the 5th century, Byzantium or Constantinople (naming Constantine who established the city as a center of the Eastern Roman empire) took the priority from the western cultural center in Rome. The proximity to Alexandria, small Asia as well as Persia, Baghdad and Damascus enriched and diversified their cultural and scientific heritage.

The Byzantines distinct themselves from the Romans. They were Greek-oriented, and their Christianity eventually separated from the Catholic establishment and the pope. The eastern Roman empire centered in Constantinople survived many centuries beyond the Western Roman Empire, throughout the middle ages.

1453 Constantinople is taken by the Ottoman Turks, and renamed "Istanbul". Massive scientists escape to Italy is one of the background causes for the Renaissance.

Scientists in Byzantium

Oribasius (320–400) Summarized medical knowledge

Anthemius of Tralles (474 –534) Geometry and Architecture – one of the designer of the dome of Hagia Sofia, the largest building in the world at his time (oversized later by the cathedral of Seville, Spain).

John the Grammarian or John Philoponus (490– 570) falling bodies are all accelerated (what he called impetus) by equal rate, and this is not, as Aristo believed, the effect of air. Sunlight heats the air by friction. Discards existence of vacuum.

Simplicius of Cilicia, (530) Interpret Aristo, that attraction of bodies to their “natural” position depends on distance from it.

Paul of Aegina (625–690) Physician and a surgeon

The Venerable Bede (672–735) Studies tides. From recognition of sounds in speech he derives “atom” of time = $1/6$ second



Bede

Islam Golden Age

The Caliphs in Baghdad

The Abbasid Caliphs succeeded the Umayyad Caliphs at 750.

Haroon al-Rashid 763-809 formed international relations from China on the east till Charlemagne in the west. He employed Persian administrators to rule his empire. At the influence of the Bramakid family viziers (Wazirs), a Persian family of Buddhist origin, he supported scientists and established a paper mill to spread knowledge by books. His first son **Al_Amin** ruled for a short time and was engaged in battles, and after his death in 813 his step brother **al-Ma'mun (786-833)** ruled in period called "golden age". He build the "house of wisdom" – science academy, engaged in translation and study of science, politics and arts. He practically turned "reason" (Muatzala in Arabic) to become the state religion, strongly influenced by classical philosophy and rational thinking, a skill all governmental officials had to demonstrate.

Only a few of the great many scientists of this period were Muslim Arabs. Most of them were of Indian and Pakistan origin, and even suffered from the Muslin orthodox, but flourished in Baghdad.

Muhammad ibn Musa al-Khwarizmi 780-850

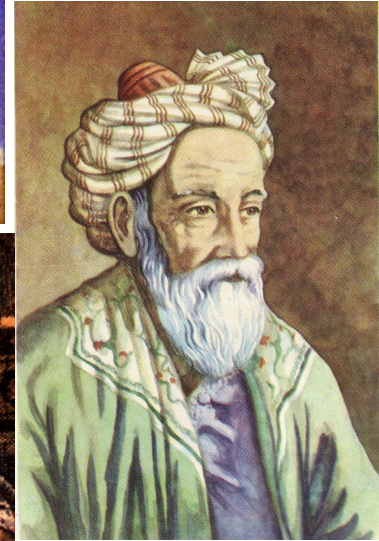
"Algorithm" is named after him.



Omar Khayyam 1050-1123

Termed the name "Algebra" "al-jabr" in Arabic,
Meaning unification of broken parts.

Solves quadratic equations,
and studies polynomial equations.



Al-Batani 850-929

Translates "Almagest" Ptolemy's astronomy,
Including exact description of earth axis precession.



al-Farabi abu Nasr 879-959

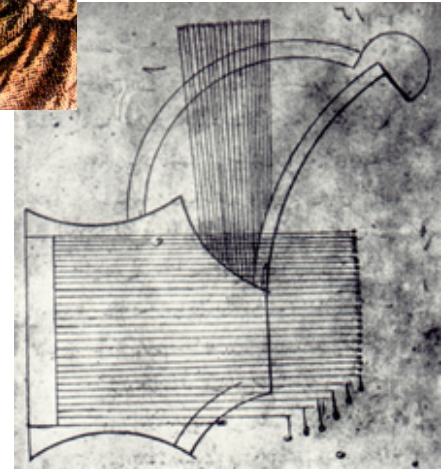
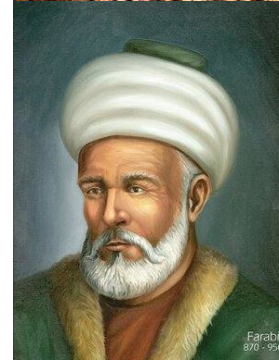
Philosopher, mathematician and musician
of Afghan-Persian origin

[a musical instrument drawn in al-Farabi's book]

Abu I-Hasan 'Ali Ibn Nafi' 789-857

Added a fifth string to the Oud.

Established a school of Music in Cordoba.



Al-Kindi - Alkindus 801-873

Imported ZERO from India, and implemented decimal

“Arabic number system” with decimal point.

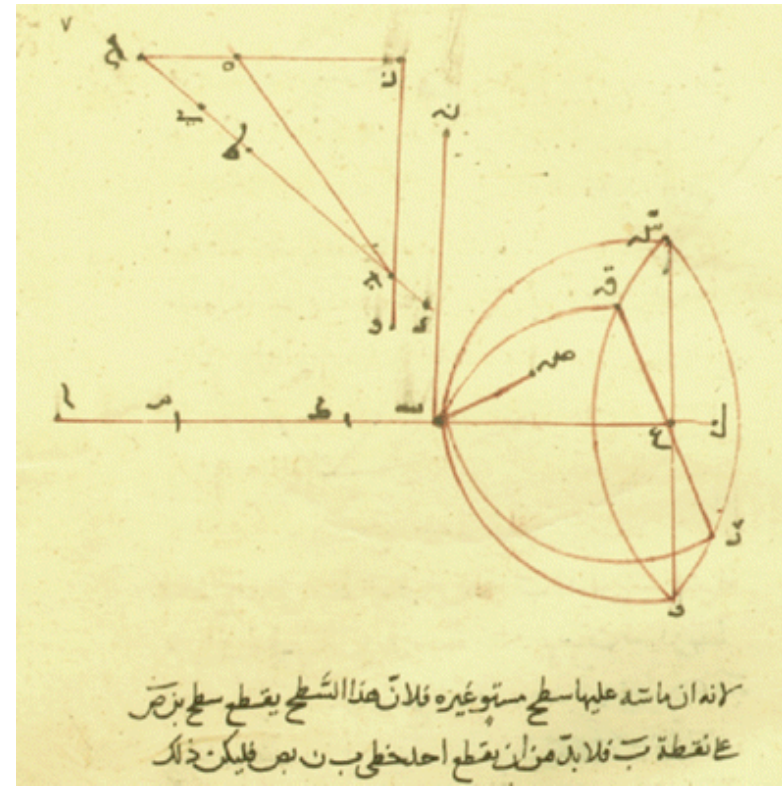
Developed encoding-decoding methods for encryption by combinatorial calculation of frequency of occurrence of words in texts.

In optics – he analyzed critically classical theories of vision. Aristo proposed three-dimensional medium (air) contact between the eye and the seen object. Euclid believed that the eye sends straight rays and detects the reflections back from the object. Euclid is compatible with seeing a circle as a line from the side, but not Aristo.



Abu Sa'd al-'Ala' ibn Sahl 1000-940

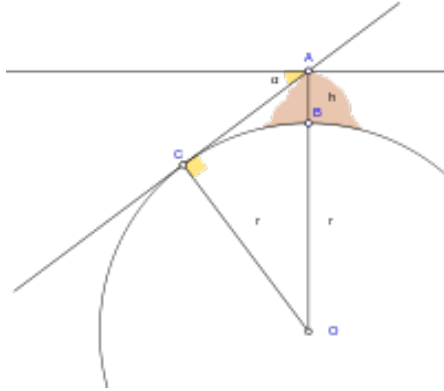
Law of light refraction in media (later: Snell's law).



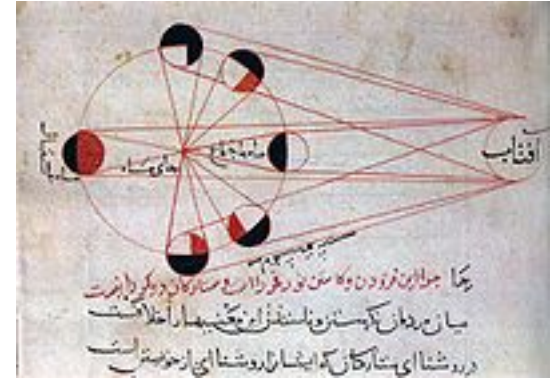
Abū al-Rayhān Muhammad ibn Ahmad al-Bīrūnī 973 -1048

Lived in Persia, Bukhara and Baghdad. Assimilated Indian sciences in anthropology, geodesy, astronomy and mechanics.

Was first to show that velocity of light is much bigger than sound velocity



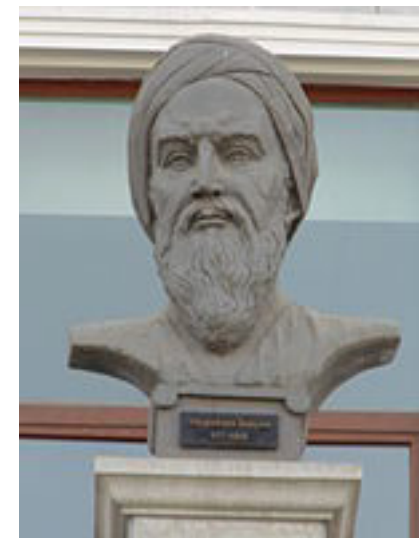
A diagram for measuring the radius of earth



Explanation for the phases of the moon



Commemorating Biruni in Tajikistan



and Tehran

Ja'far ibn Muhammad
Abu Ma'shar al-Balkhi
Abu Said al-Sijzi
Nasir al-Din al-Tusi
Mo'ayyeduddin Urdi
Ibn al-Shatir Averroes
Abu Said Sinjari
Qutb al-Din al-Shirazi
Umar al-Katibi al-Qazwini

Astronomers whose work influenced Copernicus heliocentric model

Al-Khazini 1121

Dependence of gravity and potential energy on height.
Decrease of air density with height



Ibn Khaldun

Pioneer in social and political sciences, History of cultures and politics. Demography,

Ibn Yunus~1000

In Egypt. Pendulum. Was applied right away to city tower clocks in Europe.

Al-Jazari (cizre) 1206

A Turkish engineer and mechanical inventor. Camshaft and Crankshafts
And worm gears in automata to convert circular to linear motions
and vice versa. Double-acting water pump.
First blood measurement devices.



Abu Yusek Yacob ibn Ishak al-Kindi (Alkindus) 850

Optics, medicine, rejected transmutation of metals

Omar Khayyam 1079

Calculated length of a year = 365.24219858156 days
Shortens in 6th place after the decimal point during man's lifetime. Today: 365.242190
Solved 3rd order (cubic) equations algebraically and graphically.

Awhad al-Zaman Abu'l-Barakat al-Baghdadi 1120

Abraham ben Meir Ibn Ezra 1145

Spread the use of ZERO in Spain and then into Europe (via Fibonacci)

Al-Kashi (born 1380)

Algorithm for computing n^{th} order root, based on successive approximations

Nasīr al-Dīn al-Tūsī

Mathematician

$$(1+x)^n = 1 + nx + \dots$$

Abu al-Qasim

Medical books. Surgical tools: scalpel, syringe, clamps. Extended on the Indian **Sushruta**.

Al-Jahiz

Describes struggle for existence in nature, and food chains.

Al-Dinawari

Father of Botany in the Arab world. Describes 637 plants, plant life cycle, flowers and fruits.

Modern optics was born following works of Islamic scientists, among them:

Ibn Sina 980-1037; al-Bīrūnī 973-1048; al-Shirazi 1236-1311 ; Al-Farisi

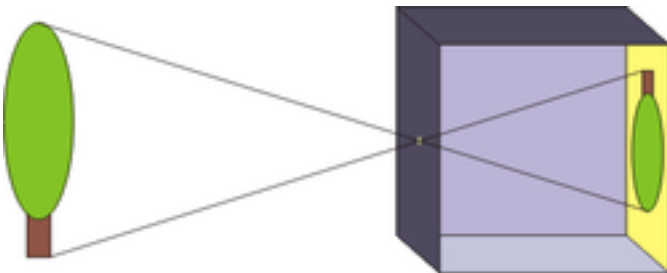
And above all: **Ibn al-Haytham (Alhazen) 965–1040**

Alhazen is considered the greatest scientist of his period. With other contemporaries they were first to explain vision correctly: light rays emerge from the sun as straight lines in all directions, hit objects and reflected rays enter the eye, acting as an optical lens. They complemented Euclid and Ptolemy's geometric optics. Alhazen demonstrated that light move in straight lines, With **Ibn Sina** they measured reflection and refraction in various materials and determined their laws, forming the basis for focusing lenses and mirrors. He drew ray traces in spherical lenses and parabolic mirrors, and understood spherical aberration.

He describes the principle of least time for light path.

Alhazen invented Camera Obscura (pinhole camera). He published his findings in "**Optica**", considered with Newton's "Principia" the most influential books in physics.

Alhazen rejected Ptolemy's description of star movements.



Al-Farisi (born 1260) Qutb al-Din al-Shirazi

Common division factors and combinatorial methods
Were first to correctly explain formation of rainbows

Ibn Bajjah (Avempace) ~1150

discovered the reaction to action of force – Newton's third law

Hibat Allah Abu'l-Barakat al-Baghdaadi (Nathanel) ~1150

Linked force to acceleration, not to speed as the classists believed. Preceded Newton's second law by 570 years (1687).

In his book "Kitah al-Mu'tabar" he describes personal impressions. Unlike Aristo, correctly describe the projector path of a stone, slowing horizontally and accelerating downwards vertically.

Ibn Rushd (Averroes) ~ 1150

Links force and kinetic energy. Denies Ptolemy's epicyclical star migration.

al-Jazari 1206

Applies pulley and Archimedes law of moments to practical machines. Pioneer in understanding momentum in body motion. Was also a physician.

Avicenna 980-1037

Bodies move due to applied forces, and not, as Aristo believed, by “ambition”

- Bhaskara 600-680

Indian mathematician that developed the basis for differential calculus. The concept of tangential and slope (was previously treated by the Greeks).

Writes an excellent approximation to sinus function:

$$\sin x \approx \frac{16x(\pi - x)}{5\pi^2 - 4x(\pi - x)}, \quad (0 \leq x \leq \frac{\pi}{2})$$

Spanish scientists (mostly from Maori Spain)

Abbas Ibn Firnas 810 – 887

Mathematician and inventor of eyeglasses. Manufacture glass from stones. Jumped at 875 from a tower in Cordoba using a wide coat and invented parachute and glider from wood and feathers that provided control on height and direction of gliding. Invented the metronome, and building clocks is attributed to him.

Maslamah al-Majriti (died 1008)

Mathematician, astronomer and chemist. Used Mercuric Oxide to show conservation of mass (500 years before Lavoisier)

Arzachel 1028–1087

Designed an astrolabe that was independent on latitude (quoted by Copernicus)

Rogerius Salernitanus: 1140 – c. 1195

Physician and surgeon. His book "Chirurgia Magna" was studied in the medical schools of Bologna in Italy. Use egg yolk for curing open wounds. Teaches that injured nerves cannot regenerate.

Ibn al-Baitar -1248

Botany and pharmacy. Registered 1400 plants for nutrition and medications,

Jewish scientists in Spain

Maimonidas 1190

A physician and philosopher. He composed medical books that critically reviewed Galen's heritage, including guides to nutrition, gymnastics, sexology, mental health and affects on physical health, and preventive medicine. He discuss the digestive system and snake bites. Collected 2000 drugs and causes of death. Criticized superstition and false believes (preceding Paracelsus).

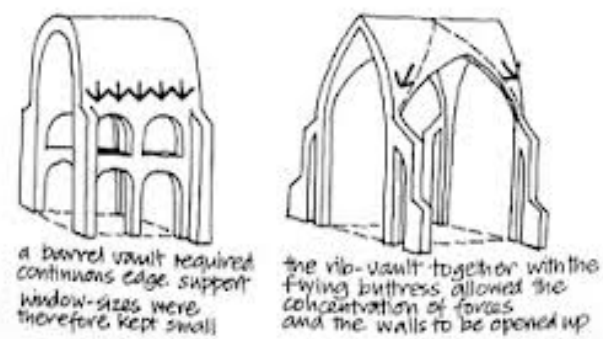
Maimonidas wrote an extensive summary of the Jewish code of laws ("Mishne Tora") and searched logical basis in Jewish religion ("Guide for the Perplexed"), attempting to reconcile with Aristo's philosophy.

Solomon ben Judah Ibn Gabirol, or Avicebron 1050

Was a poet and a philosopher discussing metaphysics, matter and the universe.



ARCHITECTURE and ART

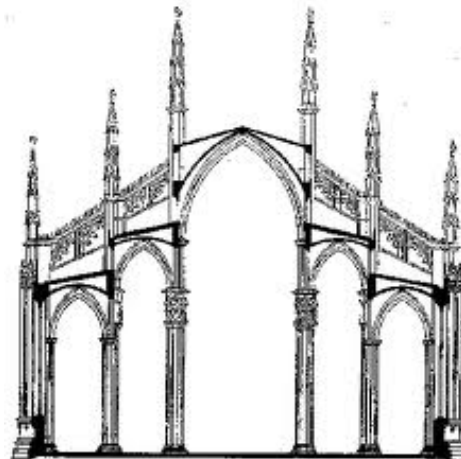
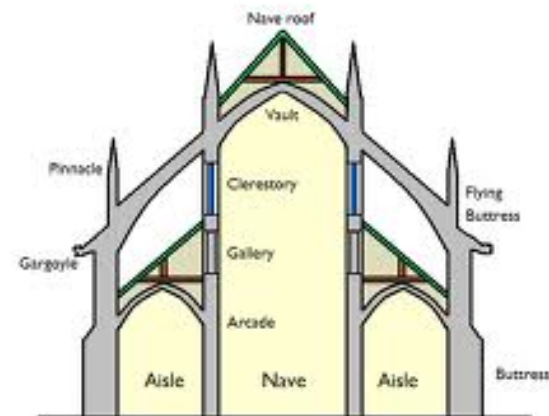
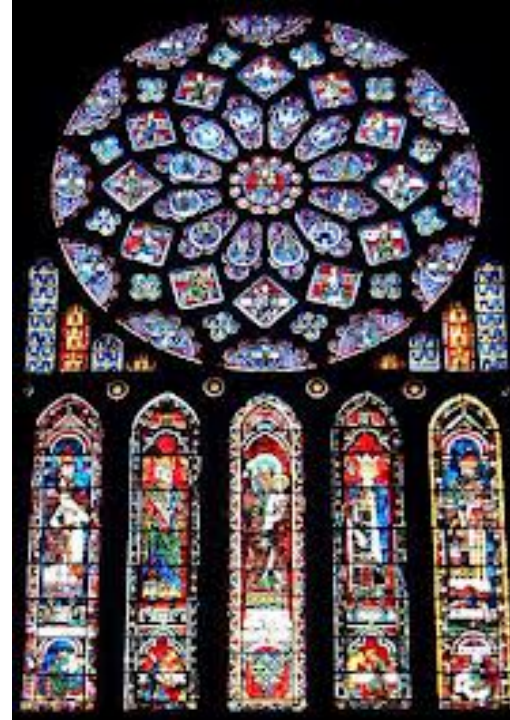


Romanesque architecture (till 1100)

Very bulky walls and columns

Gothic architecture (from 1100)

Note the supporting wings providing thinner columns and huge high space inside the church.





The static forces supporting the roof in a cumbersome crusades castle (Acre, Israel), versus the fine inside columns due to the supporting wings in Gothic Notre Dam de Paris

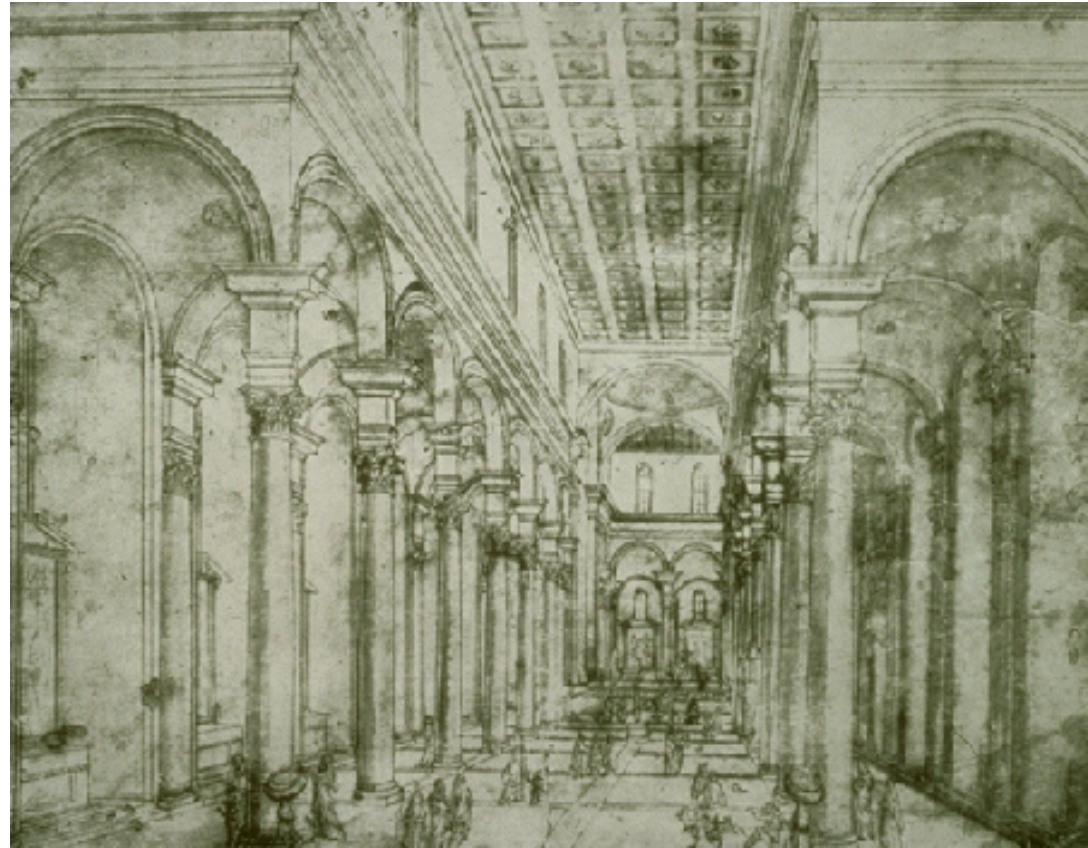
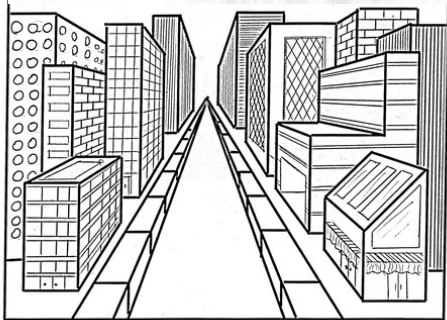
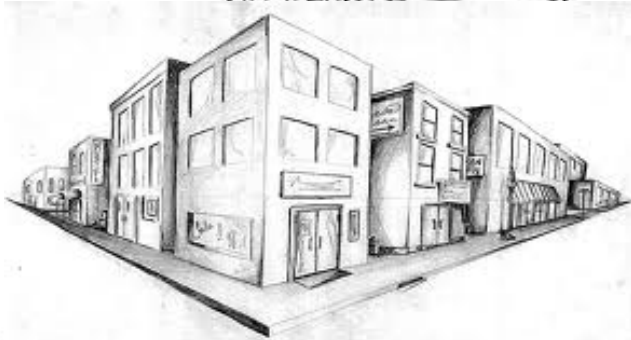
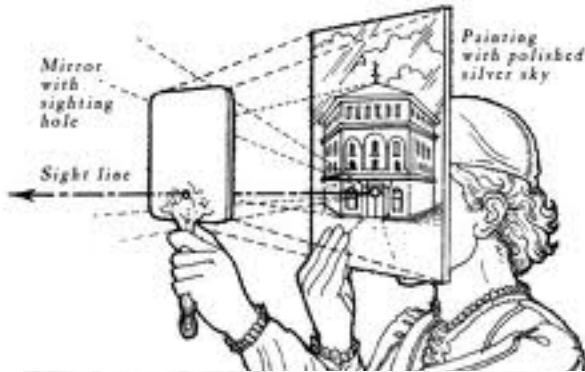


Hagia Sofia in Constantinople, the largest dome of its time in the world.
Below: Alhambra in Andaluc a, Spain. Forest of thin supporting columns.

The first minarets appeared during the 9th century under the Abbasid rule.



1420 Filippo Brunelleschi use in his drawings perspective based on optics. Depending on viewer position with respect to the building he sets one or two meeting points to parallel lines. (see: Geometry in the ancient and classical world))





Giotto di Bondone 1266-1337

Florentine painter and architect. Painted chapel Scrovegni in Padua. Revival of the classical Roman style in fresco, drawing naturally bodies and space.

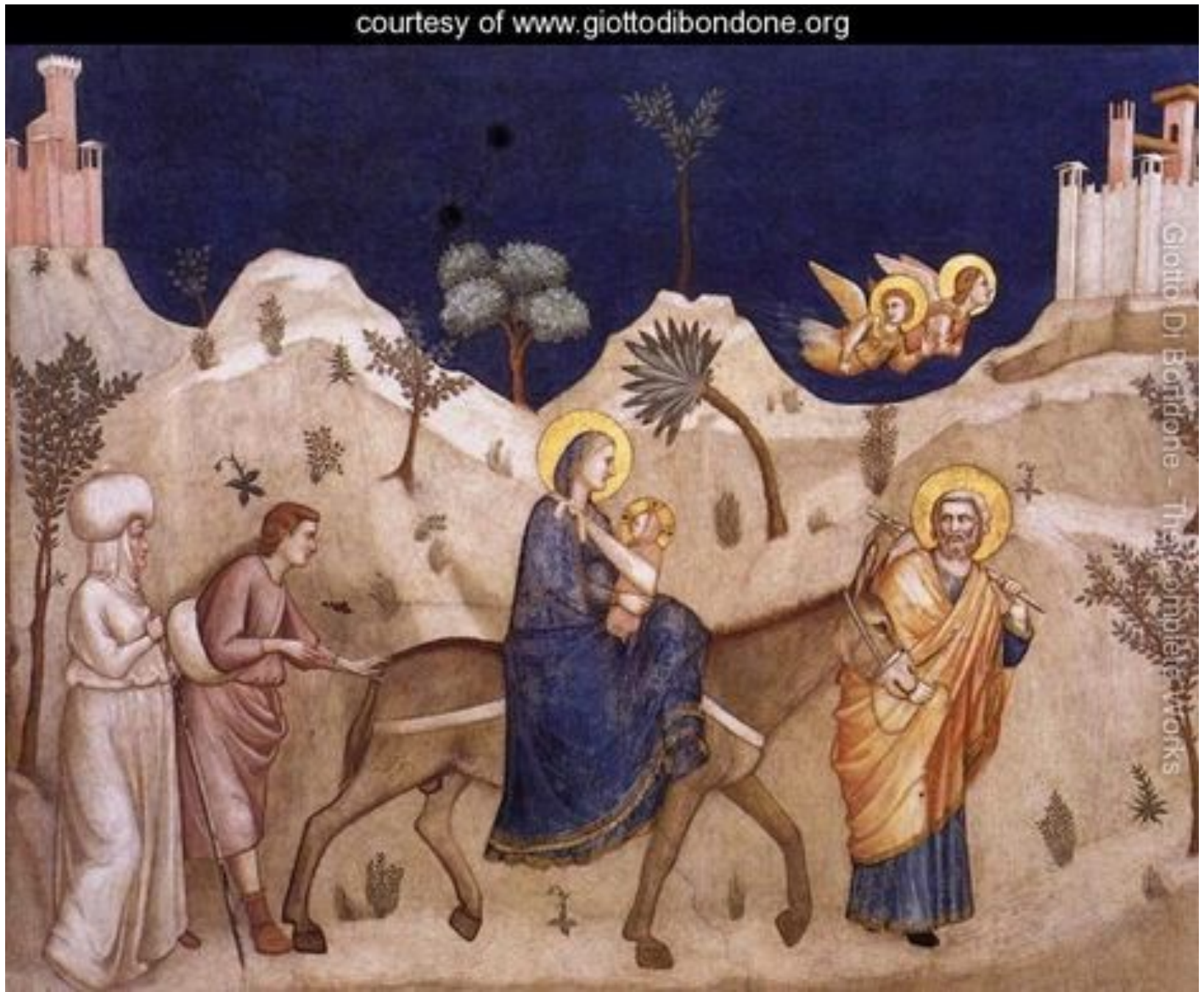
Portrait of
Dante Alighieri
By Giotto



Giotto's
bell tower
in Florence

Giotto – Jacob go to Egypt

courtesy of www.giottodibondone.org



Islamic Pottery and glazing

Quite distinct from the Chinese porcelain with blue and white paintings, the Islam world pottery developed colorful decorations with opaque white background and colorful metallic paintings and patterns that were typical to Persia, Baghdad and Spain.



Decoration in Mosques (both sculptured relics in stone, and ceramic tiles patterns) carried some parallel features to pottery decorations, with repeating lattice of shapes matching the structural ones.



LITERATURE



Durante degli Alighieri- Dante 1265-1321

An Italian poet and writer. Wrote “Divine Comedy”, three volumes [Hell (Inferno), Purgatory (Purgatorio), and Paradise (Paradiso)] describing a journey in human nature, morality, compassion and religious obedience. The books are written in Florentine Italian (not Latin), in poetry structure.

Dante was a public servant who fought for freedom from the pope's control. He was sentenced to exile, died in Verona, and became a national hero. His grave was recently moved to Florence.



Francesco Petrarca 1304 –1374

A poet considered with Boccaccio and somewhat with Dante as the founders of modern Italian. His rediscovery of Cicero's letters are considered one of the initiators of Italian Renaissance.

Petrarca is described as the father of humanism.

His sonnets to Laura refers to a woman not as a spiritual symbol, but a person with feelings.



Giovanni Boccaccio 1313-1375

Poet and writer, worked in Florence and Milan, and was influenced by Petrarca. He wrote "Decameron" (10 days) including stories told by men and women who fled from the plague into a castle in Florence. He decorated some copies.

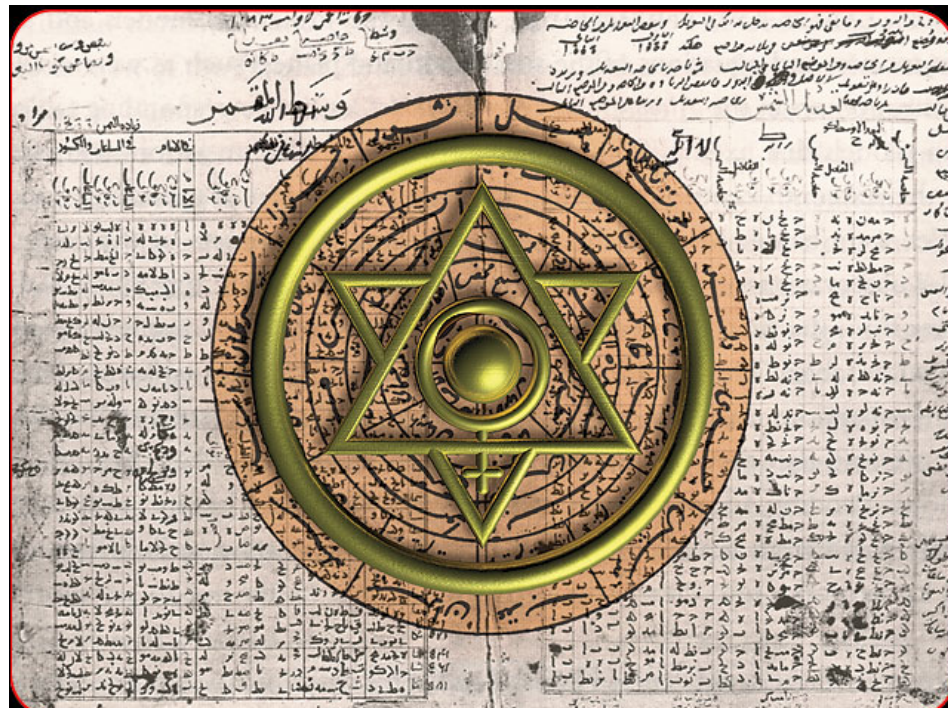
Decameron illustrations



CHEMISTRY

Aristo believed in four “sources” that make all compounds: Fire (hot and dry), Earth (cold and dry), Water (cold and wet) and air (hot and wet). Since metals are purified from ores, he believed that by changing ratios of the above four sources all metals can be purified, including gold. They believed the “Philosopher stone” or powders from this stone facilitates the variation in source ratios, and help transmutation of one material to another.

This Greek and classical chemistry models of four “sources” was falling apart in hands of chemists of the Arabic world. The increasing number of acids, bases and alcohols in use and the improvement in furnaces to melt and purify metals provided means to carry experiments that rejected the four sources theory.



Arabic chemists

Abu Musa Jabir ibn Hayyan (aka Geber), 815

Considered the father of chemistry. He advanced theoretical and experimental basis to chemistry.

He purified acids from grapes, fermented fruits, lemons, and burning sulfur and ammonium chloride:

e.g.: Hydrochloric acid HCl , nitric acid (aqua regia) HNO_3 , Citric acid $\text{C}_6\text{H}_8\text{O}_7$, acetic acid CH_3COOH & tartaric acid $\text{C}_4\text{H}_6\text{O}_6$

He studied the chemical reactions of mercury and sulfur.

Geber implemented a scale to evaluate effectiveness of drugs.



Muhammad ibn Abu Bakr Muhammad ibn Zakariyā al-Rāzī (Rhazes) 854 – 925

Classified materials: salts, metals, acids etc. (Sulfuric & Nitric acids and aqua regia). Described distillation of kerosene and other

hydrocarbons from crude petroleum using an apparatus called alembic, (kerosene lamps) and distillation of alcohols (ethanol), glycerin from olive oil and manufacturing of solid soaps with nice smell.

Studied steel hardening.

Based on experiments he carried, he rejected Galen's 4 humors theory and Aristo's 4 sources.

Wrote chemical prescriptions: "Kitab al-Asrar" (Book of Secrets)



Other scientists that studies chemistry:

Al-Farabi 872-951 from Persia and Damascus

Avicenna 980 –1037 wrote the medical Canon

Al-Ghazali 1058–1111 philosopher

Al-Kindi (Alkindus) 9th century – rejected metal transmutations

In India the **atomic theory** of matter repeatedly appear and discussed.

-

Chemists in Europe

Robert Grosseteste 1220

Publishes remarks about Aristo and scientific research methods.

Roger Bacon 1267

Publishes about research methodologies. Experimented with gun powder.

Albert the Great (1193–1280)

1250 First to isolate pure Arsenic by heating soap with the mineral orpiment (As_2S_3).

Pseudo-Geber, 1310

Anonymous Spanish chemist who published under Geber's name. He claimed that all metals are composed of Mercury and Sulfate in various ratios.

Arnold Bochmove de Villanova, (1235 – 1310)

Distilled alcohol by cooling the vapor. 1332 prescribed liquor and gin production

COPPER 850 The Moors in Spain precipitated pure copper from copper salts by iron - Sort of electrolysis.

BISMUTH purified **1500**

ZINC purified **1500**

Paracelsus 1530

Was first to use the term "Chemistry". The basis of pharmacology. Burnt Galen's writings as being false, so are Hippocrates and Aristo. Understood that diseases are caused by extra body factors – anticipating discovery of bacteria

Andreas Libavius 1597

Published "alchemy"

13th century and on: Cistercian monks develop melting furnaces in towers heated by blowing hot air (1300°C) driven by water flow. Such a furnace was found in Laskill, England, and was probably copied from similar Chinese furnaces.

Using such furnaces Metals were purified: **Copper, Arsenic, Bismuth and Zink.**

Mercury reaction, producing Mercuric-oxide and back was the experimental system to study preservation of mass in chemical reactions. (see **Maslamah al-Majriti**)

The purification of these metals emphasized that transmutations are impossible.

Steel hardening and sword production was an art specialized in Damascus.

Alcohol purification was applied by liquor producers starting with low alcohol wines, but provided also solvents for extracting drugs and dyes from plants.

900 Explosives – are prepared by Chinese monks for fireworks, and composed of mixture of Sulfur, Potash and Coal. Moved to Europe motivated by cannon technology.

Giles of Rome –1290 Supports atomism.

Albertus of Bollstadt-Magnus 1250 In Germany, isolates **Phosphorus**

1260 The Mamluks use hand cannons in Ain Jalut battle against the Mongols.

1453 The Ottoman Turks use “fire balls” in the conquest of Constantinople

In central America smoking Tabaco and chewing Coke was common. It was brought to Europe and North America, and spread fast.

Coffee was brought to the Arabic world from Africa. It evaded Europe slowly due to the church opposition (first coffee house in London 1572).

Tea was only imported at the 17th century by the Dutch and British from China.

PHYSICS

During the whole of the middle ages Aristo was the “great priest” of natural sciences: his doctrines were strongly supported by the church (most of the intellectual activities were concentrated in monasteries), and only a few disputed some of his theories (typically secretly, from the fear of the religious authorities). It is sarcastically told that Aristo describes 6 legs for spiders, and simple observations were not a part of the scientific method to dispute this description.

The Franciscans in Oxford denied some of Aristo’s doctrines, but mainly on religious grounds. **Roger Bacon**, who was not extremely religious (at least compared to his contemporaries), studied Aristo thoroughly. **The Dominicans in Paris** e.g. **Albertus of Bollstadt-Magnus (1250)** **Thomas Aquinas** accepted Aristo, excluding only determinism (all events are determined in advance) which was incompatible with religious principles. Others, e.g. **Siger de Brabant** accepted even determinism. In south of France (Montpelier) and in Italy (Salerno, Padua, Bologna) religion was less dominant, and Arabic and eastern influence were stronger, affecting more critical reading of the classicists.

The middle age sciences were engaged with scientific methodologies, including description of scientific laws and their verification logically and experimentally. Reflecting the sites of scientific activities, monastery scientists emphasized philosophy, while university people attempted experimental as well as applied-science approaches.

Science philosophy:

Thomas Aquinas 1227–74

Italian theologian and Dominican monk. Developed methods of proving religious claims in accord with scientific findings, therefore announced as a saint after his death.

Jordanes de Nemore 1230

Writes a mathematical expression to Archimedes' law of moments:
$$fm = \text{constant}$$

William of Moerbeke 1260

Translated Archimedes and Hero scripts into Latin

Ramon Lull 1274

Human brain is finite, Nature is infinite

John Duns Scotus 1290

The sense of existence.

Roger Bacon 1267 -1268

Specification of inspection and measurement methods.
Experimental and scientific verification.



1348 William Swineshead

“The computer from Oxford”. Claims that every physical entity is measurable.
Undefined ratio between infinity and finite quantity.

William of Ockham 1287–1347

1328 Ockham Razor, principle of parsimony.

“Entities should not be multiplied without necessity.”

Simple scientific explanations are preferable over complex ones.
This is a milestone in science, turning over centuries of scientific argumentations of mythical and complex nature that tended to conceal simple facts, just because they were incompatible with prejudice.

Ockham investigated kinematic and dynamic of body motion.



Relativity "sapekshavad" in space and time, was a philosophical concept in **India** discussed logically:

“just as a man in a boat sees the trees on the bank move in the opposite direction, so an observer on the Equator sees the stationary stars as moving precisely towards the west”.

It was only at the 20th century that Einstein, also first by “gedanken experiments”, predicted mechanical laws in inertial systems, based on the experimental finding that speed of light is constant, and these predictions were confirmed experimentally (shortening of time and distance in a moving system as measured by a standing observer).

Mechanics and light

1328 Thomas Bradwardine, William Heytesbury, Richard Swineshead, John Dumbleton

All from Merton College, Oxford, studies velocities and accelerations of bodies.

Jean Buridan 1298-1358

1350 proposed, in conflict with Aristo, that body motion is due to “momentum” he had acquired, and is proportional to his mass and velocity.

Nicole Oresme 1361

Moving distance = average velocity * time
Used coordinates.

Nikolaus von Cusa 1401–1464

1440 A German astronomer, who claims that astronomy conform with physical laws, and they can be expressed mathematically. Nature is infinite but brain is finite, therefore our understanding of nature is asymptotic.

Earth moves around the sun, stars are suns.

He performed an experiment demonstrating that plants use air in their metabolism.

Vincent of Beauvais 1230

Speed of light is finite, and light spreads in a way similar to sound waves.

Maximum angle of the rainbow, close to sunset, is 42°.

Kitah al-Shifa

Disputed Aristo's mechanics theory that moving bodies are driven by air. He teaches that motion is coming from the body itself (momentum).

517, John Philoponus

Also analyzes body motion.

530, Simplicius of Cilicia

Understand gravitational force – tendency of bodies to fall, and teaches that acceleration is inversely proportional to distance.

Jean Buridan a. 1295-ca. 1358

In Paris university, develops dynamic theory for bodies in motion at constant speed, acceleration (impetus) and inertia, totally diverting from Aristo's. He believes that heavy and light bodies fall with same acceleration. The high towers built in many cities allowed testing this revolutionary concepts, but it was not until Galileo that a public demonstration made impact and turned the course of mechanics towards Newton's three mechanical laws.

Valerius Cordus 1540

Extended the theory of aether.

Magnets and Compasses:

1190 Magnetic needle compass in Italy

1269 Pélerin de Maricourt

Describes magnetic poles, and notes that they cannot be separated

Petrus Peregrinus of Picardy, Italy

Iron needles turn in direction of the poles of a magnetic stones.

Clocks:

1335 The first mechanical clock in Milan.

1389 Pendulum and clock on a tower in Rouen, rings every 1/4 hour

1364, Giovanni di Dondi

1370 Builds a clock with dates.

1370 Parisian clocks are synchronized.

.Optics:

Roger Bacon 1266

Polishing lenses for eyeglasses and magnifying glasses. Studies refraction and dispersion of light. Used camera obscura to image the sun. Studies spherical and parabolic mirror aberration.

Vision is from light falling on the eye (not emerging from it, according to Aristo)

Alessandro della Spina 1285

Eyeglasses for short sight people.

Dietrich (Theodoric) von Freiberg 1305

Used glass balls and water bottles to study internal reflection and color dispersion, and applied it to explain the formation of the rainbow.

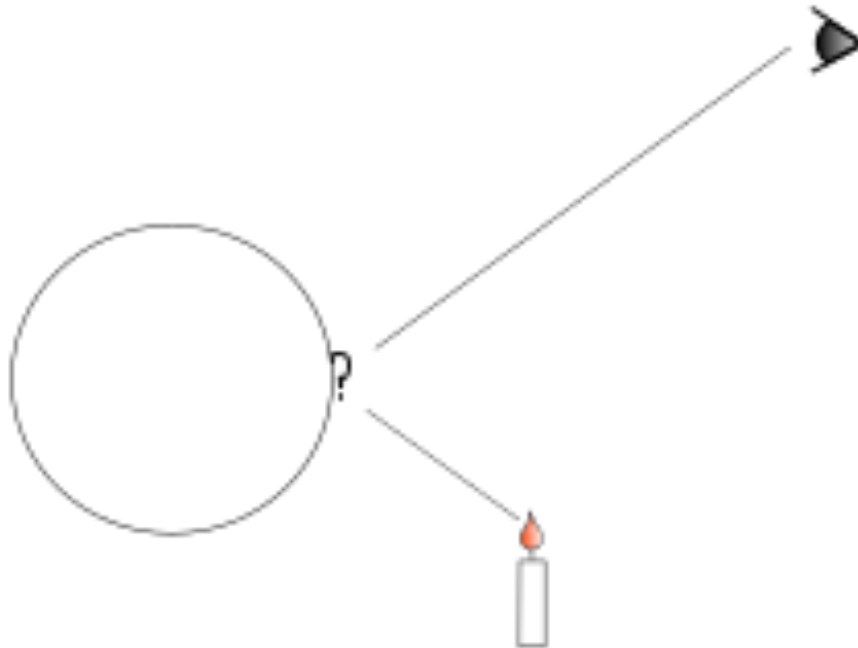
Nicholas of Cusa 1451

Convex lenses for eyeglasses, probably imported Chinese technology.

.Optics:

Alhazen's problem

Seems to be simple, this problem posed first by Ptolemy was only solved recently. It asks: what is the point on a reflecting sphere that will reflect back light from a source to an eye positioned at two arbitrary points.



ASTRONOMY and EARTH SCIENCES

Astronomers - performed measurements on inclination of earth rotation axis and on star locations with increasing precisions, laying the ground for Copernicus work.

Latitude measurements became more precise because of clocks.

Compass— brought from China (Earliest record by the English scholar Alexander Neckam) **and Actrolabe** – from Arabia (al-Biruni), enabled marine navigation in open oceans and geographic explorations.

Supernova explosions demonstrated that the sky is not made of solid crystals.

Walcher of Malvern 1091

Observed sun eclipse in Italy and calculated latitude of England from the time the eclipse was observed there.

Omar Khayyam 1079

Determines day length at 11 digits accuracy



Nicolas Oresme 1320/1325-1382

1377Proves that earth is rotating around the sun, and stars are resting.

Ibn Sina (Avicenna) 980-1037

Geology: Propose two reasons for creation of mountains and valleys: Rise of earth crust shell, e.g. during earth quakes, or in erosive processes by water, preferentially eroding soft rocks and leaving hard rocky hills.

Albertus 1260

A geology book, based on fossils explorations of Avicenna. Was first to purify Arsenic.

Siger 1260

Study of tides. Correlated with the position of the moon, but puzzling irregularities.

Albert of Saxony 1350

Was probably first to notice the difference between center of mass, and geometrical center. Noted that the earth center diverts from its center of mass.

Geology: Heat and drying create deserts and hills.

Logical games with infinite series.

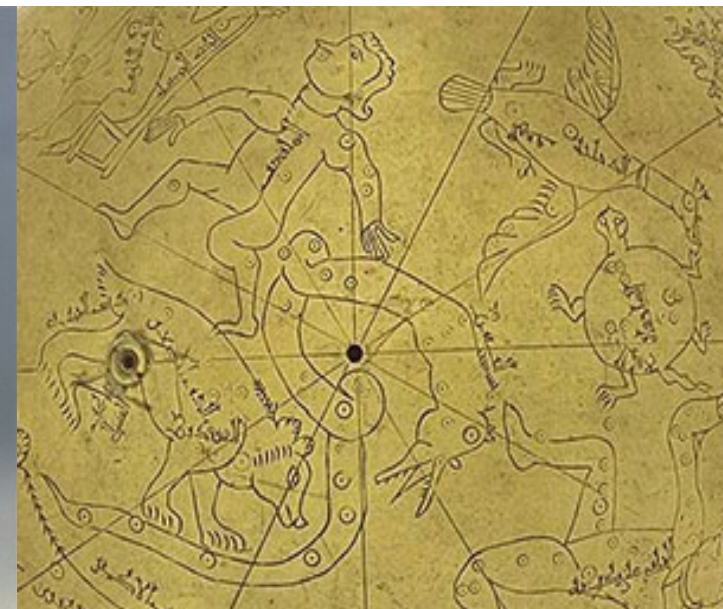
Islamic Astronomy

Astronomy serves in all religions to set holiday times. In Islam - timing of the 5 daily prayers, the full moon for a new month and assigning the direction of Mecca are essential. But astronomy was central for navigation in land and ocean commercial convoys (unlike the Mediterranean shipping). In order to read star positions, measuring instruments were invented (astrolabes, quadrants, equatorium torquetum) spherical trigonometry was developed, and astronomical tables were composed.

al-Zarqâlî from Toledo 11th century

Distributed astronomical tables in Europe, and spread Arabic terminology e.g. zenith, azimuth, nadir (the direction opposite the zenith), almucantar (a circle parallel to the horizon, defining star altitude)

The picture shows a sky globe, signed by Ibrahim Ibn Sa'id and his son, Muhammad from Valencia, at the year 478 of the Hagra (1085 AC). It marks 1025 stars from Ptolemy's star catalogue. A ring of the sun course (the ecliptic) is tilted with respect to earth axis and the equator. The globe is fixed on two rings allowing to set longitude and horizon and read the time of rise of a star from the sun-to-star angle.



The two faces of an astrolabe

**An earlier
instrument to
measure
latitude:
the Kamal**



For the first time in history, Astronomy and Astrology are separated. Observatories in Iran, Baghdad and Damascus recorded deviations from Ptolemy's model in his "Almagest" as translated to Arabic.

Arabic scientists started to believe that space is less dense than air, found evidences for changes in intensity indicating that stars are not solid, and described attraction between masses and applied it to star motion.

Ibn al-Shatir

Motion of the moon and planets.

al-Battani, Ibn al-Haytham

Corrections to the geocentric model.

Ja'far ibn Muhammad Abu Ma'shar al-Balkhi, Abu-Rayhan Biruni, Abu Said al-Sijzi, Qutb al-Din al-Shirazi, and 'Umar al-Katibi al-Qazwini & Averroes

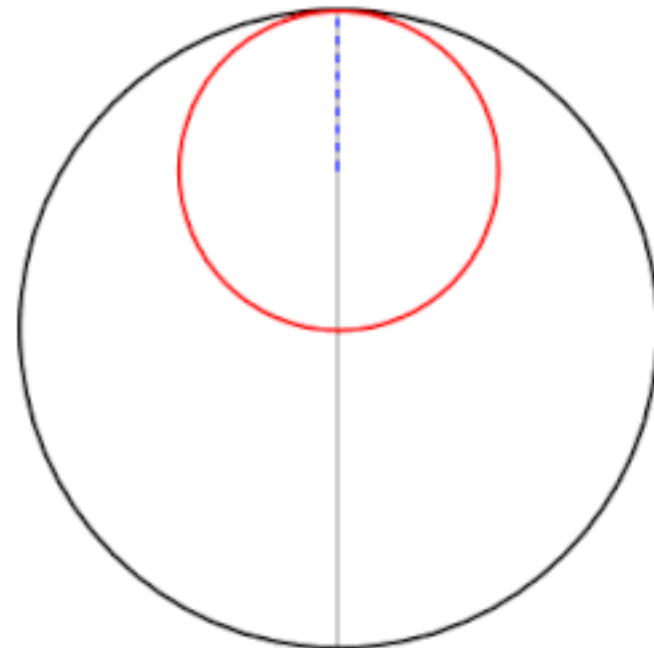
Discussed the heliocentric model.

Maragha astronomers: Nasir al-Din al-Tusi, Mo'ayyeduddin Urdi and Ibn al-Shatir

Influenced Copernicus. Al-Tusi described linear motion of a point on a circle rotating inside twice its radius circle:

Tusi Couple:

.



In India:

Aryabhata 476-550

In his Aryabhatiya (499) and Aryabhata Siddhanta he develops a heliocentric model with gravity and elliptical orbits. Describes the perimeter of earth and planet years of rotation around the sun. Moon light is reflection of sunlight. He defined trigonometric functions, and calculated tables for them. He used algebraic techniques.

Brahmagupta , 7th century

Recognizes gravitational forces.

Invented ZERO as a digit and a reserved place in decimal presentation of numbers. His books were translated to Arabic at the 9th century.

Bhāskara , 12th century

Siddhanta Shiromani: book on astronomy. Virtual and absolute position of planets due to earth and sun daily motion with respect to the constellations.

Kerala school of astronomy and mathematics: Madhava of Sangamagrama

14th-16th century

Contributions in mathematics, astronomy and calculus.

TECHNOLOGY

Previously we listed technologies that migrated from **China** to Europe: Shipyard docks for building and repairing ships, Calipers, Piston pump, Iron casting, Furnaces with water driven air blowers, Iron ploughs, seeding machine, wheelbarrow, Sieving machine, Air fan, Propeller, Parachutes, Hanging bridges, Natural gas for warming, Matches, Arches, rackets (including multi-stage), Bridle and Horseshoes, Embossed maps, Sky Globe turned by water.

793 Paper: **Chinese** invention brought to Baghdad then to Europe
Block printing, and **removable letters blocks** brought from **China**. 1455 **Gutenberg**
Iron and steel furnaces and steel hardening was developed in **China** and **India**

Spinning wheel for drawing threads of wool and cotton invented in **India**
Windmills appear in Persia in the 7-9th centuries

1477 Louis the 9th establishes **mail services**

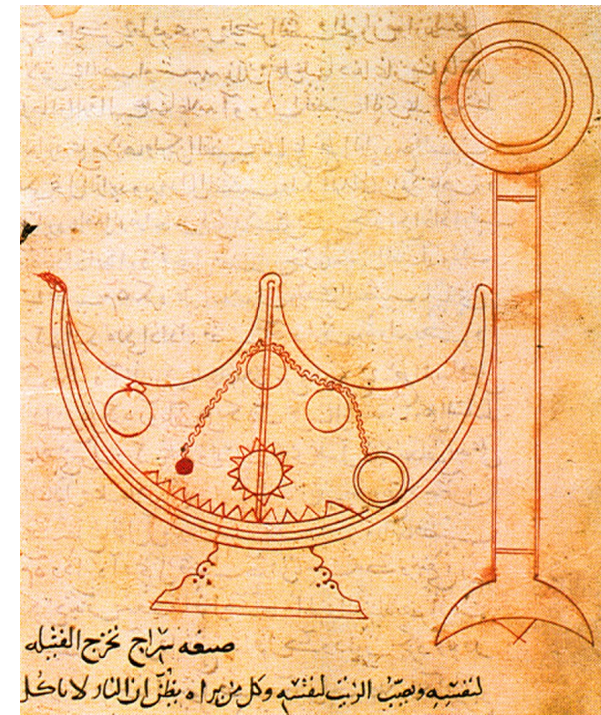
The Banū Mūsā brothers: Abū Ja‘far, Muḥammad ibn Mūsā ibn Shākir (803 –873), Abū al-Qāsim, Aḥmad ibn Mūsā ibn Shākir (d. 9th century) and Al-Ḥasan ibn Mūsā ibn Shākir (d. 9th century)

three 9th-century Persian scholars who lived and worked in Baghdad.

They are known for their “Book of Ingenious Devices on automata (automatic machines) and mechanical devices”.

Include: self-operating valves, timing devices, delay systems, Steam-driven pipe organ and music sequencer.

The drawing from this book shows a delay line for automatic trigger.



Military technologies:

Multistage rockets were invented in **China**

Iron rockets were developed in the Mysore kingdom, south of **India**

1241 Mongols used lighted kites in the battle of Legnica in Poland.

Eilmer of Malmesbury

Built a wooden glider that was launched from a bell tower and glided for 200 meters.

Roger Bacon 1250

Technical description of aviation, and a design of an ornithopter.

Marco Polo 1300

Writes about parachutes, as well as ceremonial and manned kites,

Giambattista Danti della Porta 1496

A mathematician who hovered from a tower. 1558 he publishes theories and drawings to build kits

Hieronimus Bosch 1500

Paints a battle between air ships over a burning city (see next slide)



A triptych by
Hieronymus Bosch
depicting air ship battle
over a burning city

Iron processing and steel hardening for swords enabled by hot air furnaces with water-driven blowers copied from the **Chinese**.

Bridle and Horseshoes, Pulls and cranks, and armatures for horse and man, starting of 8th century (in **China** from 477, and in **Persia** from 694, taken forth by the **Crusaders**)

Improvements in bows, arrows, cannons and rifles. Starting at the 11th century bows with aiming loops and triggered release. Their lethal effect brought an attempt at **gun control** by the second Ecumenical session...

Burning projectiles were already in use with Catapults by the Romans.

The trade with China imported to Europe and Arabia gunpowder and associated arms (cannons, guns). “**Black Berthold**” 1313 was probably first to use bronze cannon, maybe captured from the evading Mongols.

Rockets propelled by solid fuel, were also brought from **China**.

673 the Ottoman fleet use “Greek fire” during the siege on Constantinople. They were explosive cannon shells containing kerosene, quicklime, sulfur and tar.

1260 The Mamluks use hand cannons in Ain Jalut battle against the Mongols.

1320 Gunpowder start to be used by the military.

1346 English archers defeated French Armored knights in battles during the 100 years war, and ended the age of knighthood in Europe.

1326 Pot-de-Fer and Baton-a-feu are early cannons or mortars brought to use by the French during the 100 years war.

1387 in **China** – Arquebus - a rifle, used by three-lines of soldiers: first shooting and receding back, second and third load the rifles. Second half of the 15th century, in use by Hungarians. Replaced at the 18th century by Muskets.

1326 French discover that drying of gunpowder with water forms beads with better blast upon explosion. 70 fortresses recaptured from the English using cannons end the 100 years war at 1450.

1453 The Ottoman Turks use “fire balls” in the conquest of Constantinople.

The concept of Walls and Moats to defend cities falls apart.

1500 Development of rifles back-loaded and locked with exploding primer.

15th century – the Ottoman army was first to have several standing rows of soldiers with firearms, loading and shooting in turn.

CLOCKS

Sundials:

Most ancient clocks. Maybe Stone Hedge and Obelisks were used as clock gnomons (shadow casting center stick).

Problems: hour depends on the season,
length of shadow and its speed varies during day.

Solution: Tilted dial at an angle equals latitude: the shadow creates equally spaced hours.

Abu'l-Hasan Ibn al-Shatir in 1371 was first to record doing it.



Water Clock (clepsydra):

Mechanisms driven by river flow, see Chinese drawing:
With the development of glass dishes:
water level in a vile:

Problem: Flow varies with water level in upper vile.

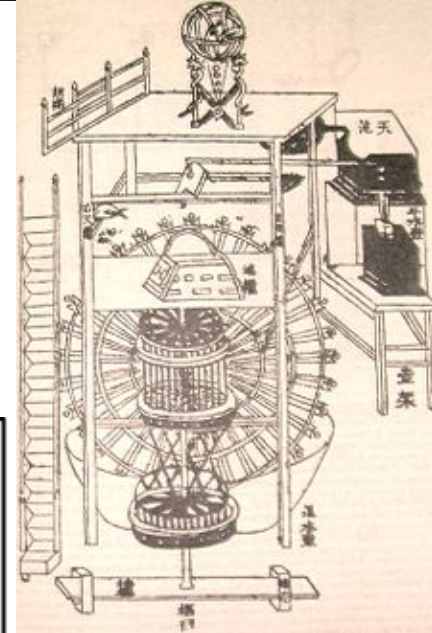
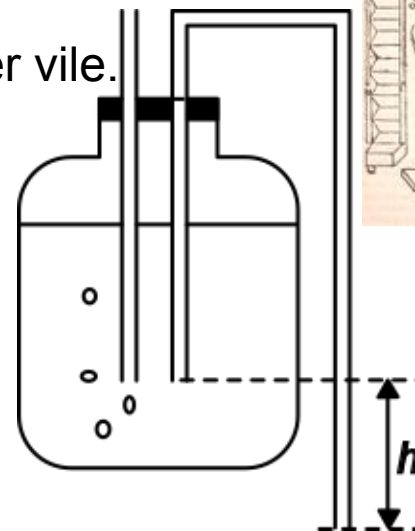
Solution: “Mariotte bottle”: constant
pressure at the level of exit pipe

Candle Clock: with hour marks.

Sand clocks (hourglass):

Friction between sand grains creates uniform
sand stream, independent on amount of
sand left.

Problem: need to turn over every hour for
long time measurements.



CLOCKS

Mechanical Clocks:

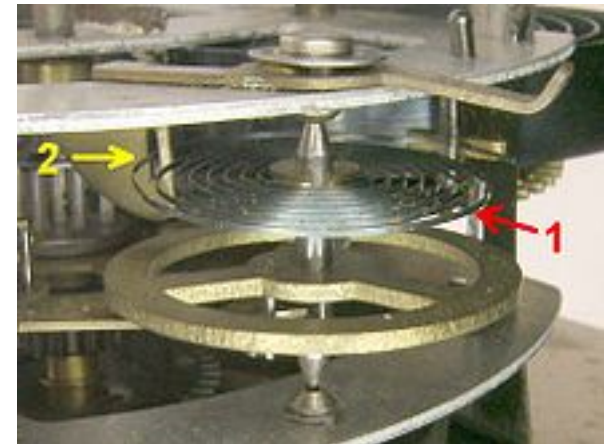
Pendulum clocks, driven by weights or springs. Proliferated following the invention of the release mechanism for one tooth at a time in a cogwheel. Early mechanisms were constructed from pegs in wood wheels, and slowly evolved into brass and steel gears.



Nautical clocks, Marine chronometers:

To determine position in sea Longitude, Latitude were needed. The measurements depended on time. But pendulum clocks could not be used in sea. Sailors tables tried to bypass the problem by making measurements of sun height at midday (highest sun point), or of the polar (north star) just before dawn or after sunset. Such measurements depended on weather...

The invention of balance wheel **1** and spiral spring **2** provided stability of the mechanisms against motion, and production of marine chronometers started.



Modern clocks: **Electrical** motor driven by **quartz** oscillator.
Radioactive clocks, counting radioactive decay events.

MATHEMATICS

Algebra:

2000 BC Babylonian tables with algebraic rules.

250 BC Diophantus in Alexandria use algebraic methods.

820 Muhammed ibn Musa al-Kwarizmi in Baghdad, writes the first systematic composition about Algebra: Al-Kitab al-mukhtasar fi hisab al-gabr wa'l muqabala – meaning: calculation, complementation and equalization of equations.

Al gabr became Algebra in Europe.

1130 Al-Samawal applies all arithmetic tools on unknowns, e.g.:

P=percent discount X=price D=discount $D=p \cdot X/100$ or: $X=100 \cdot D/p$ or: $p=100 \cdot D/X$

10th century Al-Karaji use induction to prove equations dependent on integer n

al-Banna al-Marrakushi 1256 – 1321 was a Moroccan mathematician who translated Euclid to Arabic, and wrote numerous scripts himself.

mathematical generalization steps:

Number 5 is a generalization to any five objects

or quantity of materials (grams of gold, liters of wine)

Sign X is a generalization to any number or quantity.

Algebraic equation is a relation between quantities.

Allow elimination of unknowns from equations.

Variables, expressions and equations:

Variables can accept any numerical value, so are expressions (or functions) in terms of variables, that accept a numerical value when the variables are substituted by numbers.

Equality between two expressions allow to solve for unknown value required to uphold this equality numerically.

Laws of Order of operations is a convention to prevent ambiguities:

$8+9-3$	$8+(9-3)$
$8-9-3$	$8-(9-3)$
$8+2*9-3$	$8+2*(9-3)$
$9/3-2$	$9/(3-2)$

$$3x+5x = 8x$$

$$7x=14 \rightarrow x=14/7$$

$$3x=x+x+x$$

$$x+2=5 \rightarrow x=5-2$$

Algebraic unknown elimination technique:

$$3x=y \rightarrow F \cdot 3x = F \cdot y \quad (F \neq 0)$$

$$x/3=14 \rightarrow x=3 \cdot 14$$

$$5x/12=4$$

$$20-7x=6x-6$$

$$3(x-1)=2(x+3)$$

$$z/16 = 2(3x+1)/9$$

$$x-8=x/3+1/6$$

$$\frac{3}{4}x+2 = \frac{3}{8}x-4$$

$$3/x=5$$

$$3/(x+4)=4/(x+3)$$

$$8(3x+10)=28x-14-4x \Rightarrow \text{infinite number of solutions ...}$$

Rabbi Abraham ibn Ezra c. 1140

Describes the symmetry in the coefficients of expanding $(a+b)^n$

1	$(a+b)^0$
1 1	$(a+b)^1$
1 2 1	$(a+b)^2=a^2+2ab+b^2$
1 3 3 1	$(a+b)^3=a^3+3a^2b+3ab^2+b^3$
1 4 6 4 1	$(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4$
1 5 10 10 5 1	$(a+b)^5=a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

Levi ben Gerson (Gersonides), 1321

Found explicit expression for these coefficients, (will be called Pascal's triangle)

1342 he publishes tables of sinus function at 5 digit accuracy, and writes the sine law for triangles:

$$\sin(a)/a=\sin(b)/b=\sin(c)/c$$

Abū al-Wafā' al-Būzjānī 10th century

Extensive use of all 6 trigonometric functions. Provided 8 digits accurate tables every $\frac{1}{4}$ degree. Uses the relations:

$$\sin(2a) = 2\sin(a)\cos(a)$$

$$\sin(a \pm b) = \sqrt{\sin^2(a) - [\sin(a)\sin(b)]^2} \pm \sqrt{\sin^2(b) - [\sin(a)\sin(b)]^2}$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \sin(b)\cos(a)$$

They were described in words, not equations, e.g.: "If you want to calculate the sine of a sum or difference of angles, you add or subtract the products of the sine of one, and cosine of the other".

He also knew the sine law for spherical triangles:

$$\sin A / \sin a = \sin B / \sin b = \sin C / \sin c$$

Ibn Yunus - end of 10 and start of 11th century

The Egyptian astronomer proved that

$$\cos(a)\cos(b) = \frac{1}{2}\{\cos(a+b) + \cos(a-b)\}$$

Applications of algebraic relations:

Triangulation for mapping,

Quadratic equations (e.g.: projectile path for artillery calculations)

Indian mathematics:

Aryabhata 476-550

Tables of trigonometric functions.

Place dependent number system (used points instead of zeros)

Bhāskara 12th century

Extensive use of trigonometry for Astronomy.

Madhava of Sangamagrama

Considered the creator of calculus: differential and integral operations, slopes and tangentials, numbers with power, infinite series, and power series, approximating functions by “Taylor series” (including for trigonometric functions, for which he calculated tables with 12 digit accuracy.

$$\sin x = x - x^3 / 3! + x^5 / 5! - x^7 / 7! + \dots$$

$$\cos x = 1 - x^2 / 2! + x^4 / 4! - x^6 / 6! + \dots$$

$$\text{Arctan } x = x - x^3 / 3 + x^5 / 5 - x^7 / 7 + \dots$$

His students continued his work for 400 years:

Kerala school of astronomy and mathematics 12th-16th centuries)

The ZERO

Allows to attribute a value to digits 1-9 according to its position: 2 or 20, dismissing the need for special signs for tens, hundreds, etc. Zero was imported to Persia from India, by **Al'Khwarizmi** and **Biruni**. Zero was introduced first as a digit 0 in a place-preserving decimal presentation. Zero use as a value brought about long persisting problems:

Brahmagupta 598-670

Was first to define laws for calculations with zero:

Adding or subtracting zero does not change the number

Multiplying a number by zero gives zero

And laws for calculations with positive and negative numbers

(called them credit and debit)

Debit minus zero gives debit

Credit minus zero gives credit

Zero minus zero gives zero

Subtracting a debit from zero gives credit

Subtracting a credit from zero gives debit

Multiplying credit or debit by zero gives zero

Multiplying zero and zero gives zero

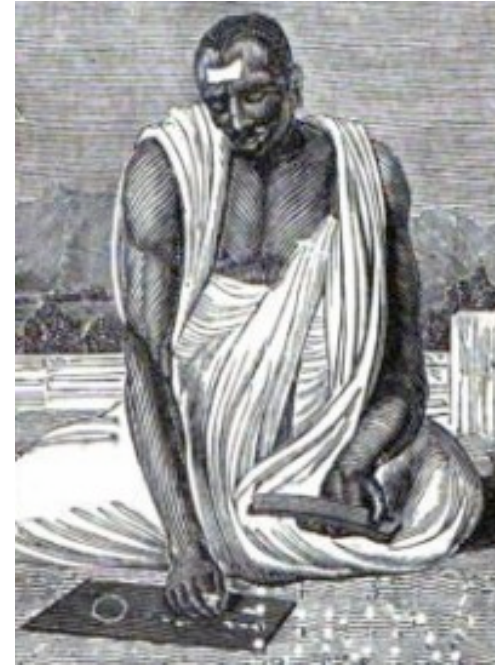
Multiplying or dividing two credits give credit

Multiplying or dividing two debits give credit

Multiplying or dividing credit and debit give debit

Dividing zero by zero gives zero

There is no solution for a number divided by zero

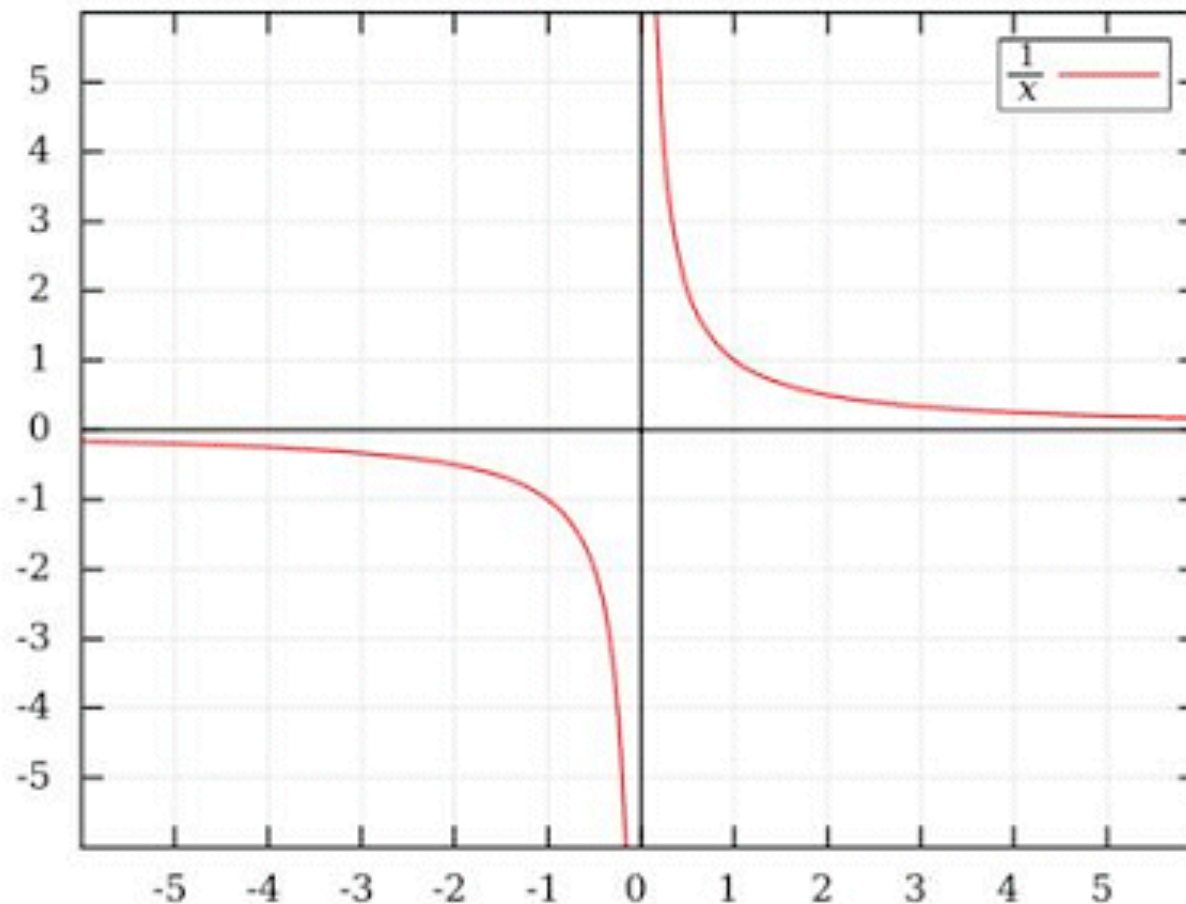


The zero calls for extending the range of numbers:

Like subtractions of positive numbers – create negatives

Division by ZERO create “infinity” (∞)

**As x gets bigger and bigger, $\frac{1}{x}$ approaches zero.
Conversely, as x gets smaller and smaller and
approaches zero, $\frac{1}{x}$ approaches infinity.**



The ZERO cont.

Mahavira 800-870

Dividing a number by zero leave its value

Bhaskara II (Bhaskaracharya) 1114-1185

Calls the result of division by zero “infinity”, a number that does not change upon adding or subtracting a number.

Mohammed ibn-Musa Al'Khwarizmi

Wrote in Iran a composition about the Indian number system including the zero

Rabbi Ben Ezra 1092-1167

In Spain, spreads the use of Arabic number system (calls zero a wheel) (in the picture: using an astrolabe)

Al-Samawal 1130-1180

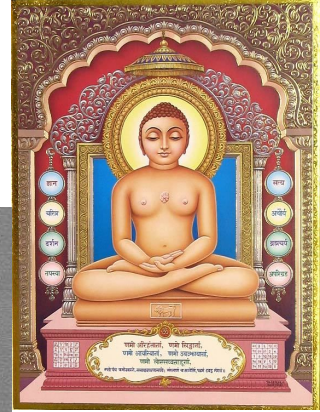
In Iran, discuss arithmetic with zero

Zhu Shijie or Chu Shih-Chieh 1260-1320

In China, writes a composition on zero, possibly by Indian influence. At this time also the **Maya** in central America device a position-dependent system, base 20, and included zero as a number.

Jamshid Al Kashi d. 1429

Composed “Book of addition and subtraction by the method of calculation of the Hindus” introduced the zero and the place value system to Baghdad.



The ZERO cont.

Leonardo of Pisa (Fibonacci) (c.1170-1250)

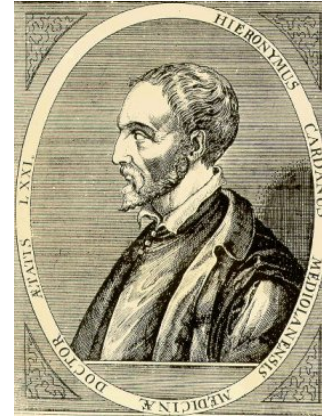
Zero is implicated as a sign in position-dependent number system, the Arabic system. Zero is not yet a value though.

Girolamo Cardan or Cardano 1501-1576

In Italy, solves quadratic and cubic equations without using zero, which greatly complicated the solutions.

Rene Descartes 1596-1650

Uses (0,0) as the origin of the coordinate system



Arabic vs. modern numbers:

Hindu-Arabic	0	1	2	3	4	5	6	7	8	9
Western Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	۰	۱	۲	۳	۴	۵	۶	۷	۸	۹
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९

The ZERO cont.

We see that the confusion about these laws of arithmetic with zero persisted for hundreds of years. Indeed, if

$$\infty = n/0$$

then algebraic laws will imply that that

$$0 * \infty \text{ is any number } n \dots$$

The correct definitions of zero came only with **Leibnitz** and **Newton**, by the development of differential calculus and the concept of incremental approach to a number, expressed in modern algebra as:

$$\lim_{x \rightarrow y} [f(x)]$$

Under these definitions the ratio of two functions $f(x)$ and $g(x)$ that get the value of zero at $x=y$, $[f(y)=g(y)=0]$ can be evaluated, and depending on the “strength” (or power) of their approach to zero, it can be 0, ∞ , or any number in between.

Number theory

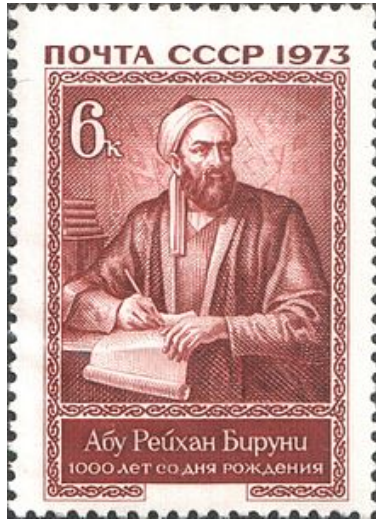
Al Haytham describes a theorem for primes:

any number n is a prime number if, and only if, $(n - 1)! + 1$ is divisible by n
This is called today “Wilson’s theorem”

To prove theorems Al Haytham used proofs by contradiction

Abū Rayḥān Muḥammad ibn Aḥmad Al-Bīrūnī 973-1048

Developed methods of deciphering encoded messages based on frequency of letters in use (good for letter-to-letter encoding).



1000 years after Biruni, he is commemorated in stamps by Afghanistan, Russia, Syria and Iran, the present country borders where he lived 1000 years ago.

Fibonacci's series

Two rabbits mate at age of 1 month, and give birth 1 month later:

Month 0 – 2 rabbits

Month 1 – 2 rabbits + 1 pregnancy

Month 2 – 4 rabbits + 1 pregnancy

Month 3 – 6 rabbits + 2 pregnancy

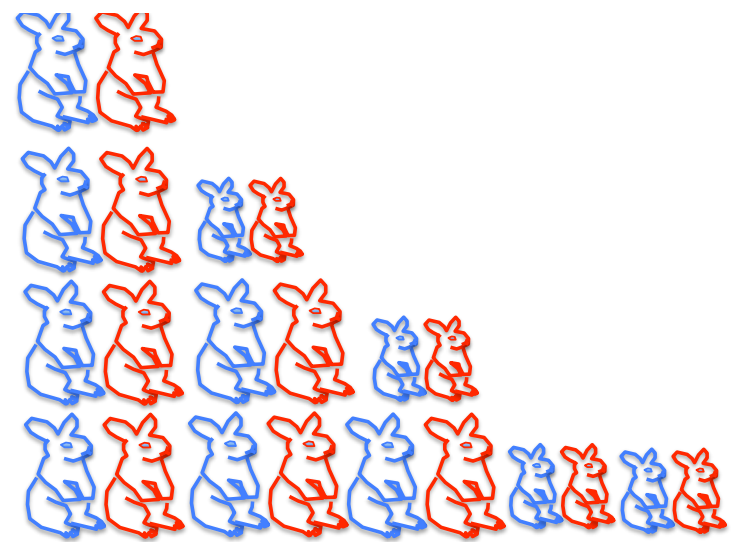
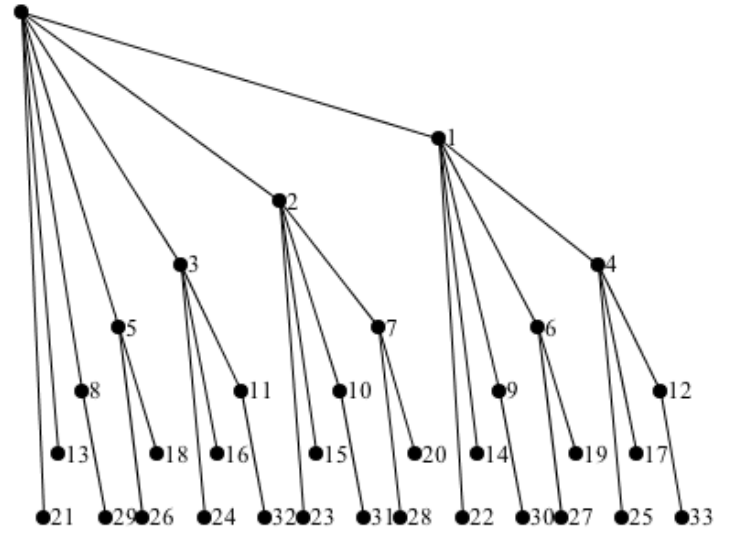
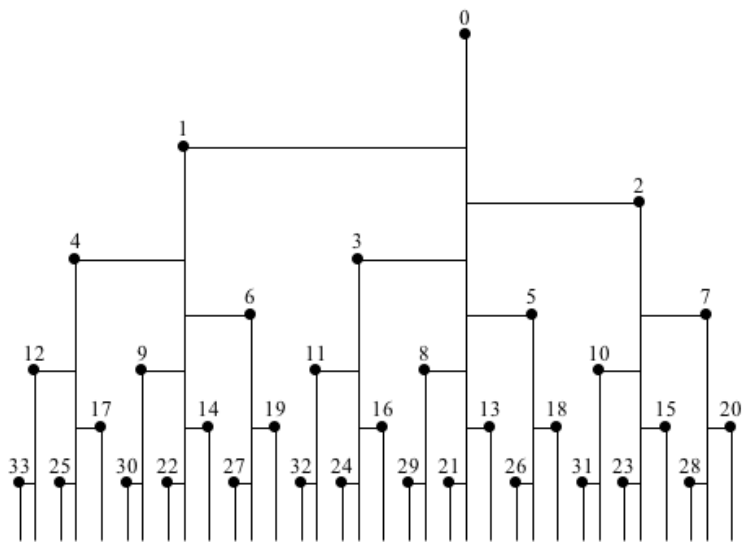
Month 4 – 10 rabbits + 3 pregnancy

Month n – A_n rabbits + A_{n-1} pregnant

Month n+1 – $A_n + A_{n-1}$ rabbits A_n pregnant

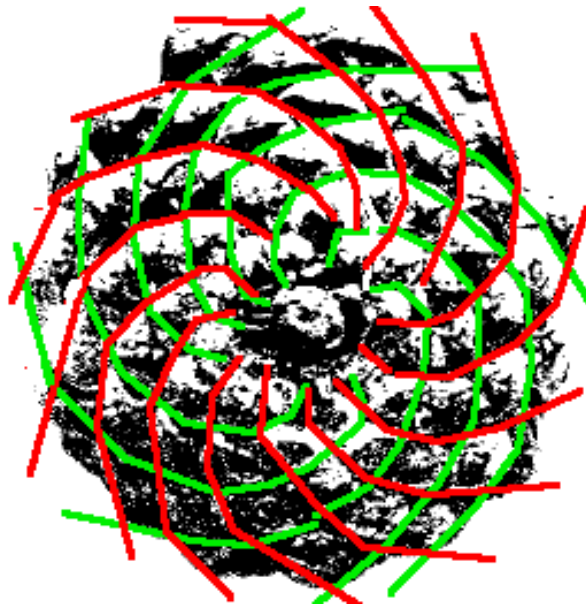
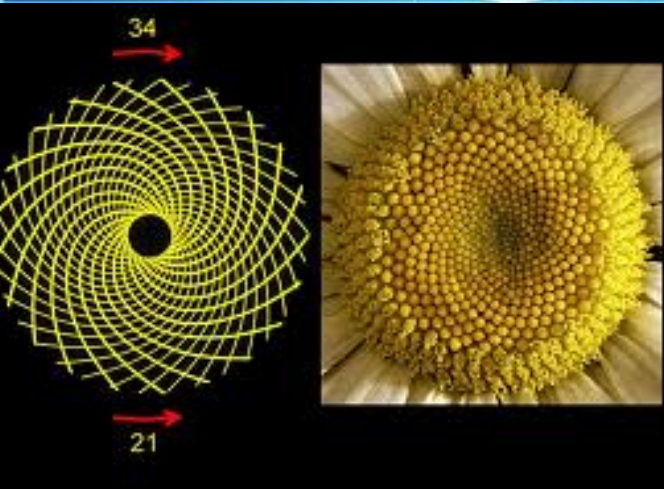
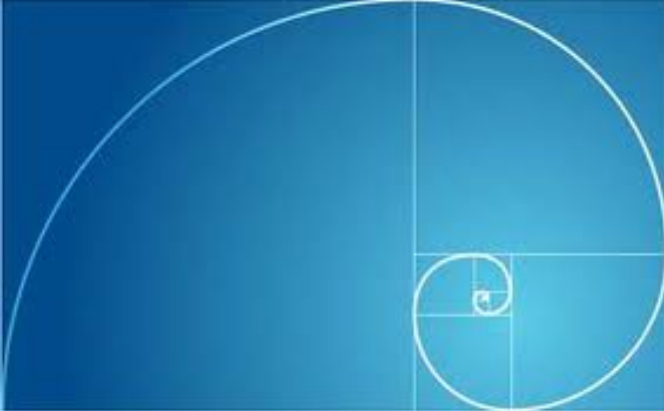
the series formed:

1,1,2,3,5,8,13,21,34, ...



Fibonacci's search is more efficient than binary for “divide and conquer” optimization algorithm (search for a lion in the desert). Interesting: the ratio between successive Fibonacci's numbers approach the golden ratio

Examples of Fibonacci's series in Nature
Result from time delayed cell division:
Sunflower, Snail, Pine fruit

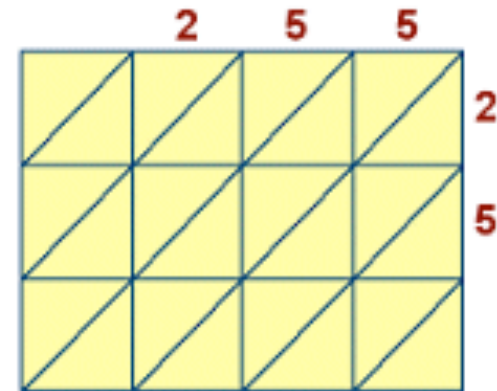


Fibonacci's Lattice multiplication method (according to Khwarizmi)

Lattice multiplication

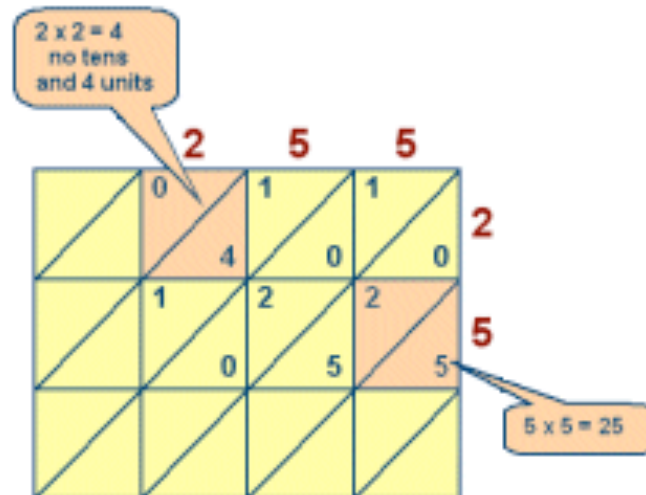
STEP 1

To multiply using the lattice method, create a grid with diagonal lines, and split the numbers to be multiplied into their place values, e.g. 255×25 (see right)



STEP 2

Multiply the columns by the rows, splitting the digits up on either side of the diagonal (see below)



STEP 3

Finally, add along the diagonals to get the answer: $255 \times 25 = 6,375$ (see below)



Add along the diagonals
 $0 + 2 + 0 + 1 = 3$

$$a = \sqrt{xYZ}$$

$$b = \sqrt{yZX}$$

$$c = \sqrt{zXY}$$

$$d = \sqrt{xyz}$$

$$X = (w - U + v)(U + v + w)$$

$$x = (U - v + w)(v - w + U)$$

$$Y = (u - V + w)(V + w + u)$$

$$y = (V - w + u)(w - u + V)$$

$$Z = (v - W + u)(W + u + v)$$

$$z = (W - u + v)(u - v + W).$$

Tartaglia formula for the volume of tetrahedron

$$V^2 = \frac{1}{288} \det \begin{bmatrix} 0 & d_{12}^2 & d_{13}^2 & d_{14}^2 & 1 \\ d_{21}^2 & 0 & d_{23}^2 & d_{24}^2 & 1 \\ d_{31}^2 & d_{32}^2 & 0 & d_{34}^2 & 1 \\ d_{41}^2 & d_{42}^2 & d_{43}^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

UVWuvw=d_{ij} distance between vertices

$$T = \frac{1}{4} \sqrt{- \begin{vmatrix} 0 & a^2 & b^2 & 1 \\ a^2 & 0 & c^2 & 1 \\ b^2 & c^2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}}$$

This is a kind of extension of Heron formula for the area of triangles given their edges

$$\text{volume} = \frac{\sqrt{(-a + b + c + d)(a - b + c + d)(a + b - c + d)(a + b + c - d)}}{192uvw}$$

$$T = \frac{1}{4} \sqrt{(a + (b + c))(c - (a - b))(c + (a - b))(a + (b - c))}.$$

Niccolò Fontana Tartaglia-1557- 1500 (the stutterer)

Was city enforcement engineer and land surveyor in Venice. Translated Archimedes and Euclid into Italian. Projectile orbit studies (confirmed by Galileo). Was injured in his face in the conquer of Brescia by the French during the War of the Holy League.



THIRD ORDER EQUATIONS

$$ax^3 + bx^2 + cx + d = 0 \quad (1)$$

$$\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2.$$

$0 > \Delta$: three different solutions

$0 = \Delta$: double real solutions

$0 < \Delta$: one real and two complex solutions

$$x_1 = -\frac{b}{3a}$$

$$-\frac{1}{3a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$-\frac{1}{3a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$x_2 = -\frac{b}{3a}$$

$$+ \frac{1 + i\sqrt{3}}{6a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$+ \frac{1 - i\sqrt{3}}{6a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$x_3 = -\frac{b}{3a}$$

$$+ \frac{1 - i\sqrt{3}}{6a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$+ \frac{1 + i\sqrt{3}}{6a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

Tartaglia-Cardano method

The solutions can be found with the following method due to Scipione del Ferro and Tartaglia, published by Gerolamo Cardano in 1545. This method applies to the depressed cubic substituting $x=t-b/3a$ in the general equation

$$ax^3 + bx^2 + cx + d = 0 \quad (1)$$

Get: $t^3 + pt + q = 0$. (2)

with

We introduce two variables u and v linked by the condition

$$p = \frac{3ac - b^2}{3a^2}$$

$$u + v = t$$

and substitute this in the depressed cubic (2), giving

$$u^3 + v^3 + (3uv + p)(u + v) + q = 0 \quad (3)$$

At this point Cardano imposed a second condition for the variables u and v :

$$3uv + p = 0$$

As the first parenthesis vanishes in (3), we get

$$u^3 + v^3 = -q \quad u^3 v^3 = -p^3/27$$

Thus u^3 and v^3 are the two roots of the equation

$$z^2 + qz - \frac{p^3}{27} = 0.$$

At this point, Cardano, who did not know complex numbers, supposed that the roots of this equation were real, that is that

$$\frac{q^2}{4} + \frac{p^3}{27} > 0.$$

Solving this equation and using the fact that u and v may be exchanged, we find

$$u^3 = -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \quad v^3 = -\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

As these expressions are real, their cube roots are well defined and, like Cardano, we get

$$t_1 = u + v = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

The two complex roots are obtained by considering the complex cubic roots; the fact uv is real implies that they are obtained by multiplying one of the above cubic roots by $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and the other by $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$

If $\frac{q^2}{4} + \frac{p^3}{27}$ is not necessarily positive, we have to choose a cube root of u^3 . As there is no direct way to choose the corresponding cube root of v^3 , one has to use the relation $v = -\frac{p}{3u}$ which gives

$$u = \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} \quad (4)$$

and

$$t = u - \frac{p}{3u}.$$

Note that the sign of the square root does not affect the resulting t , because changing it amounts to exchanging u and v . We have chosen the minus sign to have $u \neq 0$ when $p=0$ and $q \neq 0$ in order to avoid a division by zero. With this choice, the above expression for t always works, except when $p=q=0$, where the second term becomes $0/0$. In this case there is a triple root $t=0$.

Note also that in several cases the solutions are expressed with fewer square or cube roots

If $p=q=0$ then we have the triple real root $t=0$

If $p=0$ and $q \neq 0$ then $u = -\sqrt[3]{q}$ and $v = 0$

and the three roots are the three cube roots of $-q$.

If $p \neq 0$ and $q=0$ then $u = \sqrt{\frac{p}{3}}$ and $v = -\sqrt{\frac{p}{3}}$,

in which case the three roots are

$$t = u + v = 0, \quad t = \omega_1 u - \frac{p}{3\omega_1 u} = \sqrt{-p}, \quad t = \frac{u}{\omega_1} - \frac{\omega_1 p}{3u} = -\sqrt{-p},$$

where

$$\omega_1 = e^{i\frac{2\pi}{3}} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i.$$

Finally if $4p^3 + 27q^2 = 0$ and $p \neq 0$ there is a double root and a simple root which may be expressed rationally in term of p and q , but this expression may not be immediately deduced from the general expression of the roots:

$$t_1 = t_2 = -\frac{3q}{2p} \quad \text{and} \quad t_3 = \frac{3q}{p}.$$

To pass from these roots of t in Equation (2) to the general formulas for roots of x in Equation (1), subtract $b/3a$ and replace p and q by their expressions in terms of a,b,c,d .

Vieta substitution

$$x^3 + px + q = 0,$$
$$x = w - \frac{p}{3w}$$

We substitute

$$w^3 + q - \frac{p^3}{27w^3} = 0.$$

Multiply by- w^3

$$w^6 + qw^3 - \frac{p^3}{27} = 0$$

second order equation in- w^3

$$W^3 = -q \pm \sqrt{q^2 - 4p^3/27}$$

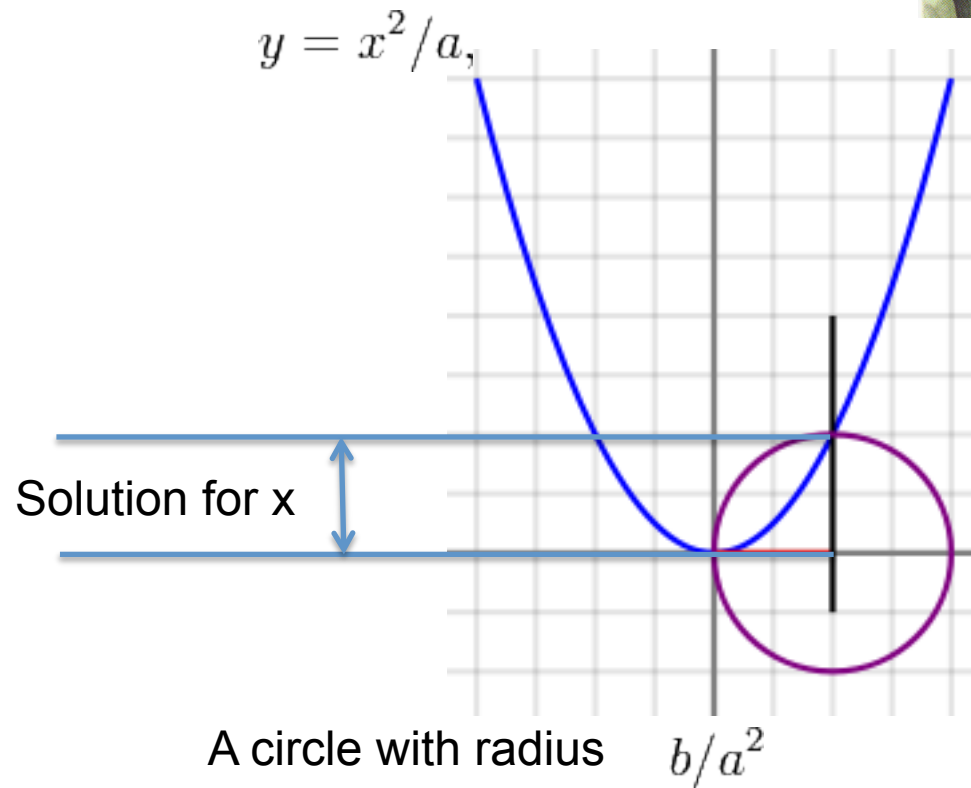
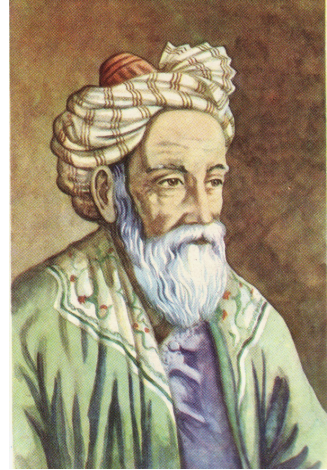
If w_1, w_2, w_3 are the cubic roots of the solutions for w^3

$$x_1 = w_1 - \frac{p}{3w_1}, \quad x_2 = w_2 - \frac{p}{3w_2} \quad \text{and} \quad x_3 = w_3 - \frac{p}{3w_3}.$$

$$\sqrt[3]{8^3} = \begin{cases} 8 \\ -4 + 4\sqrt{3}i \\ -4 - 4\sqrt{3}i. \end{cases}$$

$$\sqrt[3]{-27i} = \begin{cases} 3i \\ \frac{3\sqrt{3}}{2} - \frac{3}{2}i \\ -\frac{3\sqrt{3}}{2} - \frac{3}{2}i. \end{cases}$$

Omar Khayyám proposes a graphical solution to $x^3+ax^2=b$ for $b>0$
He classified 3rd order equations – used for non Euclidian analytical geometry



FOURTH ORDER EQUATIONS



Solving 4th order equation

Ludovico Ferrari 1540

Published with the solution for cubic equations by his teacher

Gerolamo Cardano 1545

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$



Let:

$$p_1 = 2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace$$

$$p_2 = p_1 + \sqrt{-4(c^2 - 3bd + 12ae)^3 + p_1^2}$$

$$p_3 = \frac{c^2 - 3bd + 12ae}{3a\sqrt[3]{\frac{p_2}{2}}} + \frac{\sqrt[3]{\frac{p_2}{2}}}{3a}$$

$$p_4 = \sqrt{\frac{b^2}{4a^2} - \frac{2c}{3a} + p_3}$$

$$p_5 = \frac{b^2}{2a^2} - \frac{4c}{3a} - p_3$$

$$p_6 = \frac{-\frac{b^3}{a^3} + \frac{4bc}{a^2} - \frac{8d}{a}}{4p_4}$$

Then:

$$x = -\frac{b}{4a} - \frac{p_4}{2} - \frac{\sqrt{p_5 - p_6}}{2}$$

$$\text{or } x = -\frac{b}{4a} - \frac{p_4}{2} + \frac{\sqrt{p_5 - p_6}}{2}$$

$$\text{or } x = -\frac{b}{4a} + \frac{p_4}{2} - \frac{\sqrt{p_5 + p_6}}{2}$$

$$\text{or } x = -\frac{b}{4a} + \frac{p_4}{2} + \frac{\sqrt{p_5 + p_6}}{2}$$

4th order equation solution
 $\alpha x^4 + \beta x^3 + \chi x^2 + \delta x + \varepsilon = 0$

$$x = y - \beta/4\alpha$$

$$y^4 + Ay^2 + By + C = 0$$

$$A = \frac{\gamma}{\alpha} - \frac{3\beta^2}{8\alpha^2},$$

$$B = \frac{\delta}{\alpha} - \frac{\beta\gamma}{2\alpha^2} + \frac{\beta}{8\alpha},$$

$$C = \frac{\varepsilon}{\alpha} - \frac{\beta\delta}{4\alpha^2} + \frac{\beta^2\gamma}{16\alpha^3} - \frac{3\beta^4}{256\alpha^4}.$$

Add and subtract
 $2sy^2 + s^2$

$$(y^2+s)^2 - [(2s-A)y^2 - By + s^2 - C] = 0$$

Then we factor the quadratic polynomial

$$(2s - A)y^2 - By + s^2 - C = (2s - A)(y - y_+)(y - y_-)$$

and make $y_+ = y_-$, which will impose a constraint on s (equation (4)). We will get:

$$\left(y^2 + s + \sqrt{2s - A}y - \frac{B}{2\sqrt{2s - A}} \right) \left(y^2 + s + \sqrt{2s - A}y + \frac{B}{2\sqrt{2s - A}} \right) = 0,$$

where s satisfies the *resolvent cubic equation*

$$8s^3 - 4As^2 - 8Cs + (4Ac - B^2) = 0.$$

The four solutions of (2) are the solutions of (3):

$$y_1 = -\frac{1}{2}\sqrt{2s - A} + \frac{1}{2}\sqrt{-2s - A + \frac{2B}{\sqrt{2s - A}}},$$

$$y_2 = -\frac{1}{2}\sqrt{2s - A} - \frac{1}{2}\sqrt{-2s - A + \frac{2B}{\sqrt{2s - A}}},$$

$$y_3 = -\frac{1}{2}\sqrt{2s - A} + \frac{1}{2}\sqrt{-2s - A - \frac{2B}{\sqrt{2s - A}}},$$

$$y_4 = -\frac{1}{2}\sqrt{2s - A} - \frac{1}{2}\sqrt{-2s - A - \frac{2B}{\sqrt{2s - A}}}.$$

Thus, the original equation (1) has the solutions

$$x_k = y_k - \frac{\beta}{4\alpha}, \quad k = 1, 2, 3, 4$$

It is impossible to write an algebraic solution to 5th order equations and higher orders,
proved by **Niels Henrik Abel 1824**

Abel–Ruffini theorem

1830, Évariste Galois - found the group of polygons that do have an algebraic solution: the beginning of group theory.

BIOLOGY and MEDICINE

In the following slides we list physicians pharmacologists and surgeons and their contributions to medicine.

The list demonstrates the slow departure from the classical (mainly Galen) heritage, and the beginning of independent ideas based on experimental studies.

This progress in biomedicine was centered in monasteries, and later medical schools and hospitals in main European and middle-east cities, but skipped the villages of central Europe. When the black plague hit Europe, a third to half of the population found their death, due to lack of basic understanding about minimal hygienic, and superstitious beliefs enforced by the Church relating the plague to lack of religious practices.

7000 BC Mehrgarh province, India

Bow drill for teeth cavities. Dental surgeries.

2600 BC suture, Imhotep, Egypt

Pharmaceutical creams

500 BC Sushruta, India

Plastic and cosmetic surgeries

400 BC Hippocrates, Greece

Hippocrates treatment bench

900 AD Ammar ibn Ali al-Mawsili, Baghdad

Hypodermic needle for injection of medications.

1000 Abu al-Qasim (Abulcasis)

Bandages, forceps, plaster, amputations and scraping procedures, hooks, needles and tandem threads in surgeries.

1020 Ibn Sina, or Avicenna

Thermometer, distillation from vapors, medical ointments.

Al-Quanun, or Canon of Medicine

Medicine is studied by experience and reason.

872, Ahmad ibn Tulun

built a hospital in Cairo that provided care to the insane, which included music therapy

Eyeglasses

The Arabs polished lenses for magnification and studies their optical properties. **Marco Polo** must have seen eyeglasses in China and brought the idea to Italy. They were produced at 1306 in Pizza by **Alessandro della Spina**. Soon guilds of spectacle producers were formed in Venice, including concave and convex glasses. But only **Kepler** came up with good optical explanation to vision corrections.



Biology and Medicine – during the middle ages, especially in the Arabic world, discarded Galen's heritage of four humors. Galen was translated into Arabic by **Hunayn ibn Ishaq** 830-870, about a century before **Avicenna's** medical canon was published at 1000.

Common medical procedures included dental care, plastic surgeries, syringes, scalpels, clamps and various surgical tools, as well as tools to facilitate births. Ointment creams, bandages and plasters were used for simple injuries.

Classification of diseases was related to human anatomy – heart and blood system, muscles, metabolism etc. Since bloodshed was prohibited by Catholics, “planned” surgeries were not allowed in Europe. **Mondino de Lucchi 1316** published an anatomical book based on post mortem surgeries. Surgeries on animals help gain knowledge on live body functioning and eventually to better diagnostics. Starting at the 13th century in Italy physicians register “concilium” or case study of diseases and their treatment. This helped accumulation of medical knowledge, and is used universally today.

Robert (Michael) Scot 1217 reported use of opium-mandragora-henbane based anesthesia during painful treatments. In **University of Salerno** anesthesia was applied,

Hugh & Theodoric Borgognoni brothers 1266 clean wounds with wine as an antiseptic to accelerate curing. Soaked cloth with anesthetic drugs to minimize patients pain.

1247 the first mental hospital “bedlam” (**Royal Bethlehem Hospital**) in London.

Guy de Chauliac 1363 published “Chirurgia Magna”. Advocates stretching broken limbs before fixation. Introduced teeth bridges and dentures.

Middle ages are dominated by belief (promoted by the clergymen and gave them power and control), that diseases are caused by sins, and cured by faith.

Maimonides linked the black plague to microorganisms and the immune response. Mathematical analysis of the effectiveness of drugs gradually replaces “traditions”. In Baghdad a pharmacy is opened.

Classification of plants was motivated by drugs extracted from them, but botanists become aware of the need in air and water for plant metabolism

1440 Nicolas of Cusa proves experimentally that plants get nutrition from air.

9th century Ammar ibn Ali al-Mawsili

Invented syringes for injections of drugs.

1200 Abu al-Abbas al-Nabati

from Andalusia – develops scientific methods for pharmaceutical research – testing, describing and classifying medical drugs based on empirical studies.

1225 — Ibn al-Baitar,

A student of Nabati, writes a botanical encyclopedia for pharmaceuticals: “**Kitab al-Jami fi al-Adwiya al-Mufrada**” including 1400 plants, nutritional materials and drugs, 300 of them were discovered by him. The manuscript was in use by botanists in Europe during the 18-19th centuries.

1242 — Ibn al-Nafis

In commentaries about the canon he describes pulmonary & coronary blood cycles.

1250, Gilbert the Englishman

Finds loss of peripheral sensory system (in the skin) as an early stage of diseases such as leprosy. Became a common diagnostic procedure.

1543 – Andreas Vesalius (1514-1564)

Publishes “De humani corporis fabrica” dismissing many of Galen’s theories, such as that the blood circulation is driven by the heart. Assembles a complete human skeleton.

Arabic Muslim Medicine:

The basis is Greek-Roman, Animism-from popular medicine- some living in every disease: a devil that need to be manipulated. Matbib- someone who was affected by a curse of pressure. Tabib- a physician

Alexander from Treles

Mixed Galenic medicine with paganism – belief in opposing forces.

Harun al Rashid and his son Ma'amun

Translations of Greek and Latin science. Ma'amun supported Mutzila-a circle of religious intellectuals.

Beit el Hikma

Academy (or University) of sciences in Baghdad.

Hunyan Ibn Ishag

A Christian that translated and composed original medical scripts on ophthalmology. Compandia-Organized knowledge by subjects. **Ali Ibn** writes such book “Paradise of wisdom”

Rabbanal Tabari

Used Indian sources. Managed a hospital in Baghdad, and wrote ~200 books not only medical.

Ali ibn a-abbas

Composed “complete medical records”

Ibn Sina (Avisenna)

Was a lawyer who was engaged with medicine. Wrote 250 books, merged the heritage of Galen and Aristo. Was most influential on western medicine.

Arabic medicine contributed a lot in pharmacology. More than 3000 plants were registered by **Ibn Baitar**, as compared to 850 by **Diocorides**.

Another important contribution was in surgery and Chirurgical procedures, such as stomach injuries (stitches from cat's guts). Interesting to note is that a majority of the physician were Christians and Jews, with fewer Muslims.

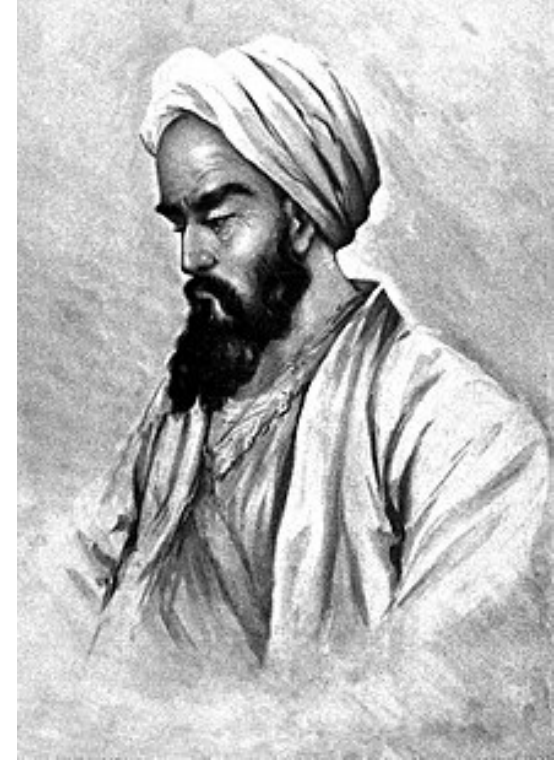
The first hospital (called **Maristan**) was established in Damascus then Baghdad.

Al-Kindi

Applied mathematics in pharmacology to grade effectiveness of drugs.
Developed a method to determine the critical time in disease progression by the phase of the moon...

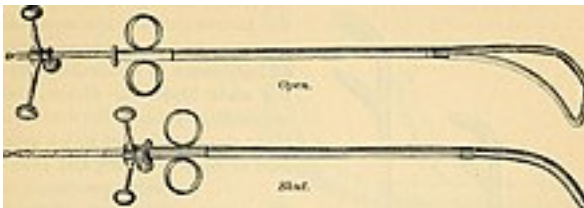
Abū Bakr Muhammad ibn Zakariyyā al-Rāzī (Rhazes)
(865-925)

Published **al-Hawī**, history of case studies he treated, which became a main stream medical procedure as a study method. He discriminated between smallpox and measles, recognized allergies (hay fever), mixes ointments and invents the spatula and medicine bottles (probably with understanding of sterility?). He believed in infinite flow of time and described reaction of eye iris to light as a reflex. The most famous medical composition is by Razi: “Al Kitab el Mansuri Fitbib” .



Abū al-Qāsim Khalaf ibn al-‘Abbās al-Zahrāwī al-Ansar (Abulcasis) (936 – 1013)

In Spain. The father of modern surgeons. Write “Kitab al-Tasrif” 30 volumes of medical encyclopedia, studies by both Arabic and European physicians till the 17th century. He developed tools for surgery, birth, needles and tandems for sawing surgical sections, clamps, scalpels, scrapers, retractors, spoons and hooks for surgeries, as well as used saws for bones and plasters. Invented Lithotrite. to crush stones inside the bladder with minimally invasive surgery. Invented inhalation anesthesia with **Ibn Zuhr**: Used a sponge soaked with narcotic drugs and placed it on patients face. Describes ligatures on blood vessels. Proposed that hemophilia was an inherited disease. Treated Migraines by surgery, dislocated shoulder by fixation (what was later named Kocher’s method”) and recognized “Walter position” in child birth. Treated Hydrocephalus by draining the brain fluids



Ibn al-Haytham (Alhazen) (1021)

From his eye surgeries he understood the optical construction and the physiology of the vision. Invented camera obscura (pinhole camera), understanding that light emerges from objects to all directions at straight lines. Studies parabolic and spherical mirrors, and optical aberration.

Ibn al-Nafīs 1213-1288

In remarks he noted on the Canon, he described blood cycle from the right chamber of the heart, through the lungs, to the left chamber (rejecting Galen who described connections through veins). Only 300 years later **Michael Servetus (d. 1553)** and **Realdo Colombo (d. 1559)**, described the blood cycle correctly in Europe. He started to study metabolism in the human body, Listed body parts: bones, muscles, nerves, esophagus (food pipe), guts etc. He rejected Galen's four humors theory.

Ammar ibn Ali al-Mawsili (9th century)

In Iraq – Hypodermic needle for injections.

754 – The first pharmacy in Baghdad, offering 2000 drugs.

1242 –Ibn al-Lubudi (1210-1267)

Dependence of body on blood cycle: the first limb that develops in embryogenesis, (not brain as Hippocrates wrote). Describes skull bones cancer.

Mansur ibn Ilyas (c. 1390)

Composes “Tashrif al Baden” (Body anatomy) including detained diagrams.

al-Andalus, Ibn Khatima and Ibn al-Khatib 14th century

During the great plague discovered that diseases are caused by microorganisms that enter the body immune system. `Tested on animals.

Abu Ali al Hussein ibn Abdallah ibn Sina - Avicenna (980-1037)

Father of modern medicine. Writes “. Kitab al-Qanun fi al-tibb” the medical canon 1020. Test and measurements in physiology and medicine. Clinical trials. Contamination of diseases via water and soil, and usage of isolation to prevent disease spreading. Classification of similar diseases (measles and smallpox). Uses ice to reduce high body temperature. Clinical pharmacology.

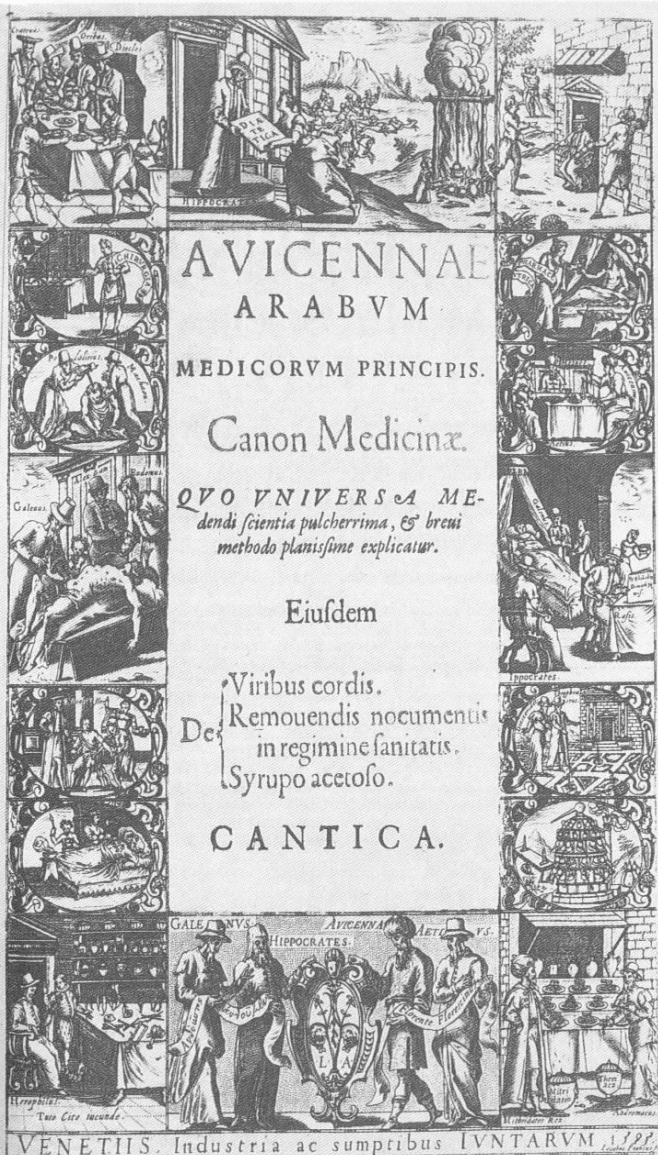


Абуали Сино - Avicenna

Avicenna's medical canon (code of rules).



Miniature du Pr. Suheyl Unver éminent historien de la médecine turque représentant le Cheik Errais Ibn Sina El Bûkhârî.



Frontispice de la version latine du Canon de la Médecine, imprimée à Venise en 1544

The Latin translation of the canon by **Gerard of Cremona** 12th century. Was the main medical book till the 17th century. The chapters:

1. General rules of medicine
2. 2. Alphabetic dictionary of medications
3. Therapy from head to foot (22 sections)
4. Diseases that are not confined to a single body part (e.g. high fever)
5. Drug prescriptions

Was written in Iran, but the Latin translations were made by Venetian delegation to Damascus

1150 — Avenzoar

Applies surgery and biopsies to prove that skin diseases are due to a parasite – rejects humorism. Animal studies test surgical procedures before applying them to people.

1200 — Abd-el-latif

Studies skeletons during starvation in Egypt. Finds that Galen was wrong regarding development of lower jaw and back bones.

1316, Mondino of Luzzi

Published "Anatomia" after applying surgeries and biopsies ordered by physiologists

1348, Gentile da Foligno

Uses Galen's terminology: seeds of diseases in discussions about plagues

1410, Benedetto Rinio

Published a illustrated catalogue of medical plants with names in several languages, with remarks and quotations.

Emperor Gaozong (r. 649–683) -Tang Dynasty (618–907)

Funds the publication of a compilation of medical materials from stones, minerals, plants, animals, fruits and grains.

Su Song (1020–1101)

Bencao Tujing ('Illustrated Pharmacopoeia')

A Zoological book about animals and their habitat.

Ouyang Xiu (1007–1072) Zhao Mingcheng (1081–1129) Hong Mai (1123–1202)

Archeology

Post Galen medicine in Europe

Gargillus Martialis 260

Catalogue of drugs from fruits and vegetables.

Magnus Nisbis of Alexandria, 4th century

A composition about the Kidney and the Urine bladder.

325-400 –Oribasius

70 volumes encyclopedia

362 Julian

Builds hospitals managed by the church in Byzantium (“xenones”)

Basil from Caesarea in Capedonia 365

Builds a hospital called “Bazylias” that includes sections for the sick, for nurses and physicians, and a medical school.

375 - Ephrem the Syrian

Established a hospital and charity institution for the poor

400 – Fabiola

Starts a hospital in Rome.

420 - Caelius Aurelianus a doctor from Sicca Veneria (El-Kef, Tunisia)

Catalogue of chronic and acute diseases in Latin

447 - Cassius Felix of Cirta (Constantine, Ksantina, Algeria),

Medical Almanac based on Galen’s books in Greek

480 -547 Benedict of Nursia

Founder of medicine in monasteries, that became hospitals and clinics. Starting at the end of 14th century hospitals became a common section in the religious establishments, as a medical and social service to the poor. The rich invited physicians for home visits.

536 Sergius of Reshaina (died 536) A Christian theologian-physician

Translated Galen's books and composed medical books of his own.

525-605 - Alexander of Tralles Alexander Trallianus

500-550 - Aetius of Amida

Medical encyclopedia. The Necrotic church in Antioquia, under the rule of the Assassins in Iran, built guest homes that became hospitals (xenodocheions/bimārestāns) in Galen's spirit.

550-630 Stephanus of Athens

560 – 636 Isidore of Seville

c. 630 - Paul of Aegina

Encyclopedia with detailed descriptions of surgery, that served Albucasis.

790-869 Leo Itrosophist also *Mathematician or Philosopher*

Wrote "Epitome of Medicine"

c. 800–873 – Al-Kindi (Alkindus) *De Gradibus*

820 - Benedictine hospital founded,

The medial school in Salerno evolved from this hospital.

857d - Mesue the elder (Yūhannā ibn Māsawayh) Syriac Christian

c. 830–870 – Hunayn ibn Ishaq (Johannitius)

A Siriaki-speaking Christian (Arameic dialect) who translated medical books.

c. 838–870 – Ali ibn Sahl Rabban al-Tabari,

Composes a medical encyclopedia in Arabic

c.910d - Ishaq ibn Hunayn

9th century Yahya ibn Sarafyun

Johannes Serapion, Serapion the Elder

Christian Siriaki-speaking physician

c. 865–925 – Rhazes

Pediatrician, first to discriminate smallpox from measles.

d.955 - Isaac Judaeus Ishāq ibn Sulaymān al-Isrā'īlī

An Egyptian-Jewish physician.

913-982 - Shabbethai Donnolo

The founder of the Salerno medical school. Wrote in Hebrew.

d. 982-994 'Ali ibn al-'Abbas al-Majusi Haly Abbas

1000 – Albucasis (936-1018)

A surgeon and inventor of surgical tools. Composed “Kitab al-Tasrif”

d.1075 - Ibn Butlan

A Christian physician in Baghdad, Composed “Tacuinum sanitatis” in Arabic, and translated to Latin.

1018-1087 Michael Psellos or Psellus

Byzantine monk who wrote several medical scripts.

c. 1030 – Avicenna

The writer of the medical canon – university main text book in the Islamic world and in Europe until the 18th century.

c.1071-1078 Simeon Seth or Symeon Seth

A Byzantine Jew who translated into Greek Arabic literature.

1084 First hospital in Canterbury, England.

1085-1087d - Constantine the African

1083-1153 Anna Komnene, Latinized as Comnena

1095 "Congregation of the Antonines" was established to treat a common skin disease ("St. Anthony's fire")

late 11th early 12th century Trotula

1123 - St Bartholomew's Hospital

A hospital in London founded by Henry I court clown Rahir, and ran by Augustine nuns. Treated mentally sick patients.

1127 - Stephen of Antioch

Translated the books of Haly Abbas.

1100-1161 – Avenzoar

Averroes's mentor.

1170 Rogerius Salernitanus

Wrote "*Chirurgia*" or "*Surgery of Roger*"

1126-1198 – Averroes

c.1161d - Matthaeus Platearius

1204 - Innocent III

The pope who established Santo Spirito hospital in Rome inspiring hospital building all over Europe.

c.1210-1277 - William of Saliceto also known as Guilielmus de Saliceto

1240 Bartholomeus Anglicus

1242 – Ibn an-Nafis

Proposes that the left and right ventricles of the heart are separated, and linked to the pulmonary and coronary blood cycles.

c. 1248 – Ibn al-Baitar

Wrote about botany and pharmaceuticals, animal anatomy and veterinarian medicine.

1249 – Roger Bacon

Reports about convex and concave lenses to correct far and near-sight vision.

1257 - 1316 Pietro d'Abano also known as Petrus De Apono or Aponensis

1260 - Louis IX

Built a hostel for the blind “15-20 Les Quinze-vingt” that became a ophthalmological hospital till today (near the Bastille).

c. 1260 – 1316 Henri de Mondeville

1284 - Mansur hospital of Cairo

c. 1275 – c. 1328 Joannes Zacharias Actuarius

A physician who wrote the last medical composition in Byzantium.

1275-1326 Mondino de Luzzi "Mundinus"

Performed systematically autopsies after 1500 years that Herophilus of Chalcedon and Erasistratus of Ceos performed such surgeries.

1300 – concave lens spectacles to treat myopia developed in Italy.

1292-1350 - Ibn Qayyim al-Jawziya

1306-1390 John of Arderne

d.1368 Guy de Chauliac

f.1460 Heinrich von Pfolzspeundt

1443-1502 Antonio Benivieni Pathological anatomy

1493-1541 Paracelsus On the relationship between medicine and surgery
surgery book

Vienna Dioscurides, 512

A physician in Rome, who collected all knowledge about medical substances: “De Materia Medica”, containing 600 medical plants and their applications. A copy of the book prepared for the daughter of Anicius Olybrius Caesar, Anicia. A page from this copy is about blueberries, and include remarks in Arabic by the owner of this copy. As a military physician, he must have learnt from his own experience, and from **Crateuas**.



Frontispiece of Vienna Dioscurides, 512: *Seven Physicians*

The luxurious format of this book “Vienna Dioscurides” indicates its value: Seven famous pharmacists are depicted, including seated Galen, to show reference to the most prestigious professionals in the field. The book became unreadable from much use, and reconstructed.



Manuscript from Byzantium, 15th century, in Greek Bologna, University, MS 3632, folio 51 Theophilus Protospatharius, *On Urines*

This Byzantine manuscript illustrates methods of treating urinal tracks. Theophilus, the Greek who wrote it in the 6th century, is handled by his assistant a vile of urine for testing. Diagnostics based on visual and even tasting of urine was common in the France till late 18th century.



Fra Angelico, 1449, Museo di San Marco, Florence

In this painting by Fra Angelico the camel enforces burial of the two patrons of the physicians, saints Cosmas and Damian, despite controversies during their lives.



Medical instruments and surgical equipment and tools.

A significant amount of medical instruments from the Roman times survived, indicating the development and spreading of surgery as a profession. Galen and Celsus (2nd century) emphasize in their texts proper education for surgeons. With time, refined metallurgic processing helped produce better and sharper knives for general use (including swords...) as well as scalpels, some found in Pompeii at a “surgery hospital”. Latin discriminates between *medicus* (physician) and *magister* (a surgeon).

A relief from the 1st century shows a surgeon with a knife extruding an arrow from a soldier.



In an illustrated text from medieval ages a treatment of skull fractures is shown: the position of the fracture is found by touching, the open wound is cleaned by a feather, The assistant handles a cloth as a bandage for fixating The skull, and has a vile of ointment named “*apostolicon Chirurgicum*” in his other hand.



Celsus method of kidney stone removal

Reported in “Chirurgia” by John Arderne’s from the 14th century. Following a few days long diet, the stone descends to the urinal track. If failed to descent, it can be sensed by a finger inserted via the rectum. Removal is done from the rectum, as todays procedure. The patient is held by two assistants, showing the painful procedure.



Treatment of a spinal dislocation

Apollonius at the 1st century BC, refers to Hippocrates, 5th century BC for a inverted hanging procedure, with a strong pull or fast removal of the ladder.





Oribasius 320 – 40

The physician of Julian Caesar the Apostate (since he rejected Christianity). Published a collection of previously written medical books, including Galen's. The illustration is a fixation for jaw injury.

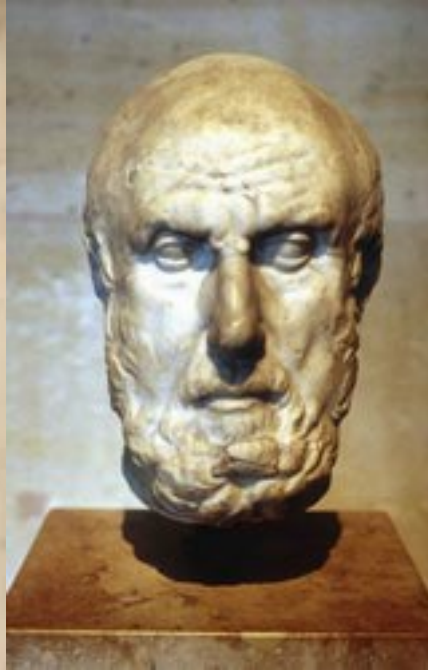
Claudius Galen 129-100

A physician, surgeon and Greek philosopher (Pergamum) who saw the plague during Antoninus Caesar 168-9. Strongly supported Hippocrates four humor theory, and blood shedding. Studies body anatomy on animals, since human studies were banned in the Roman empire. He therefore made wrong conclusions about blood cycle. Yet, his heritage was highly acknowledged in the Arabic world and in Europe, and was the basis of medical education.



1360, Guy de Chauliac

Performed autopsies under special permit from the pope in order to stop the black plague. The “father of modern surgery”. Composed “Chirurgica Magna”. Recommends to treat fractures in limbs and jaws by weights, replaced lost teeth by bones fixed with gold wire tied to the nearby teeth.



Hippocratic corpus

A collection of Greek medical scripts from unknown authors, probably all linked to Hippocrates' school during the years 350-450 BC in Cos. The collection was kept in Cos and Alexandria, and translated into Hebrew, Latin, Arabic and Cirri. The original is lost, but the collection survived via the translations.

Soranus of Ephesus (1st-2nd centuries)

A Greek physician who moved to Rome. His books "Gynecology" and "Chronic and Acute diseases" survived. A book about drugs is cited by **Galen**.

Aetius of Amida (6th century)

A Byzantine physician who wrote 16 volumes about medicine, based on **Galen** and **Oribasius**

Collections of medical instruments from the middle ages. Claude Moore Health Sciences Library, University of Virginia. A beautiful collection is also under display in the Welcome Trust, London

Vaginal opening for uterus internal inspection.

Mentioned by Soranus and was in use till the 19th century. Casted from Bronze and include an impressive screw mechanism.



Rectum opening forceps

Mentioned by Hippocrates.



Various levers to replace teeth, skeletal or skull bones after fractures.



Bone forceps

Soranus, from the 1st century, describes removing bone fragments from the skull. Similar descriptions also from the 7th century by Paul Aigenita or Paul of Aegina



Blood shedding collection vile



**Pipes for clearing cuts from blood and pus
Preservation of breathing tracks after surgeries
And insertion of ointments and drugs**



Pipes for enema



Tile Cautery

Used against skin irritation, to stop bleeding, extruding cancers, etc.

Hooks

Used, as today, to hold, move and keep aside tissues. Blunt hooks are used to hold blood vessels and nerves.

Forceps and Tweezers

For fine grabbing, including hair removal and aesthetic applications.

For grabbing and smashing glands for removal with minimal blood shed (Hippocrates description)



Scalpels



Scissors

Oribasius describes haircut as a medical procedure.

Celsus performs an haircut as a part of the curing procedure.

The production of very sharp scissors was impossible, therefore almost no applications in surgery are discussed.



Spatula

Used mainly to mix drug compounds for medications, and for smearing creams on wounds.



**LATE
MIDDLE AGES**

Scale 1:22 500 000

Miles



Charles the Great, Charlemagne 800

Takes over most of Europe. The English monk **Alcuin of York** describes an educational reform to revive the classical heritage: Seven science fields. The Tritium of humanities: Grammar, Art of speeches: Rhetoric & Dialectic, Logics, Art and search for truth. And the Quartet of Natural Sciences: Arithmetic, Geometry, Astronomy & Music.

840 A new wave of Barbaric evasion

12-13th centuries: Most of the schools established by Charlemagne that were connected with the church became universities.

~**1000** town clocks, driven by chain and weights, show local time.

1010 Eilmer of Malmesbury who studies math in England, floats in a glider to a 200 meters distance from a bell tower.

946-1003 Pope Sylvester II disseminates in Europe the Arabic number system, the Abacus, and the Geocentric model with earth at the center (**armillary sphere**).

1100 Pierre Abelard rejects Plato's doctrines.

1100 Triggered Bow rifle spread over Europe.

The Second Ecumenical assembly 1139 announced the first attempt at arms control

1020-1087 **Constantine the African** (born in Cartago)

Translates from Arabic to Latin originally Greek scripts.

1190 **Compass** becomes used by Italian navigators.



Robert Grosseteste (1168–1253) Bishop of Lincoln in Oxford

Optics, Astronomy & Geometry. “The Scientific Method” a logical analysis of deduction from experiments, and how a cause effects the result. Similar to Aristo’s decomposition and assembly theory, but added verification and rejection as a result of experiments. He believed that light is the source of all explanations of nature. He analyzed mathematically light reflection in mirrors, refraction in lenses, and dispersion of colors in the rainbow. He even discussed propagation of light as waves.



Albert the Great (1193–1280)

Held a university degree, and was a member of the Dominican order. Believed in coexistence of science and religion. Was the teacher of **Thomas Aquinas**. Wrote a geological book about minerals, metals and fossils following **Ibn Sina**. At 1250 was first to purify Arsenide.



Adelard of Bath (1126)

Translates from Arabic to Latin the scripts of **Euclid** and **Al-Khwarizmi** , Including geometry, arithmetic and Astronomical tables.

Robert of Chester (1145)

Translated Al-Khwarizmi’s “Algebra”

Gerard of Cremona (1175)

Translated from Arabic to Latin most of **Aristo’s** compositions, **Ptolemy’s** “Almagest” **Al-Kindi** and **Al-Hazen’s** optics, **Ibn-Sina’s** “medical canon”, and **Razi’s** medical books.

Burgundio of Pisa (1185)

Translated from Greek to Latin **Galen’s** books and **Hippocrates’** fables

William of Moerbeke (1269)

Translates from Greek to Latin **Archimedes**

Hermann of Reichenau, Walcher of Malvern, John of Sacrobosco, ???

Leonardo Pisano (Fibonacci) (1202)

Spread the Arabic number system and negative numbers in Europe. Solved Algebraic problems. Most known from the Rabbit-multiplication series where next term is the sum of two previous numbers 2,3,5,8,13,21,34,...

Jordanes de Nemore (1230)

Equilibrium in Archimedes law of moments

Vincent of Beauvais (1230)

Assembled an encyclopedia of 6000 pages “**Speculum majus**” from Arabic and Greek scripts about philosophy, science and math.



End of middle ages

At the beginning of the 13th century most significant compositions of the classical world and the Arabic additions were translated into Latin, including:

Aristotle, Ptolemy, Geber, al-Khwarizmi, Alhazen, Avicenna, and Averroes.

The translations and studies in universities revived scientific thinking, and not only spread classical and Arabic knowledge, but advanced critical scientific analysis, and mainly assimilated the Arabic tradition emphasizing experimental testing and confirmation of theoretical mathematical models of natural sciences.

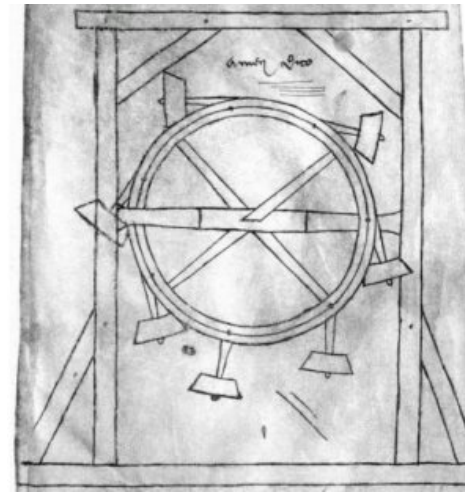
Roger Bacon, Albertus Magnus, Michael Duns Scotus, Grosseteste

Detached themselves from Aristo. The Condemnation of 1277 in Paris University of Aristo's heritage was motivated by theological arguments, but initiated renewed argumentation about scientific theories not challenged since the classic era.

Starting from the Mongol evasion of Genghis Khan at the beginning of the 13th century until the Pax Mongolica a communication system covering Asia and Europe was established, consisting of posts 1 day riding apart (Yam), that enhanced trading (e.g. the silk route used by Marco Polo), and directly exposed Europe to the cultures in China and India. The bad side of this was the import of the black plague.

Villard de Honnecourt ~1230

An illustrated book, mainly architecture, but also drawings of machines: here a “perpetuum mobile”. The falling weights drive the rotation of the wheel...



Alessandro della Spina 1285-

1306 Spectacles for far sighted people, and use of lenses in telescopes & microscopes.

Roger Bacon 1214–94

From the Franciscan order in Oxford. Studies laws of nature: Mechanics, Geography, Optics. Velocity of light is finite, and it propagates as waves, much like sound waves. The opening angle of the rainbow is maximal close to sunset: 42° . Experimented with gun powder, and discussed imaginary trapping inside a solid mass.



1267 published “Opus Majus”, a revolutionary concept in scientific education – only experiments and measurements can confirm scientific hypotheses and theoretical models.

Albertus of Bollstadt (Magnus) 1250

From Paris university anchored in Aristo’s heritage, but does not believe everything is predictable. Assembled a catalogue o plants, emphasizing classification according to shape (as of **Aristo, Theophrastus & Cesalpino**)

Universities in Montpellier, Salerno, Padua, Bologna

Centers of medical schools and anatomical studies based on surgeries and autopsies.

Pélerin de Maricourt 1269

Describes magnetic poles, and that fact that a single pole cannot be separated.

Petrus Peregrinus of Picardy, Italy 1269

Naturally found lodestones align iron needles between the two magnetic poles.

William of Ockham 1285-1349

Best known for “Ockham’s razor” (Franciscan minimalism), a scientific principle claiming that if two explanations are equivalent, science should prefer the simple one, avoiding unnecessary assumptions. For example, no need in medium to transmit light from an object to the eye, or motion is not an independent object, but the property of the moving body.

1323 discriminates between a body in free motion, and a body driven into motion (dynamics and kinematics).



Thomas Bradwardine (Oxford) 1328

Also separates kinetic and dynamics of bodies. Law of falling bodies (much before Galileo): A body moving in constant speed pass equal distances in equal times. Accelerated bodies (i.e. falling bodies) pass the same distance as a body moving in speed half the maximum of the accelerated body (uses algebra):

$$vt = \frac{1}{2} gt^2 = (\frac{1}{2} gt)t \qquad v = \frac{1}{2} gt$$

He calls Force “a potential”.

Nicole Oresme 1320–82

From Paris university, Bishop of Lisieux. Rejects astrology. Earth is moving, not the sky. Studies trajectories of freely shot objects, Without dealing with cause of the motion or forces.

Influences **Galileo and Copernicus**.

Discovered light refraction in the atmosphere, and its color changes. Influences **Hook**.



Thomas Aquinas 1227–74

Dominican Italian order. Writes a book on Alchemy:

“Aurora Consurgens”.

Introduces inertia: the resistance of body mass to change in motion.

Influences **Kepler** and **Newton**.

1267-73 writes “Summa Theologica”: a true hypothesis – is mathematical and compatible with factual physics.



John Duns Scotus 1266–1308

From the Oxford Franciscan order

Theodoric (Dietrich) von Freiberg 1305

Uses hexagonal crystals, crystal balls and glass dishes to study light refraction, and explain internal reflection and color dispersion of light in water droplets forming primary and secondary rainbows.

William of Saint-Cloud ~1250

Applies camera obscura to inspect sun eclipse.



Giles of Rome 1290

Atomistic theory of matter, following Ibn Sina

Giovanni di Dondi 1364

A clock with the date.

1370 Parisian clocks are synchronized

14th century - **Oxford Calculators**: the mean speed theorem- **Merton Rule**

The path that an accelerated body moves equals the path of a body moving at the average speed: $S = \frac{1}{2} (v_0 + v_f)$

Nicholas of Cusa 1401-1464

Our mental capacity is finite, and can only reach the truth asymptotically.

Treats infinitesimal and infinite quantities – seeds of calculus.

Plants get nutrition from the air.

Earth moves around the sun. The constellations are far away suns.

1494 - Luca Pacioli

Established a double-accounting income & expenses book keeping system.

13th century in Biology and Botany

Italian physicians write Consilium – a summary of symptoms and treatments of diseases, facilitating accumulation of medical information.

Encouragement to learn anatomy from surgeries and autopsies in Italian universities. In France and England surgery is not allowed due to church prohibition of “blood shedding”. Interesting that in Italy, the Vatican country, more academic freedom. Maybe the ruling of merchants (e.g. Medici) and their philanthropic support of sciences and arts counterbalanced the church influence, and was one of the reasons that the renaissance emerged from Italy.

Michael Scot 1217

Salerno University – a prescription for anesthetic drug (opium, mandragora and henbane plant extract).

Hugh and Theodoric Borgogoni 1266

Surgeries are performed on patients inhaling through a cloth absorbed in anesthetics. Advice to clean open wounds with wine, stitch wide-open cuts, and let the body heal itself.

Gilbert the Englishman 1250

Describes loss in skin sensual feeling at early stages of leprosy (due to injured peripheral nervous system). Became a diagnostic tool for early and more effective treatment of leprosy and other contagious diseases.

End of 13th century

Royal Bethlehem (Bedlam), mental health hospital in London

Mondino of Luzzi 1316

Publishes “Anatomy”, and institutionalizes public surgeries in education.

Guy de Chauliac 1360

Stretches fractured limbs with pulleys and weights.

Replaces lost teeth with bone enforced in situ by gold wires to nearby teeth.

Benedetto Rinio 1410

Publishes 450 plant drawings with botanical explanations.

Second half of the 13th century

Gunpowder arrives from China via the Mongols: The explosive behavior of the mixture of sulfur, coal and tar.

SUMMARY

Despite of the great engineering and practical achievements of the Roman empire, scientific research activity was not common, unlike its central status in Greece. This fact left middle-age Europe with simplistic compilation of the classical sciences, and strong acceptance of theological explanations of nature, as created by God's truth.

The Arabic world, on the other hand, absorbed the classical heritage together with independent research and mainly experimental activity, covering physics, optics, medicine, mathematics and astronomy (with Indian and Chinese influence) with applications for navigation, chemistry, health and uses in metallurgy, medicine, and more. With no church inhibitions on surgery, they developed procedures and anatomical knowledge, as well as pharmaceuticals backed up by advanced experience in chemistry.

Leonardo da Vinci's drawings of inventions is so widely known, and considered a milestone of the Renaissance, yet the Banu Musa brothers inventions and the mechanical devices of Al-Jazari well preceded the Renaissance and were almost forgotten.

Starting at the 12th century we evidence increased interest in measurements and experimental sciences in Europe, as well as analysis of theoretical-logical science methodologies. This positive trend was abruptly arrested by the black plague of 1348, that exterminated 1/3 of the European population and badly hurt centers of knowledge and progress. The authorities conduct of the plague as a Godly punishment for the sinful, inhibited any effective measure to arrest the spreading plague, painting the entire era as the "dark middle ages".

SUMMARY cont.

Yet the scientific progress recovered with the increasing number of universities all over Europe. It was further enhanced by the flow of scientists from Byzantium after its capture by the Turks at 1453. Byzantium kept the classical Greek heritage by its proximity to Alexandria, which was enriched by the enlightened Arabic intellectuals in Baghdad. Byzantine scientists had books in Greek, Arabic (translated from Greek and originals), as well as Indian. These literature arrived to Europe, translated into Latin, and widely spread after print from China was implemented ~1450, democratizing education, facilitating acquisition of new knowledge, and, with the import of Algebra, established quantitative science. This rich literature was rediscovered by Renaissance scientists and during the scientific revolution. Newton based his findings at the 17th century on the work of late middle ages scientists such as Alhazen.

Therefore, despite its darkness, the middle ages established the foundations necessary for the blooming of Renaissance sciences.

APPENDIX: LIST OF MIDDLE AGE SCIENTISTS

Red: India
Blue: Europe

Lalla

Alcuin

Al-Khwarizmi

Al-Jawhari

Banu-Musa Muhammad & brothers

Mahavira

Govindasvami

Al-Kindi

Banu-Musa Al-Hasan

Al-Mahani construct Astrolab

Prthudakasvami

Ahmed 835-

Thabit Ibn Quarra 836-

Sankara

Abu-Kamil 850-

Al-Battani 850-

Sridhara

Al-Nayrizi

Ibrahim Ibn Sinan 908- treaties on Astrolab

Al-Khazin 900- treaties on Astrolab

Ibrahim

Al-Uqlidsi 920-

Aryabhata_II

Al-Quhi 940- treaties on Astrolab

Al-Khujandi

Vijayanandi

Abu'l-Wafa 940-

Al-Sijzi

Yunus

Al Uqlidisi decimals from India

Al-Karaji 953-

Al-Haytham 965-

Abu Nasr Mansur 965- treaties on Astrolab

Al-Biruni 973- treaties on Astrolab, projection of earth for maps

Al-Baghdadi 980-

Avicenna

Al-Jayyani

Al-Nasawi

Hermann of R***

Sripati

Shen

Omar Khayyam 1048-

Brahmadeva

Abraham

Adelard

Ezra

Jabir ibn Aflah

Gherard
Al-Samawal 1130-
Al-Tusi Al-din Sharaf 1135-
Grrosseteste
Fibonacci
Sacrobosco
Albertus Magnus
Al-din Al-Tusi Nasir 1201-
Ch'in
Bacon
Al-Maghribi
Campanus
Jordanus
Llull
Tibbon
Yang
Al-Samarqandi
Al-Banna
Al-Farisi 1260-
Chu
Ockhan
Levi
Bradwardine
Albert

Al-Khalili

Oresme Nicole 1320–1382

Narayana

Mahendra Suri

Madhava

Qadi Sada

Paramesvara

Al-Kashi 1380- - Trigonometric tables

Ulugh Beg 1393- Trigonometric tables

Al-Umawi

Cusa

Alberti

Francesca

Al-Qalasadi

Peurbach George **1423-1461** “New theory of the planets” 1472

Borgi

Regiomontanus

Nilakantha

Chuquet

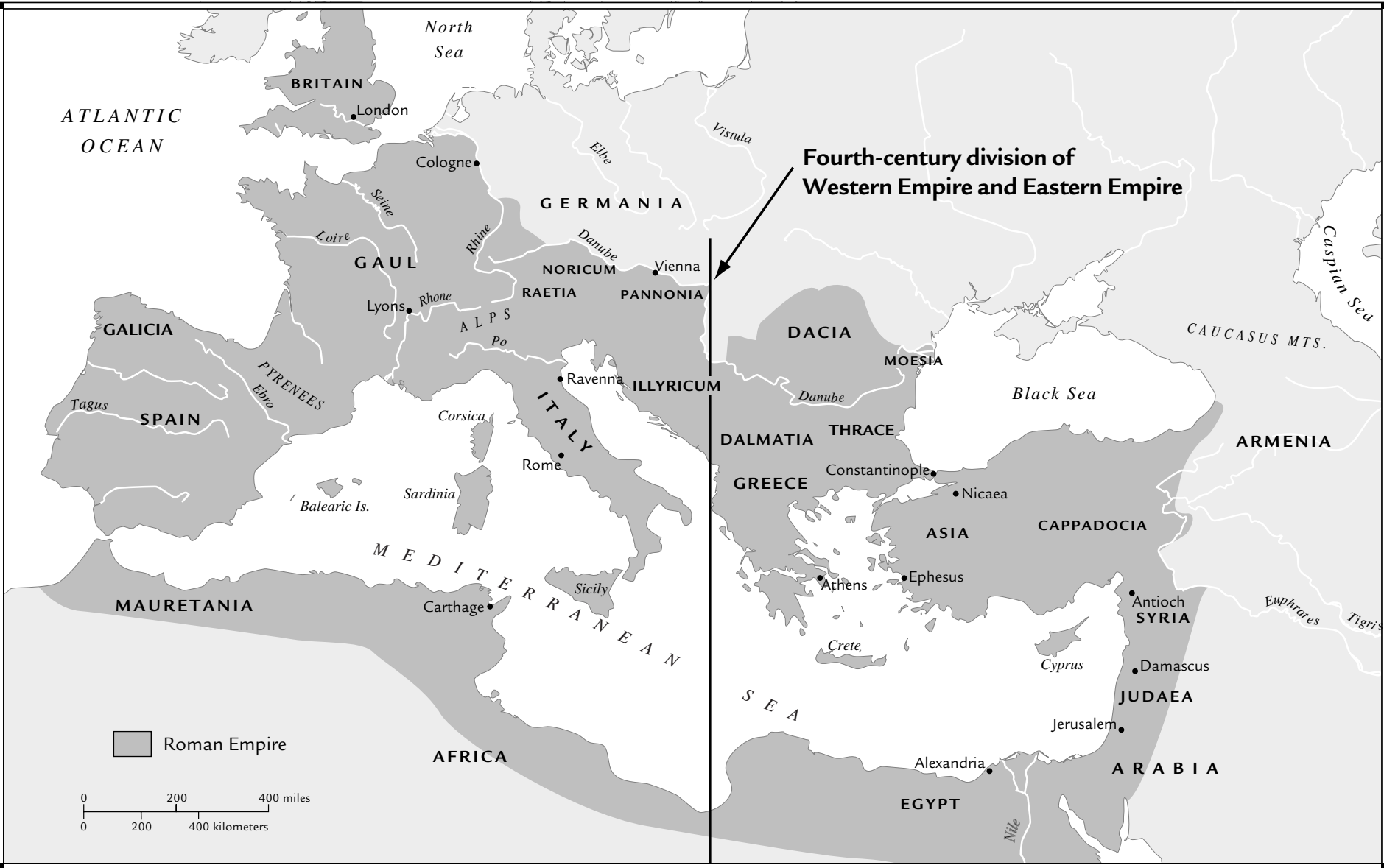
Pacioli

Robert Grosseteste (Grossetete) [1175 -1253](#)

Jean Buridan French [1295 – ~1358](#)

APPENDIX: MAPS OF EUROPE DURING THE MIDDLE AGES

<http://www.medievaleuropeonline.com/maps.html>



Roman Empire during the “Pax Romana”

Barbarians
expansion
~500



500-700 Europe



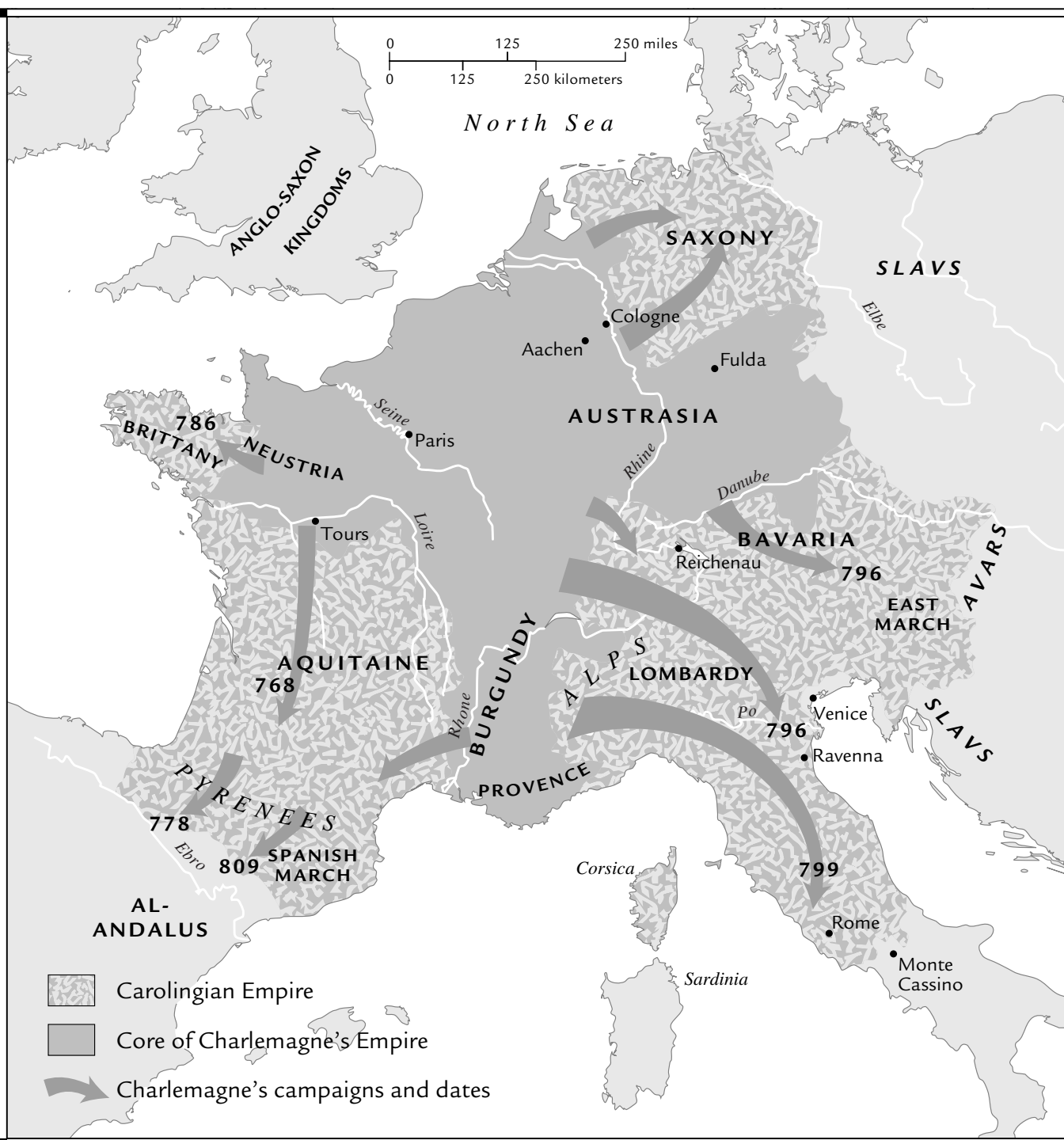
0 500 1000 miles
0 500 1000 kilometers

Byzantine Empire and the east: 800 and extension ~1000. Umayyad rulers replaced by the the Abbasid Caliphs ~900 with independent Caliphs in Spain, Egypt and Arabia





The Islamic empire ~850. Expansion under Muhammad (Higra-622) to Arabian desert. First Caliphs 661 Egypt Umayyad ~750 Persia, North Africa and Spain The Abbasids from ~750 and the Seljuks from 1055. 1055–750 והסלז'וקים מ-750 האבסידים מ-1055



Carolingians 800~

Charlemagne conquests:

France, Belgium,
Holland, Luxemburg
And Switzerland

Italy, Austria, Germany

But not
Spain and England



Grandsons of Charlemagne ~843
Divided his empire to France, Germany (the holy Roman Empire)
And the center (Italy, Swiss, and the Netherlands)

The Capitan Dynasty in Paris ~987 lose control on the Dukes and Counts in the French countryside.

The Vikings evasion (Danes, Norwegians, Swedens) Hungarians and Muslims 800-1000





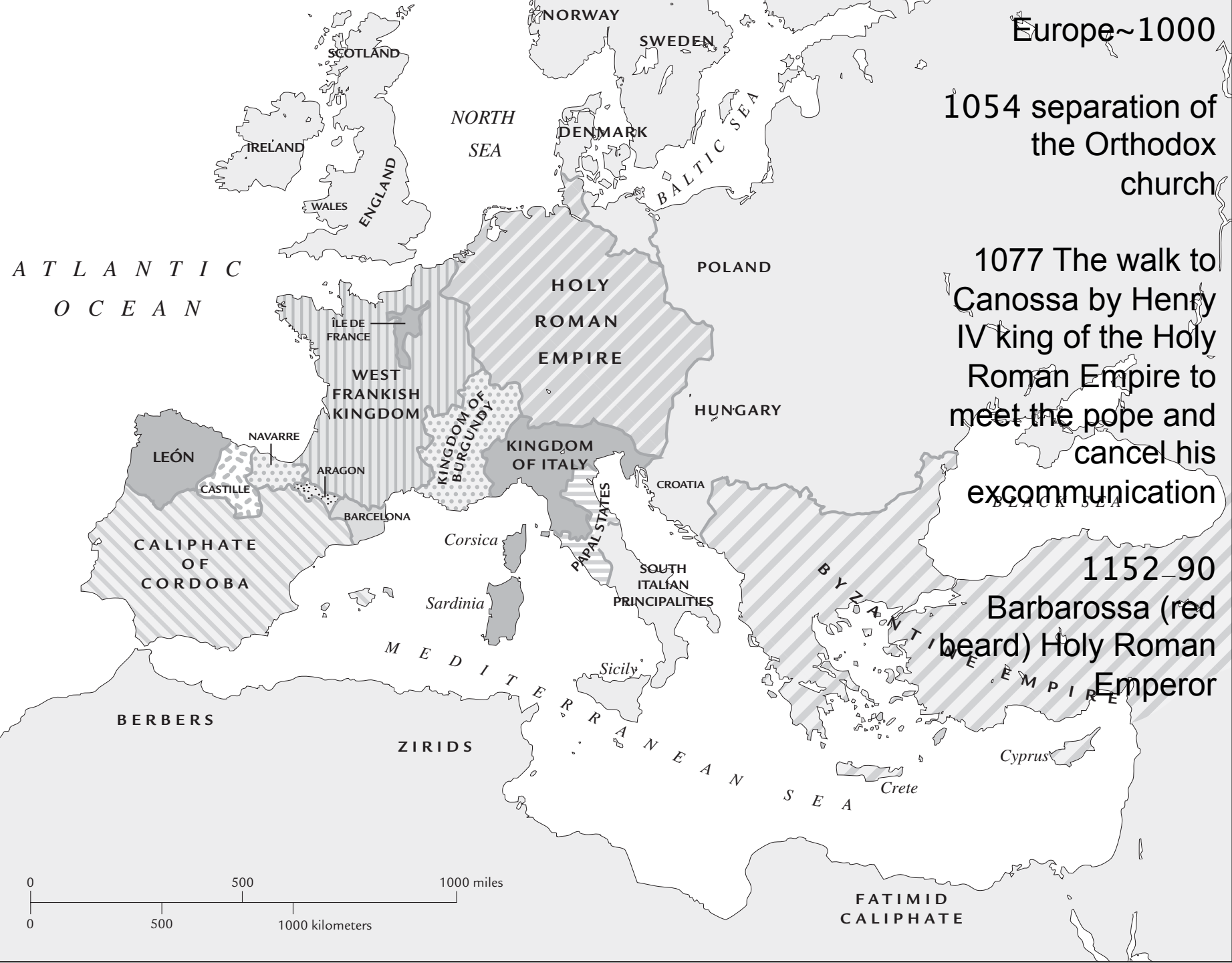
King Alfred the Great 871–899

Liberates London and south
of England from the Danes.

From 875
United English Kingdom



Otto ~962
King of the Holy Roman Empire.
Rules over
Burgundy and Lorain at the
west,
the Marshes at the north-east
and North Italy at the south



Europe~1000

1054 separation of
the Orthodox
church

1077 The walk to
Canossa by Henry
IV king of the Holy
Roman Empire to
meet the pope and
cancel his
excommunication

1152-90
Barbarossa (red
beard) Holy Roman
Emperor

0 500 1000 miles
0 500 1000 kilometers



Universities in the
middle ages ~1200

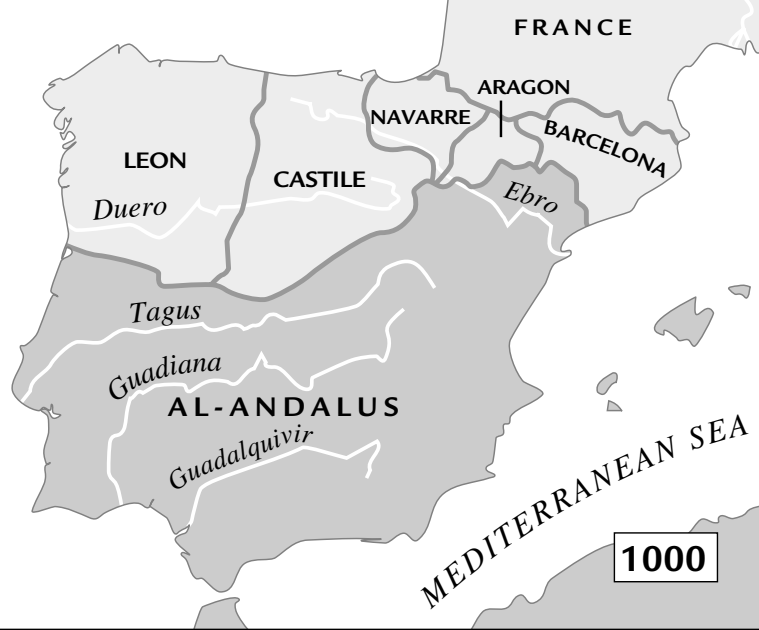
Oxford, Salerno,
Bologna, Paris,
Montpelier

By 1500 there were
universities in every
region in Europe

1215 Magna Carta
1300 Parliament in
England

The Reconquista in Spain. Jews expulsion from Spain, 1492 and Portugal, 1497.





Stages in the Reconquista of the Iberia peninsula, and the strengthening of Castile and Aragon

Christian States
 Muslim States

0 100 200 300 400 miles
 0 100 200 300 400 kilometers



German expansion east of the Elbe
and south of the Danube

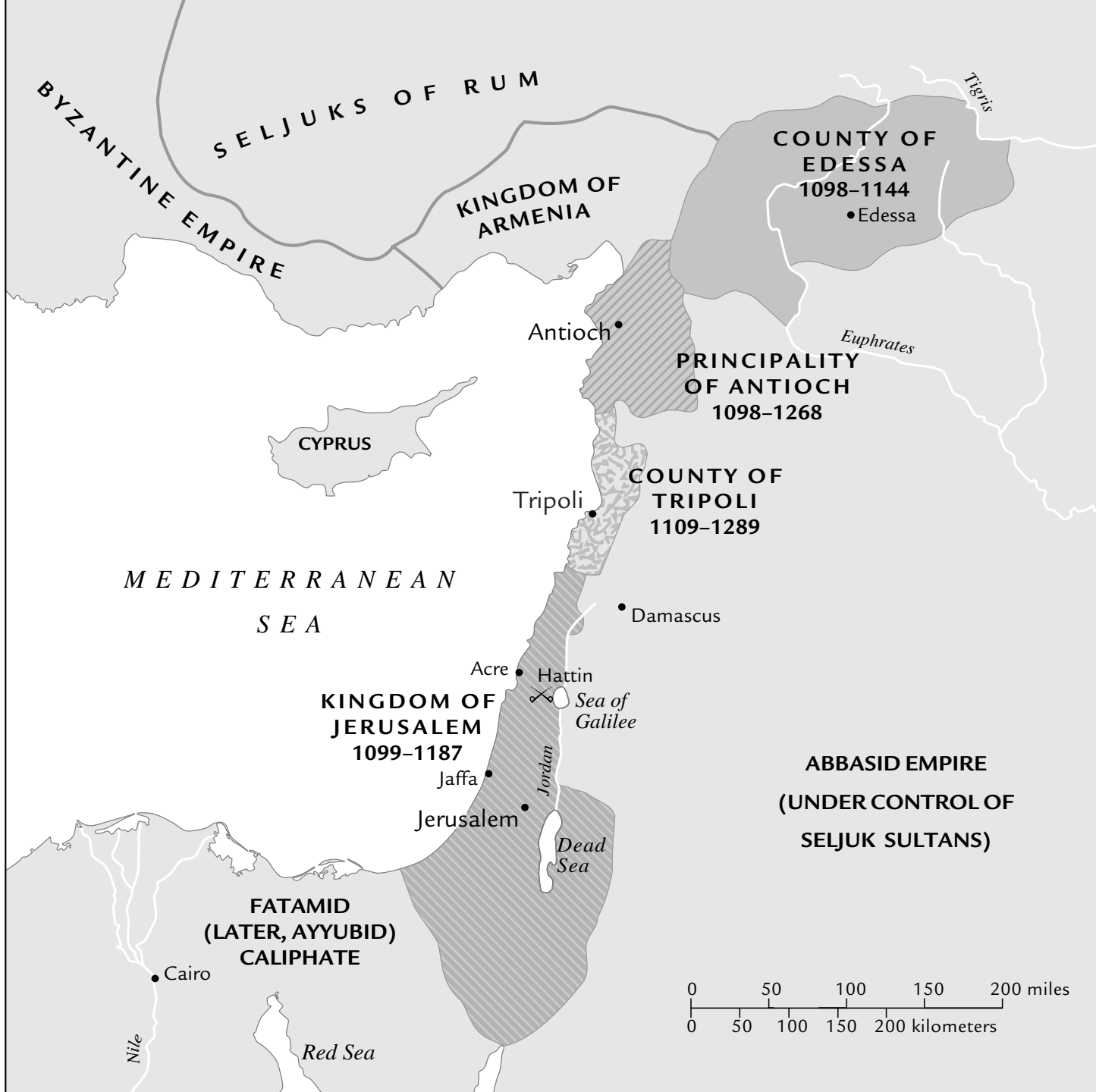
The reign of the Teutonic knights in the
Baltic sea ended 1410 at the battle of
Totenberg against the Poles and the
Lithuanians

1480 Ivan III defeats the Mongols

The Crusades kingdom 1100–1300

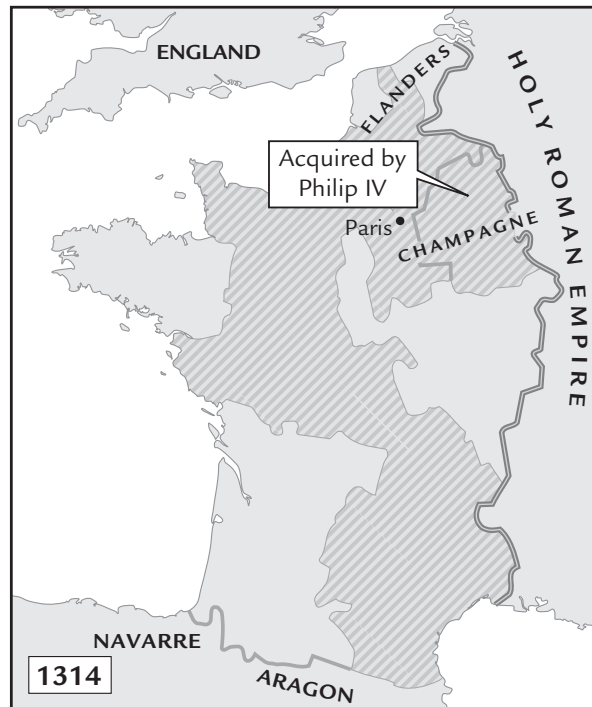
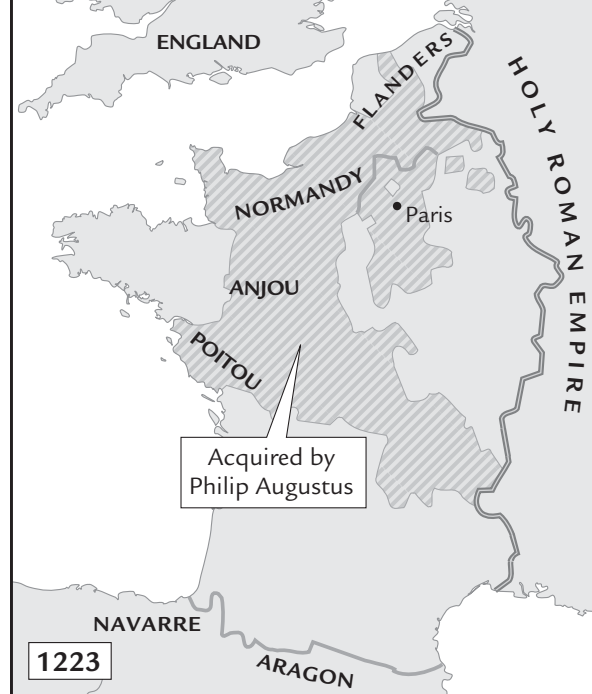
Edessa,
Antioch, Tripoli
and Jerusalem

Surrounded by
Seljuks,
Abbasids and
Fatimid
Caliphates



Holy Roman Empire and Italy ~1300





Strengthening of the central reign in France after Henry II was defeated in the battle of Bovines 1214

1309–1377 The popes in Avignon



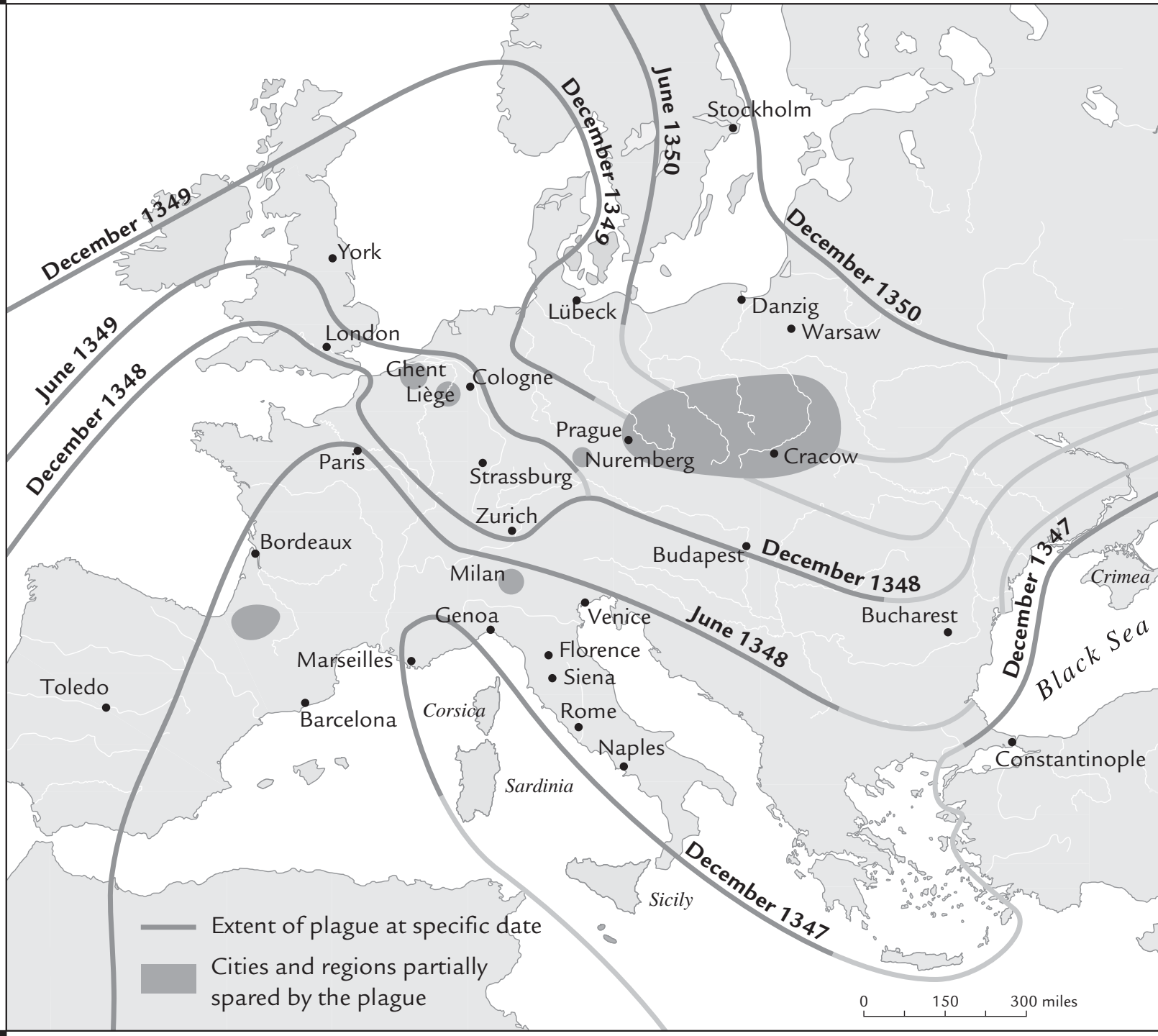
~1300
Byzantium shrinks
Granada shrinks and
Castile growth
Holy Roman Empire
growth

1453 Fall of
Constantinople
to the
Ottoman Turks

0 500 1,000 miles

Spread of
the Great
Plague
1347–50
From Sicily,
Sardinia,
Corsica,
Then Paris.

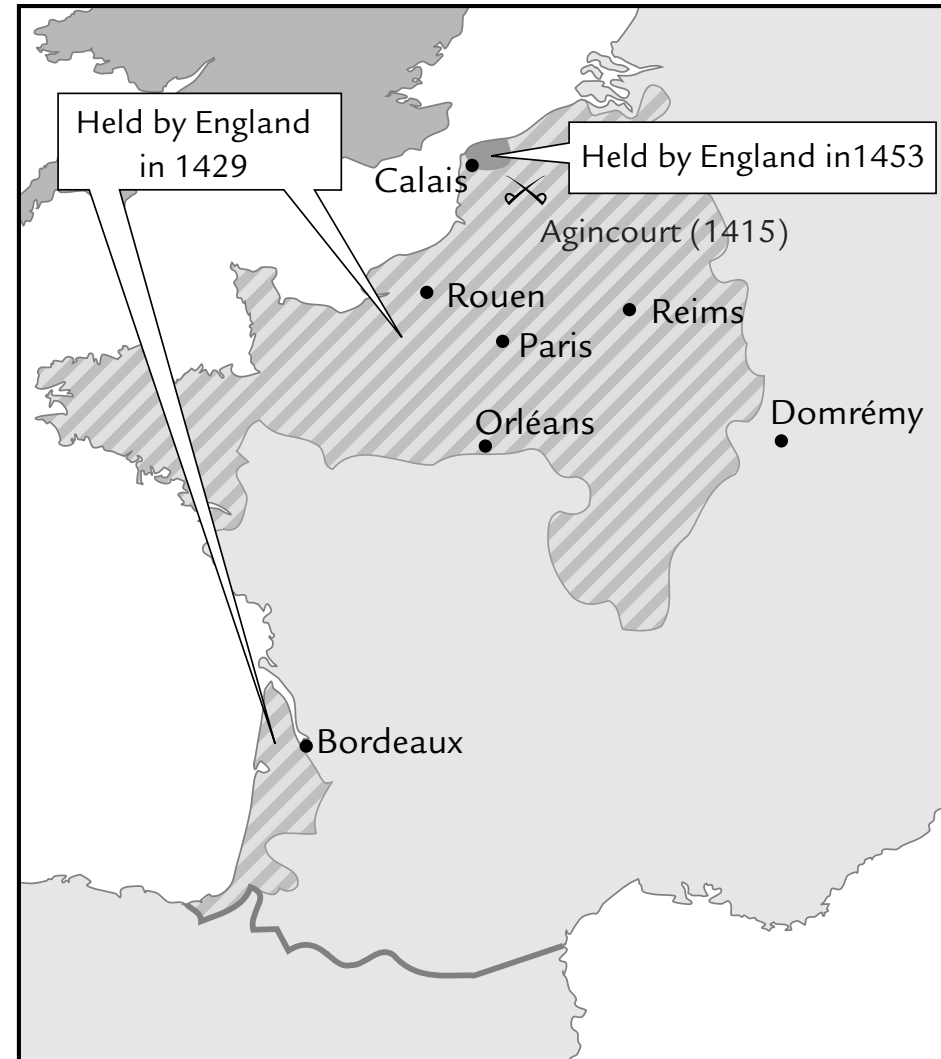
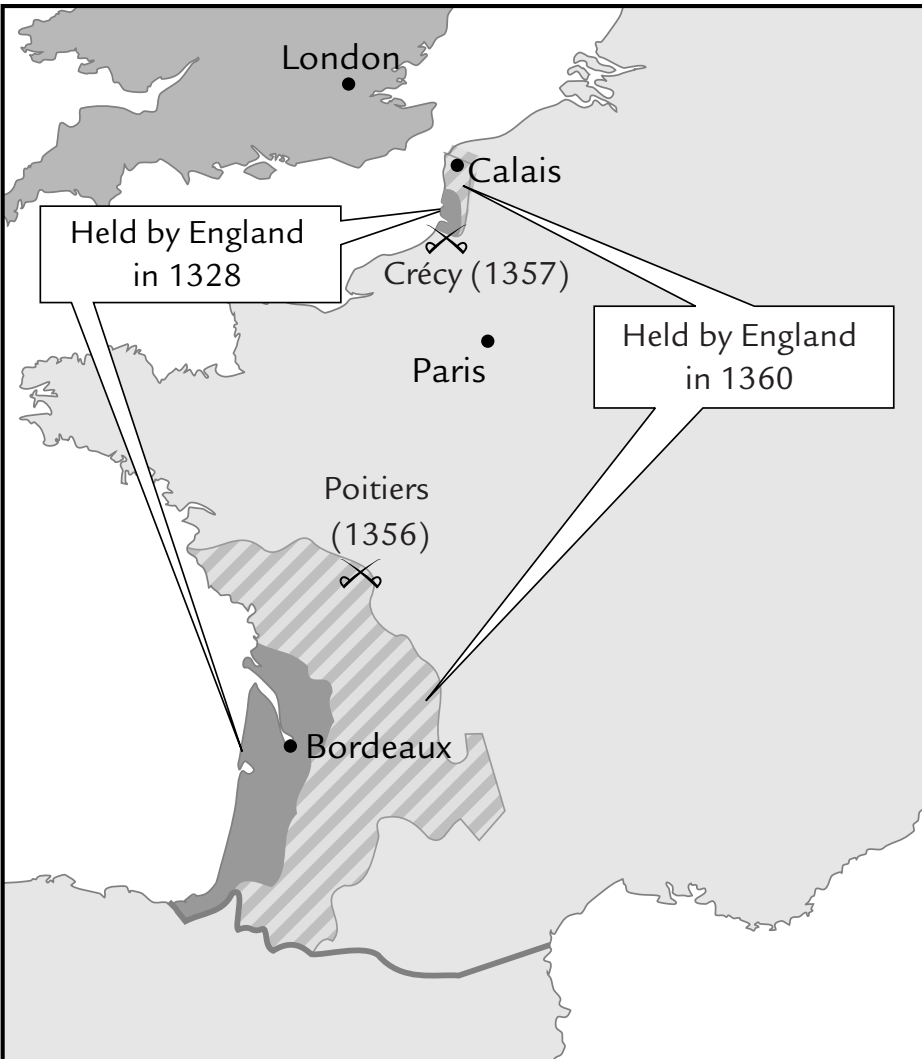
There were
a few areas
(dark grey)
that
escaped the
plague.



100 years war between England and France

From the capture of Calais 1347 to ruling of the whole north of France and the Netherlands, Bordeaux, till the loss of all this area excluding Calais – 1453.

Jeanne d'Arc 1431



Spain during Isabella (Castile) who married Ferdinand (Aragon) ~1469



Center and eastern Europe~1490



Europe 1500

— Boundary of Holy Roman Empire



