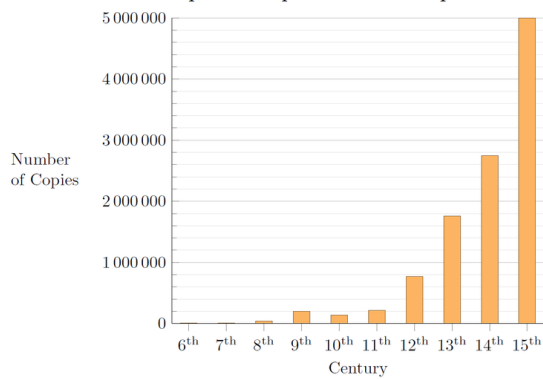


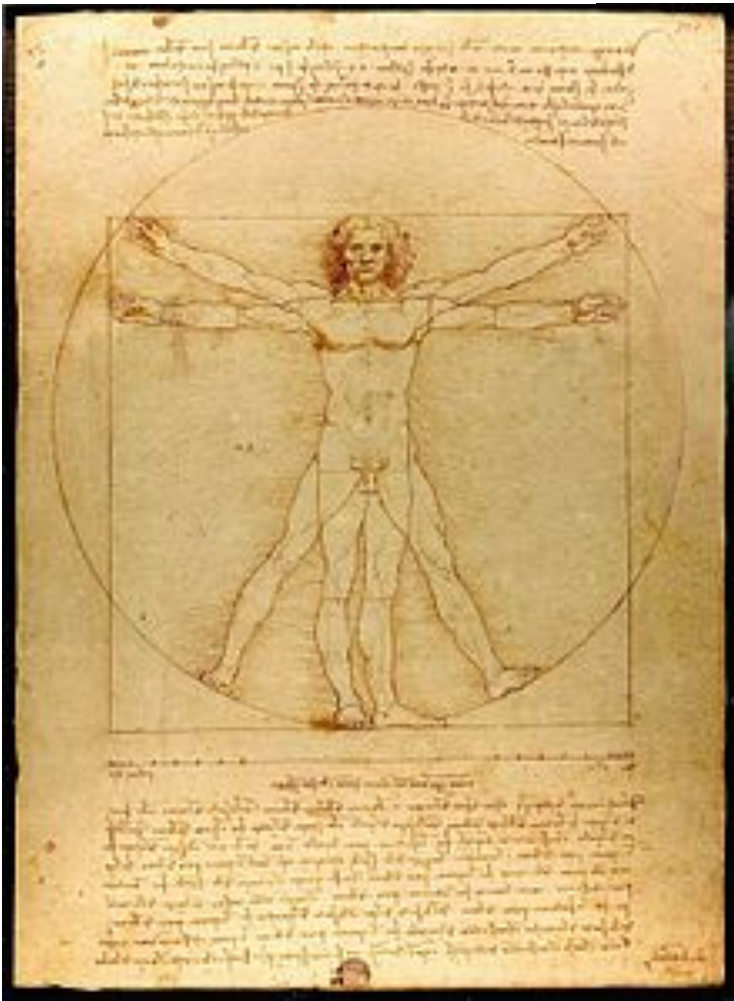
RENAISSANCE

Collected and edited by Prof. Zvi Kam,
Weizmann Institute, Israel

European Output of Manuscripts 500–1500*



*without Southeast Europe (Byzantine realm) and Russia



RENAISSANCE and the scientific revolution

Contents:

- Renaissance art: Italy, Netherland, pain, Germany
- Astronomy
 - Leonardo da Vinci 1486-1513
 - Copernicus 1473-1543
 - Ticho Brahe 1546-1601
 - Johanes Kepler 1571-1630
 - Galileo Galilei 1564-1642
- Telescopes and Microscopes
- Clocks
- Aviation and military technology
- Medicine
- Mathematics
- Chemistry. Vacuum.

Shakespeare 1564-1616

Elizabeth I – English golden ages 1533-1603

Demolishing the Spanish Armada 1588

Gothenburg Printing Press 1450

Historical Background

The middle ages are called “dark”, nevertheless, under the surface they prepared the Renaissance. Historians tend to assign dates to revolutions, for example the “scientific revolution” from Copernicus till Newton. In fact, history display gradual trends that anticipate the revolutions. Without such background changes, typically built from many infinitesimal steps, revolutions could not have occurred. This is the reason we reviewed the middle ages scientist contributions. Although many of them are hardly recognized in the halls of fame, we see the seeds of the great ideas of Copernicus and Galilei in their works, which they were probably exposed to.

The crusades, 1096-1272, created mostly hostile liaison, but also cultural interface between Europe and the middle east and the Arabic world. The contacts not only taught fortification building and hardening of steel swards, but also navigation technologies, and revival of the classical knowledge that migrated from Alexandria to the wisdom centers in Baghdad, but was lost in Europe at secluded Monasteries. Arabic science strongly influenced Byzantium scientists.

Spain, a naval empire, degraded at the renaissance. Italy and Germany were segmented into many competing small kingdoms. France and England were two united kingdoms fighting to control the Vatican and the world. In all these kingdoms sovereigns' pride was not reflected by military power but also by architectural and art projects, as well as support of science via hospitals and universities that proliferated towards the end of the middle ages. In these institutions classical heritage was slowly recovered through Latin translations from Greek and Arabic of Plato, Aristo, Archimedes, Euclid and Galen.

These signs of a renaissance were quenched due to the **black plague**, 1346-1352, when millions of people died, creating shortage in farmers, blacksmiths, carpenters etc. The society after the plague was enforced to improve workers status, to weaken Feudalism, as well as caused loss in the power of the church. The religious solutions offered no cure for the spreading plague, and seeded the **Protestant reform**.

The renaissance emerged from Italy at the 14th century, and brought revival and renewal in all fields of culture and science. The scientific revolution was first of all a revolution in methodology of scientific research: following 1000 years of relying on the classical scripts to learn about laws of nature, scientists performed experiments and quantitative mathematical models in order to support their theories, and dared to reject classical laws that did not match the experimental data.

Scientists freedom grew when they were supported by enlightened sovereigns who wanted to gain prestige as well as benefit technologically. Although France and England were larger and wealthier countries, the renaissance started in the city states of north Italy, where science was supported by wealthy open-minded merchants. The sprouts were nourished by the commercial links of Venice and Florence with the east. At the conquer of Constantinople, an influx of scientists and professional workers reached Venice, and brought the influence of the Islamic science and experimental approach. Using equipment that was not available to the Greeks (optics, precise protractors, pendulum clocks, gears) they performed measurements to define laws of optics, mechanics and astronomy, and dared to reject theories of Aristo, Ptolemy and Galen.

The historical landmark was the migration of the world model from geocentric to heliocentric. The process was painful, and included prosecution of its proponents (Bruno and Galilei) by the Church. Yet, clergymen decaying power could not stop the scientific revolution, and the unification of celestial and terrestrial mechanics happened by the careful measurements of Tycho Brahe, the phenomenological detailed model of Kepler and the physical laws of mechanics and gravitation by Newton. Modern optics was another field that had huge impact on astronomy (Kepler and Newton's telescopes that revealed a dynamic universe incompatible with the crystal spheres of Ptolemy) on biology (development of microscopes), and on people's vision via spectacles. The latter together with the spread of printed scripts provided wider readership of all ages even at dim candle light, undoubtedly an important component for the flourishing of knowledge. In future years many "amateur" great scientists appeared.

The revolution in physics and astronomy was associated with revolutions in medicine, biology and chemistry. Medical schools and hospitals were centers of research in human anatomy. Despite restrictions from the Church here too, the failure to arrest the black plague was a vivid reminder of the false doctrines relating sickness with sins, and enforced rational analysis of the causes of diseases and search for treatments and medications to cure them.

What are the drivers for the scientific revival towards the renaissance ?

Italian cities, especially Venice and Florence, were in fluent contacts with Arabic, Indian and Chinese merchants. They absorbed and imported rich scientific and technological knowledge.

During the middle ages European science studies were tightly linked to theology, and was based at monasteries. Their methodologies relied on ancient Greek philosophy, mainly Aristo, of logical induction heavily biased by religious beliefs, with no attempt to challenge them by the real world. On the other hand, Arabic scientists (e.g. Ibn Al Haytham) applied experimental evidence and measurements in order to substantiate their scientific theories. Unlike the dogmatic monks, the Muslim scientists were critical of the Greek heritage, and their Arabic translations were modified and enriched by notes of added wisdom and questions of the scientist-translator.

The Romans preserved the classical scientific and philosophical texts in Greek, the language of the educated elite, and collected all knowledge in exhaustive encyclopedia (e.g. Pliny). Their original contributions were mainly engineering, practical applications and technological developments. At the decline of the Roman empire, the center of knowledge and science education moved to Byzantium. Its conquer by the Ottomans at 1453 drove an influx of professionals and educated scientists to north Italian cities. They brought with them the spirit that is undoubtedly one of the causes of the renaissance.

“The Swerve: How the World Became Modern” by Stephen Greenblatt

An interesting book that relates the renaissance to renewed publication and distribution of a classical poem “On the Nature of Things” by Lucretius the Roman writer. The text was found at 1417 after 1000 years of oblivion in a German monastery in Polda, by Poggio Bracciolini, apostolic secretary of several popes and a book hunter. A short list of ideas proposed in this poem emphasizes why Greenblatt attribute such importance to its distribution:

All matter is composed of invisible particles

The elements of matter (“sources or seeds”) are eternal

Although their number is infinite, the number of shapes and sizes is finite

All particles are moving in infinite vacuum

The universe has no creator or designer

All things happen as a result of a swerve

the swerve is the source of free will

Nature does not stop experiencing changes

The universe was not created for humanity, nor cycles around it.

Humans have no distinction

Human society did not start in a golden age of peace and bounty,
but in a primitive struggle to survive.

Soul is mortal, there is no afterlife. Death does not exist for us.

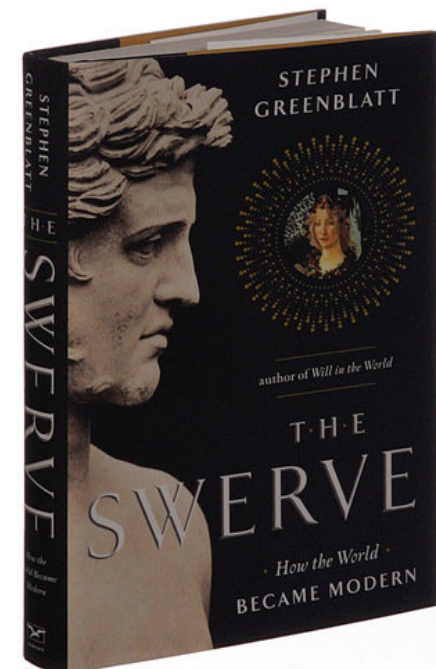
All religious establishments are illusions originated at superstitions.

Religions are cruel. Ghosts and devils do not exist.

Supreme life purpose is pleasure and minimization of pain.

The obstacle is self illusion

Understanding nature is a huge wonder.



The protestant reformation

Was one of the outcomes of the deteriorating authority of the Catholic church, and critical ideas antagonizing its fundamentals.

1483-1546 Martin Luther published in 1517 his 95 theses.

1509–1564 John Calvin humanism against the theological formality of the pope.



Although the protestant reformation challenged the hegemony of the pope and the holy chair, and opened possibilities for discussing ideas beyond the catholic conventions, one should not overestimate the reformatory freedom of thought. For example, Luther called Copernicus “an idiot who turns the bible over, since Joshua called the sun to hold in its place, not earth...”

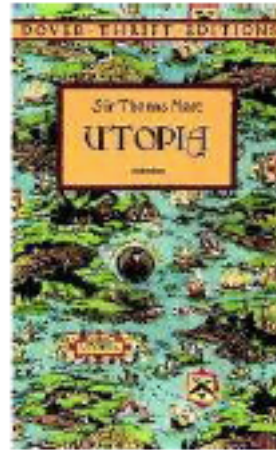
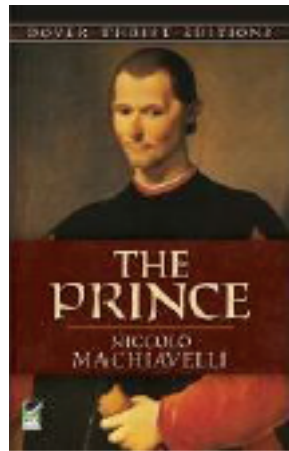
New winds also in **politics** –
deterioration in the divine authority of kings.

Niccolò di Bernardo dei Machiavelli 1469 –1527

1516 Publishes “The prince”

Sir Thomas More 1478 –1535

1516 publishes in England “Utopia”



Modern editions of “Utopia” and “The Prince”
These books are best sellers till today !

Spain is an example of missed opportunity:

Spain was divided into several kingdoms: Catalonia (Barcelona, Tarragona), Aragon (Valencia, Saragossa), Navarra (Pamplona, San-Sebastian), Basque land, Castile, Leon (Segovia, Madrid, Toledo), Astoria, Galicia (absorbed into Castile at the end of the middle ages) and in the south, under the Islamic Umayyad rule, the Moor Spain: Andalucía (Cordoba, Granada, Malaga, Seville).

The Moor Spain had a cultural and scientific golden ages with freedom and religious tolerance . The Reconquista put an end to it and freedom was replaced by in the Inquisition, established by Ferdinand II from Aragon and Isabella I from Castile in order to set Christianity on Muslims and Jews. The deportation of Spanish Jews at 1483 and the conquest of Granada, the last hold of the Moors in Spain, at 1492 completed the rule of fanatic Catholics all over Iberia peninsula, and prevented Spanish participation in the Renaissance.

The expansion of navigation in open oceans, enabled by the compass, the Astrolabe, and the triangular sails that facilitated motion against the wind and in storms, all motivated voyages to the far east around Africa. The established understanding that earth is a sphere brought **Christopher Columbus** to try and head west towards India, and he looked for funding in Venice, then the queen of seas with its large fleet of merchant ships. He may have failed since **Vasco de Gamma** has established the route

around Africa to India at 1498. He finally got his funding and support from Ferdinand and Isabella the rulers of the united Spain. He discovered America (in fact the Caribbean islands) at 1492, the year of the Reconquista and the strengthening of the Inquisition. America could have contributed to strengthen the Spanish empire, but the religious fanatics and conservatism strangled any possibility to join the European renaissance. Following 100 years of pumping America's gold to support stagnated sovereigns, the Spanish Armada was annihilated by British developed Marine technologies and war strategies.

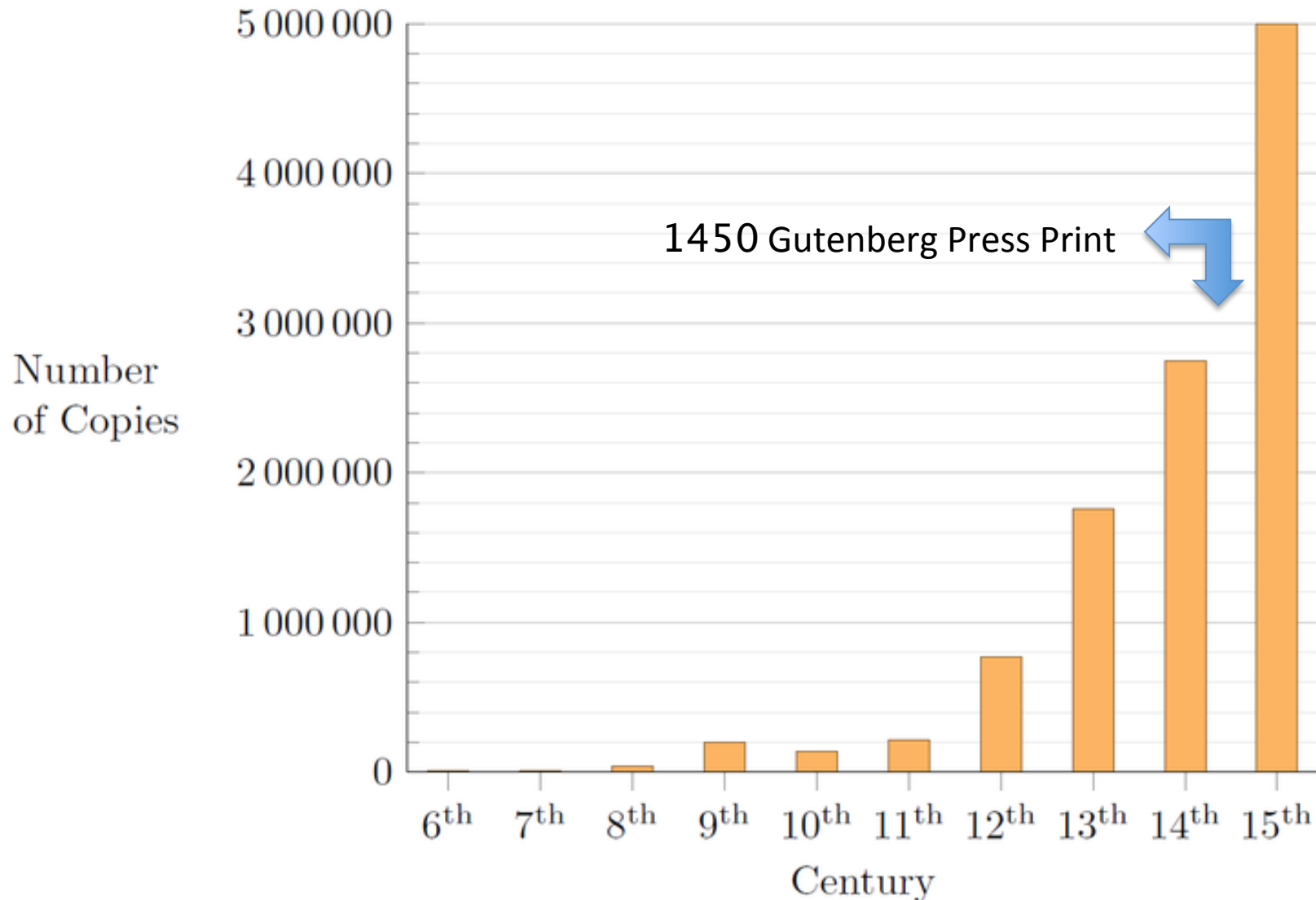
Spain was undoubtedly the closest European country to the east, but brutally erased its Islamic heritage , and stayed behind.

1450 Johann Gutenberg press print (Chinese technology) induced spread of knowledge and democratized education. Books could be possessed by larger number of people. True, the first book printed was the bible, but soon followed texts of Ptolemy (astronomy), Aristo (physics) and Galen (medicine) that exposed them to conflicting contemporary descriptions, measurements and models, and encouraged skeptic and critical scientific approaches. Starting at the 16th century one detects an explosion of intellectual activities all over Europe. The press print multiplied the number and variety of books, and people could study first hand more texts than ever. Publication of new editions of the original Archimedes scripts were followed by mathematical studies. New research could now be distributed in copies and available within a few month to colleagues in other countries. The print therefore affected dramatically scientific communication, providing collaboration across the continent. 1500 years before scientific achievements did not become widely known. Scientists reinvented ideas, and many theories were lost in time. But starting at about 1550 knowledge unification gave every scientist an account of the present status of the field, allowing him to base his work on updated information, and also exchange critical discussions with colleagues, a necessary condition to establish better science.



Is the graph indicative whether increasing demand for book motivated building the print press, or the facilitated publication and distribution of books created increase in demand?

European Output of Manuscripts 500–1500*



Writing tools.

5000 BC Pegs embedded in soft clay, and engraving with flint stones on limestone and later metal chisels on marble plates.

3000 BC Feathers dipped in ink produced from dark minerals mixed with animal fats. Scripting on papyrus and processed skins. Ink properties: need to be viscous, strong affinity to papyrus or slim surfaces, fast drying, and stable color that does not bleach in light or air.

973 Ma'ād al-Mu'izz li-Din Allah Fatimid Caliph of Egypt and the Maghreb, demanded a pen that would not stain his hands. He was provided with the first fountain pen, that held ink in a reservoir and fed it to the nib by capillary.

1495 Aldus Manutius of Venice start to use small letters for hand writing (Capitals for print).

1516-1565 Conrad Gessner describes a pencil with soft graphite core.

1636 Daniel Schwente (German) describes a pen made of a thin feather inside a thicker one.

1858 Joseph Reckendorfer registers a patent on a pencil with an eraser mounted on top.



Fountain Pen.
Nº 68,445.
Sept. 3, 1867



Nibs dipped in inkwells of different kind, then the fountain pen with a built-in ink reservoir were today replaced by the ball-pens.

1938 László Bíró sells in Argentina the ball pen “Biro”

1954 Parker, a large manufacturer of fountain pens, drives success selling a ball pen with thin tip and continuous line drawing.

Today, only snobs display and sometimes use for signing fat contracts, a fountain pen, such as a prestigious one produced by Montblanc ...



ECONOMY

1530 In Florence, Lottery funds state budget for special expenditures.

1589 William Lee The Englishman, invents a sawing machines for stockings.
textile industry flourishes in the 16th century.

RENAISSANCE ART

Renaissance Architecture

Most impressive church domes: Aya Sofia in Byzantium, built at 550, was the biggest dome of its time. Became model to Santa Maria del Fiore cathedral in Florence, with dome designed by Brunelleschi, and the Saint Peter basilica in the Vatican designed by Donato Bramante 1444-1514 and by Michelangelo.

The most impressive invention in Gothic cathedral structure was the external supporting columns (see picture below), replacing the thick heavy columns supporting the internal Romanesque cathedrals. They provided large painted-glass windows and feeling of monumental and spacious internal halls.



Saint Peter basilica in the Vatican



Aya Sofia built as a church in Byzantium,
and converted into a mosque in Istanbul,



Santa Maria del Fiore Cathedral
Florence,
designed by Arnolfo di Cambio
And the dome by Brunelleschi

Renaissance Art in Italy, Holland, Spain and Germany

Subject to pay attention to:

- Perspective, as opposed to mostly two dimensional painting before.

- Illumination: use of light and shadow in painting, to create 3D classical sculpture-like image.

- No more symbolic painting. Realistic presentations with faithful anatomy of people, horses

 - etc.

- Fresco, oil colors on canvas, and drawings on paper.

Subjects of paintings and sculptures are mostly religious, since art was funded by the church. But also subjects from Greek and national mythologies and portraits of emperors and rich families.

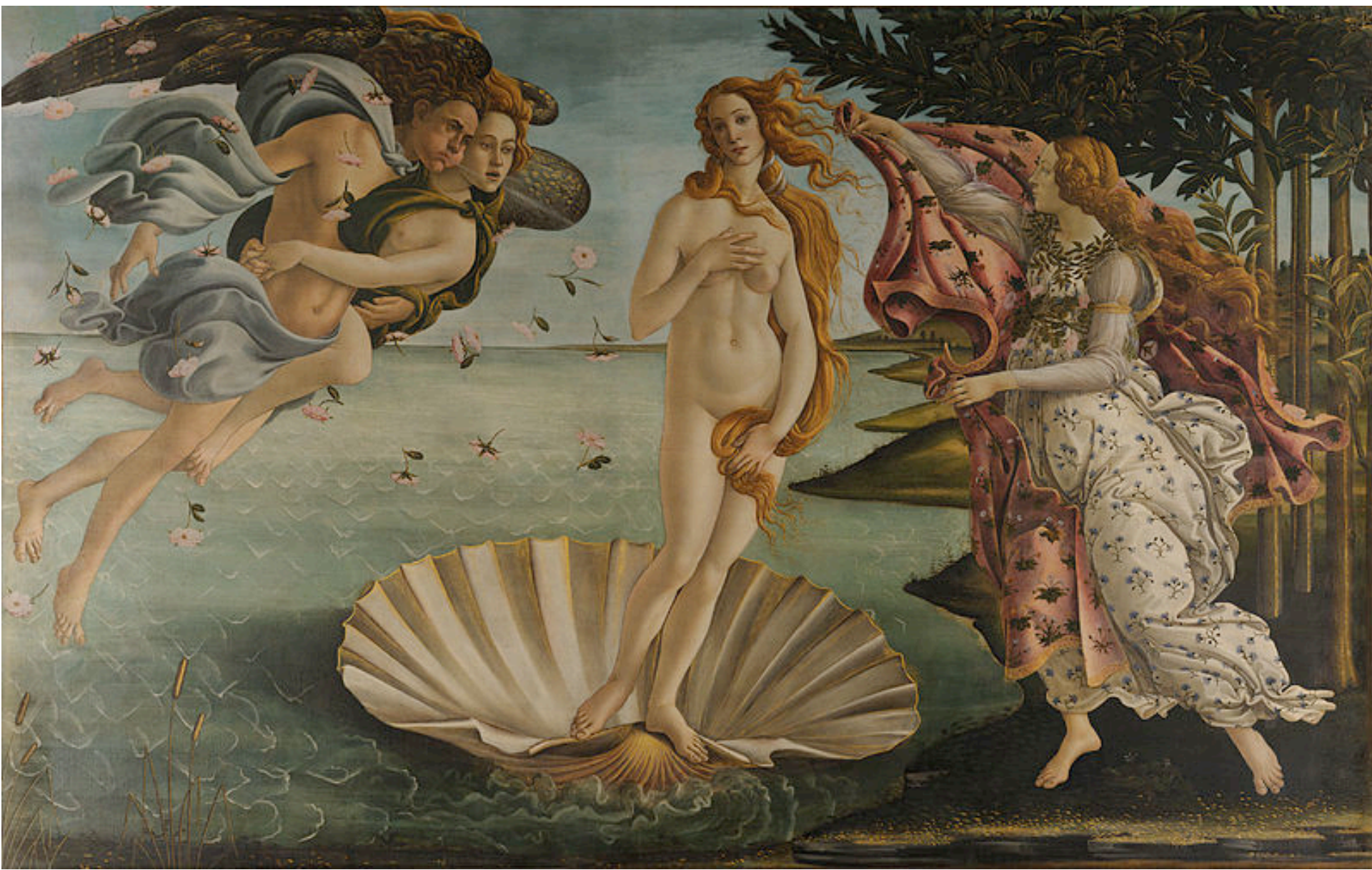
In Holland many paintings of every day life (Hieronymus Busch and Brueghel), and in Germany of animals (Dürer).

Interestingly, historical, mythological and biblical paintings reflect the views of the country where the painter lived.

In my view, El-Greco and Busch paintings are not main stream renaissance painters. El-Greco is almost an expressionist, and Busch is surrealist (reminded of Salvatore Dali).

Botticelli 1445-1510 The birth of Venus.

Note the use of light and shadow on people and dresses, and depth perception of the background



Raphael 1483-1520



Mona Lisa (La Gioconda)
Leonardo da Vinci 1452-1519





Michelangelo 1475-1565

Creation of man – Cistine Chapel, Vatican.
(Note]similarity of the painting to sculptures)

David – Florence. Moses – Rome.

Main Architect of Saint Peter's Basilica



David, Florence (note the hat...)
By Donatello, 1386-1466



The last supper,
Tintoretto, Venice, 1452-1519
(Note the dramatic use of light)



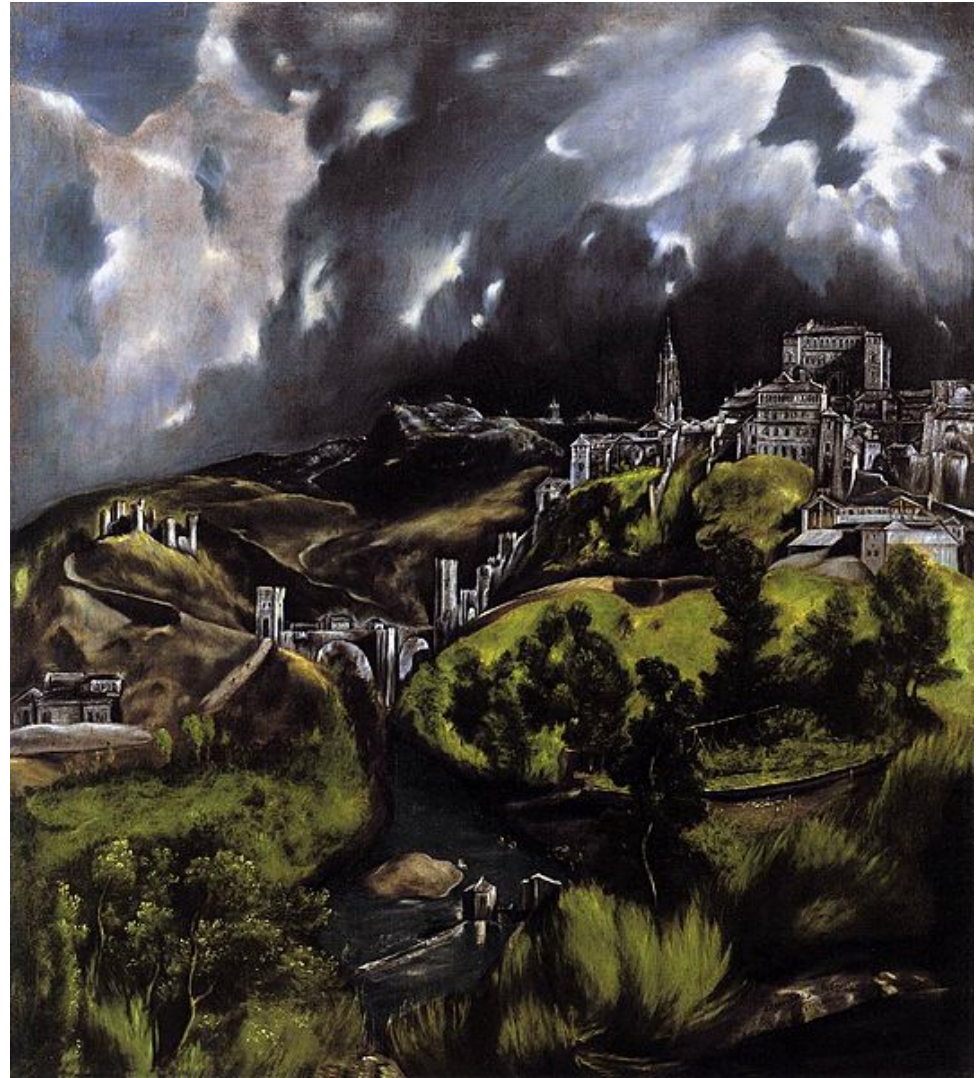
Eve handles the apple to Adam. Venice, by Tiitan, 1490-1576



Don Quixote
El Greco vs. Picasso



Toledo Spain, by El Greco, 1452-1519.



Laocoön and his sons fight the sea serpents.

Painting by El Greco

How is El Greco's depiction of human bodies
different from Michelangelo's ?



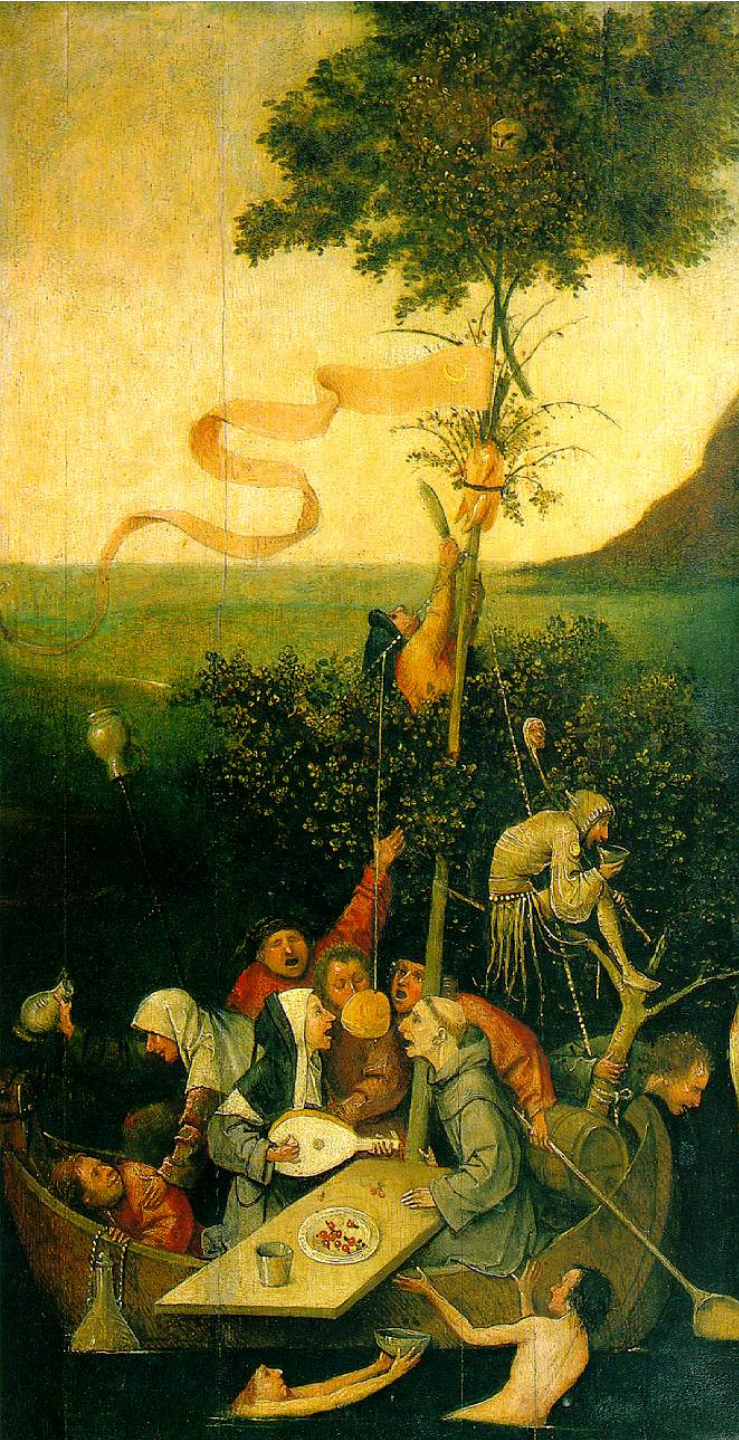


Albrecht Dürer, 1471-1578
Nuremberg, Germany
Paintings, Wood carving,
sketches
Faithful Perspective and
Proportions.

Photographic-quality
paintings of animals



, Hieronymus, or Jerome,
Bosch, 1450-1516
Surrealistic paintings



, Pieter Bruegel de Oude
Holland, 1525-1569

Every-day life scenes,
As well as mythological stories
Farmers feast

Icarus flight (note the Dutch
country scene, with only a hand
of the subject of the story seen
before he drowned.

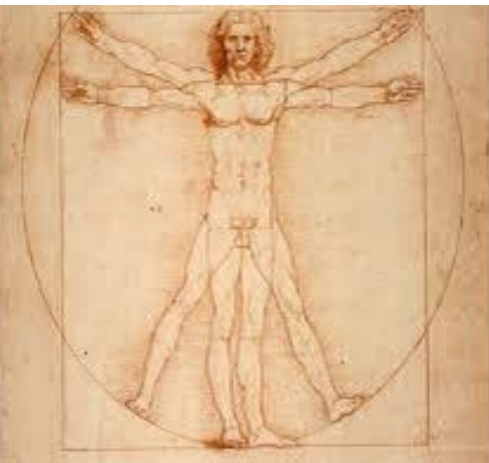




Leonardo da Vinci , 1486-1519

At the age of 15 he left his father's home to Florence, where he served as an apprentice to Andrea del Verrochio, a painter and sculptor. He later started his own workshop. 1482 he moved to Milan, serving at the court of Ludovico Sforza, organizing festivals, designing arms and fortifications (Tank, Submarine, Parachute, Helicopter, Glider with flipping wings), as an architect (Churches, homes, tunnels, bridges, including a bridge over the Bosphorus), and as a researcher in Geometry, Mechanics and mostly Anatomy. This period of his life is widely documented in note books including drawings of his inventions. Interestingly, the text in this notebooks was mirror imaged of normal letters. After the French occupation of North Italy, 1499, he wandered in Italy with little recognition. Leonardo worked for Cesare Borgia as a military engineer, and met Machiavelli.

During 1513-16 he worked in Rome. The Pope prohibited autopsied for his anatomical studies, therefore Leonardo moved to cats, horses and dragons... On 1516 Giuliano di Medici, his patron, died, and he accepts a position of chief painter, engineer and Architect at the court of Francis I, with scholarship and dwelling at royal chateau at Amboise. He died in Cloux, France 1519.

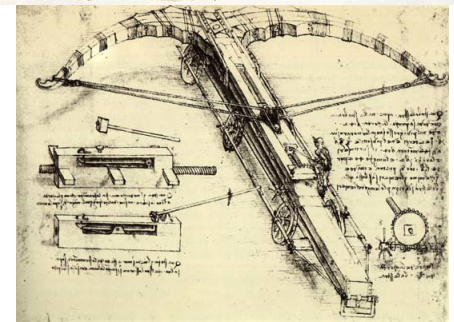
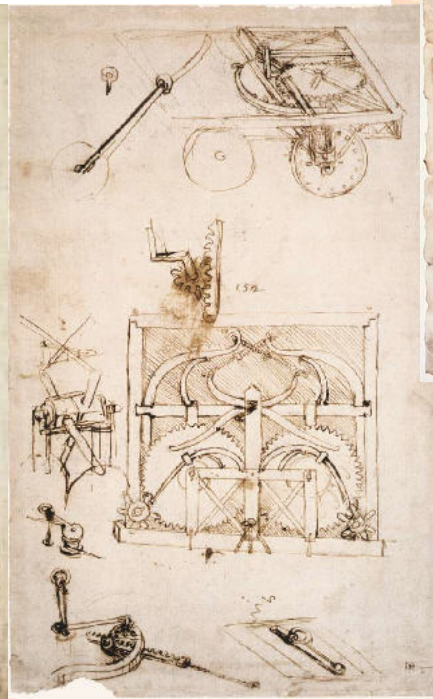
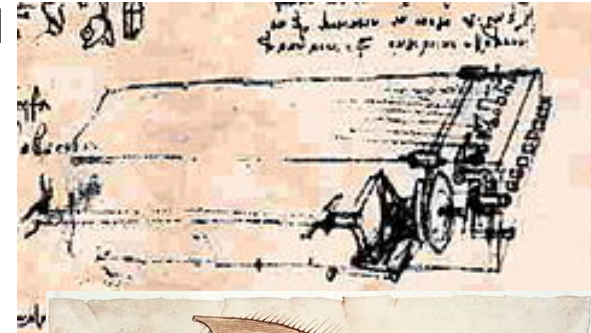


List of Leonardo's achievements:

Mechanics: Unlimited motion is impossible.

Influenced by Archimedes, he built measurement equipment to understand laws of nature. He built Clocks, Hydrometers (similar to Cusa's air humidity sensor), trip odometer (according to Hero), Anemometer (to measure wind speed and describe air flow patterns) and described the effect of capillarity.

1482 he started to study the anatomy of man and at 1510-11 he drew details from autopsies. His notebooks were mixes of anatomy, mechanical and military inventions, and explanations.



**PHYSICS
ASTRONOMY
AND
EARTH SCIENCES**

Astronomy - continue to drive all sciences: mathematics, physics as well as measurement technology (lenses, telescopes, precise angular positioning).

Science provided intellectual food for monks, but the church had tight control on them. Scientists that were supported by enlightened sovereigns were more free in their investigations. While the church prevented anatomical studies (blood shed is forbidden) they supported astronomy, to establish the calendar (move from Julian to Gregorian year). They rose up only when the centrality of earth was shattered...

1582 Pope Gregorius 13th announced the usage of the Gregorian year (The Julian year was found unfit since 45 AC).

1583 Joseph Scaligar establish a Julian day to calculate astronomical times.

The Roman Calendar was based on the moon. 355 days in a year, and as needed, a leap year was announced with an extra 22-23 days month between February and March.

The Julian year had 365 days and every 4th year 366 days (added as 29th of February). In order to close past delays in the Roman calendar, days were added every three years, causing non-uniform dates in different sites.

The Gregorian year considers the more precise number of days per year: **365.2425**: A leap year is a year divisible by 4, but not by 100 (unless divisible by 400).

Astronomy and Physics



The geocentric model of **Claudius Ptolemy**, as described in “The Almagest”, was translated into Latin at the 12th century by **Gerard of Cremona**. This drove additional studies, e.g. By **De sphaera mundi** of **Johannes de Sacrobosco** that include Ptolemy’s measurements and added astronomical Tables: **Alfonsine Tables**. Several events indicate the underflows: Published heliocentric theory “trepidation model” attributed to **Thabit ibn Qurra**, lectures at University of Wien, by **Georg Purbach 1423–1461**, collected by his students **Regiomontanus 1436–1476** and published at 1470 as a new world theory: “Theoricae novae planetarum”, new edition “Epitome of the Almagest” that included measurements of Cardinal **Bessarion** from Constantinople, all were the buds of the following blooming new terrestrial and celestial mechanics establishing the heliocentric model by:

Nicolaus Copernicus 1473–1543

Leonardo da Vinci 1486 – 1513

Giordano Bruno 1548-1600

Tycho Brahe 1546-1601

Johannes Kepler 1571–1630

Galileo Galilei 1564–1642

William Shakespeare 1564-1616

Shakespeare was knowledgeable in of astronomy, included descriptions in events in his plays, and openly ridiculed astrology. Examples:

Julius Caesar:

stable as the north star...
Beware the ides of march

Romeo and Juliet:

swear not by the moon,
..the inconstant moon, that monthly changes in her circle orb

Hamlet:

Doubt thou the stars are fire; Doubt that the sun doth move;
Doubt truth to be a liar; But never doubt I love

Conflicts in Mechanics – An arrow is shot from the arc to a free flight – but Aristo claims it moves due to external force, and its “natural” state is in rest. The new term: inertia.

Giovanni Battista Benedetti 1585

Inertia – makes a body move when free of force. He studied Archimedes and applied mathematics.

Simon Stevin 1586

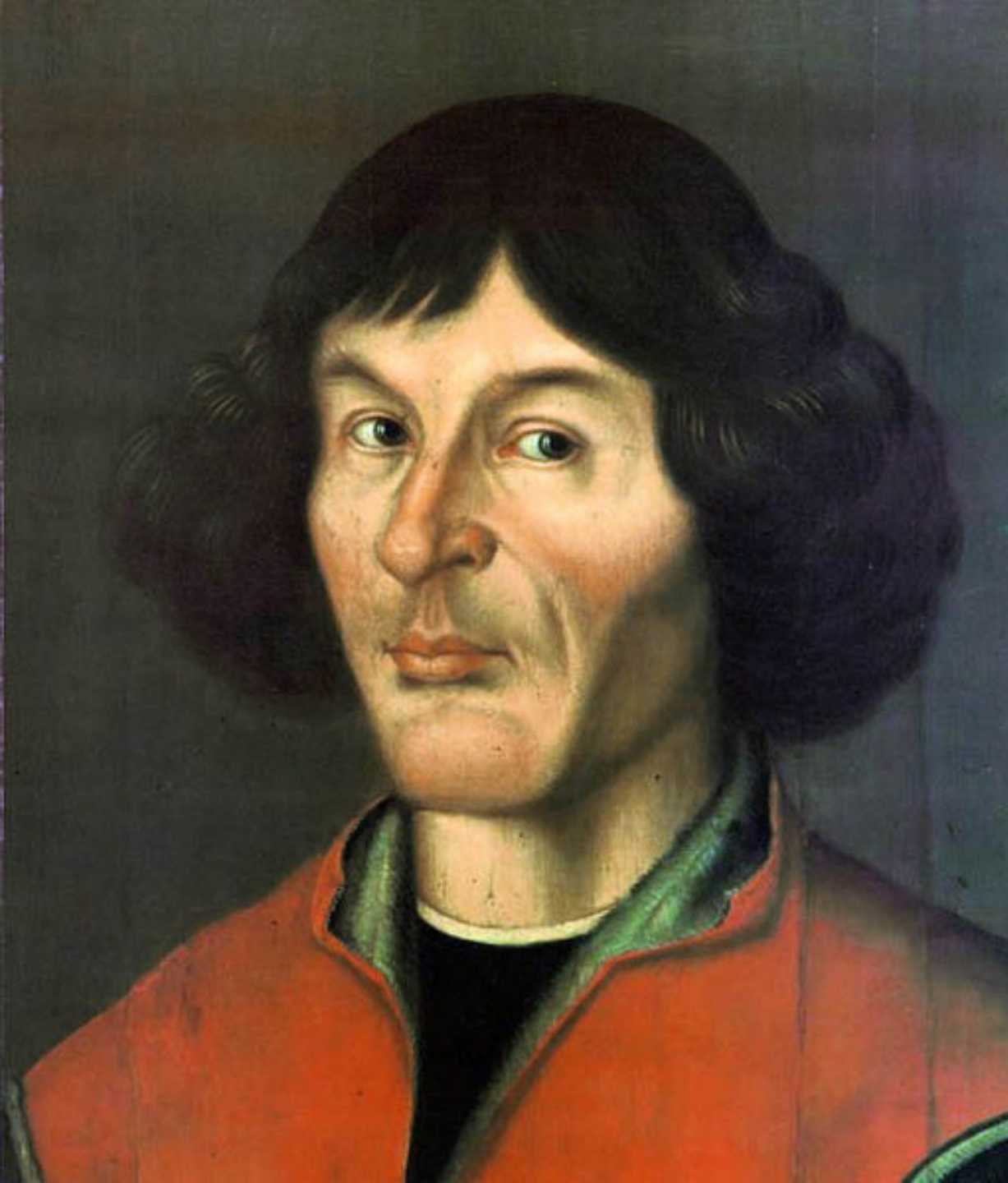
Hydrostatics – infinite motion is impossible, therefore all water is in equilibrium. Pressure at bottom of a container depends on depth only. He understood the laws of moments and centers of mass of different shapes.

1630 — Cabaeus

Discovers two kinds of electrical charges.

1637 — René Descartes

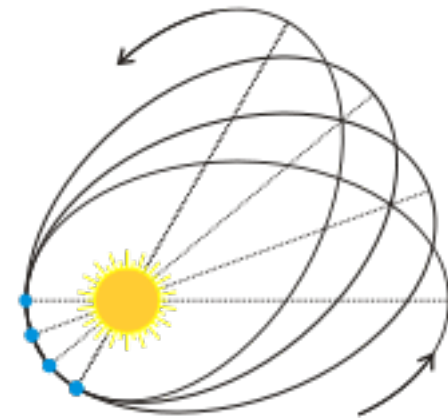
Calculates the refractive angles of sun light in rain drops, and connects to the rainbow angle.



Nicolaus Copernicus 1473–1543

The beginning of the scientific revolution is attributed to his proposed model of the planets and earth motion around the sun: the heliocentric model. His book: “*De revolutionibus orbium coelestium*” was published after his death. Copernicus applied more epicycles than in the geocentric model, to justify deviations from circular paths, but his revolutionized idea was that planets rotate around a center point, and not in a circle around a point on another circle.

Copernicus studied in Krakow, Poland 1491-5. He then studied law in Bologna, where he met **Domenico Maria Novara (1454-1504)** and heard revolutionary ideas about earth motion. At 1501 he got his PhD in law from Prada University. 1507 he finished medical school in Padua, and became the personal physician of the Polish Bishop. He worked as a land surveyor and administrator of the **Chapter of Frombork** that were loyal to **Sigismundus I** the Polish king. He performed astronomical measurements to adjust the Gregorian Calendar, and sent them to Rome at 1513. That induced him to examine the heliocentric model, and the sun motion resulting from the earth rotation in the opposite direction. He He measured the orbit of Mars and suggested its motion is in a circle around the sun. He still believed in circular orbits and the crystal sky models, but considered the stars motion resulting from earth rotation. 1514 he bought a house he used as an observatory. He found that the sun orbit diverted by 30 degrees since Ptolemy's time, thus he meant to set years from stars position, and determined the precession of earth axis of rotation. He realized earth orbit is eccentric, and designed a complex model to maintain circular orbits. 1517 Copernicus published "monetae cudendae ratione", where he states that gold coins with low gold contents drives high prices. The effect was called later "Gresham's law" 1527 he measured the eccentricity in Saturn's orbit, a year later in Jupiter and 1530 in Venus.



His book "De revolutionibus orbium coelestium": on the rotation of stellar bodies, was published in Nuremberg at the day of his death, May 24 1543



Martin Luther said about Copernicus: “this dumb wishes to turn over astronomy, but as written in the bible, Joshua commanded the sun to hold over Gibeon, not the earth to stop.”

1551 Erasmus Reinhold, the German, published astronomical calendar (the Prussian calendar) based on Copernicus measurements.

1576 Thomas Diggs rejects Copernicus model of a single sphere of stars, since they are at various distances from us.

Nocturlabe. 15th century

At night, the stars seem to rotate around the axis of the north star every 23 hours and 56 minutes, 4 minutes delay every day, creating an accumulated deviation of stars positions that (almost) reset itself after a year. This provides a way to determine the hour at night: the Nocturlabe (named after the Astrolabe). When the date is dialed on one side of the plate, the handle is aligned to the north star by a pinhole, and the other side of the disk is aligned to another star the hour is opposite the date.



Antonio Santucci's sphere. 1588

The Great Duke Ferdinand I de Medici issued from Antonio Santucci construction of a model of the universe according to Aristo-Ptolemy with earth at the center. The model is supported by gold plated pivots, axes and rings, with god overlooking from the top of the sphere.

It was completed at 1593 and displayed today in the mathematics room in the Uffizi museum in Florence.

It is a pity that so much efforts and money was invested in a model that when built was already scientifically collapsing. But it reflects the strength of belief of the public in Ptolemy's astronomy.



William Gilbert 1544-1603

1660 publish “on magnets, magnetic bodies, and the big magnet of earth”
Gilbert discriminates electric from magnetic forces, and inseparable magnetic poles.
In hydrostatics: water pressure increase with depth.

Thoma Harriot 1592

Measures angle of $2^{\circ}56'$ between the north star and the magnetic north.
Confirms Snell’s refraction law, with the same refraction index for all incident angles.
Writes the formula for the area of a spherical triangle given its three angles:

$$A=4\pi(A+B+C-180)/360$$

Marko Dominis 1611

Studies rainbows.

Giordano Bruno 1548-1600



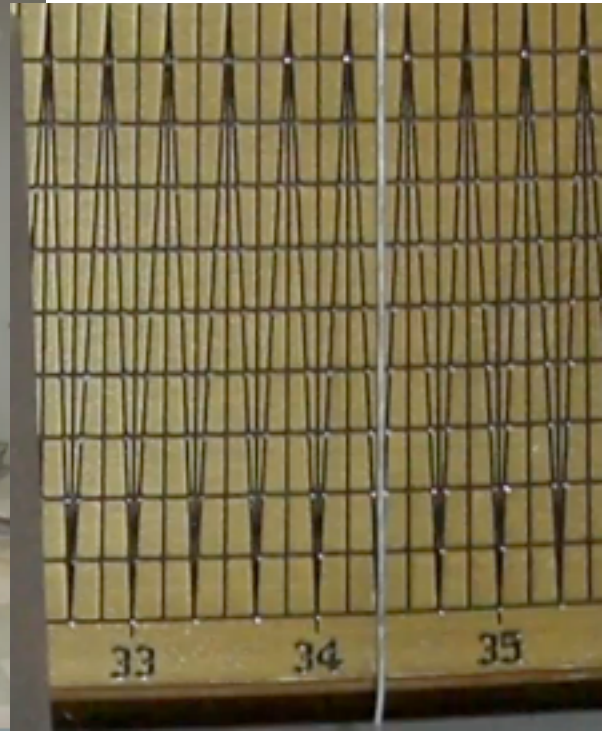
Bruno was a Dominican monk who was exposed to the recovered Plato's books, and claimed that the heliocentric theories of Copernicus conflict the perfect crystal spheres model of the universe. His astronomic model proposed that the sun is a star, and earth rotates around. He believed the universe is full of stars with intelligent creatures. Bruno exchanged the heliocentric models of Plotinus (3rd century) and Blaise Pascal (hundred years after Bruno) by a model with the center of the world anywhere in an infinite universe.

Bruno wandered in Europe, taught in Padua and lost his job to Galileo, and he moved to Venice. When he wanted to leave Venice he was delivered to the inquisition. Similar to Galilei, his acceptance of Copernicus theory had a dominant effect on his trial. But unlike Galilei, Bruno refused to deny his beliefs. When his verdict was read in the inquisitors court he said: "your fear of announcing this sentence may be bigger than my fear of its execution". Bruno was declared heretic and was burned at the stake at February 17 1600 at Campo di Fiori in Rome. The inquisitors showed mercy and tied a bag of gunpowder to Bruno's neck to shorten his pains. But his tongue was nailed to his jaw to prevent him from talking on the stack. His ashes were scattered over the Tiber.



Tycho Brahe 1546-1601

The last great astronomer in the pre-telescope era. He built and perfected a viewing tube with precise lateral and height angle measurement capability (see the nonius-like scales mounted onto his tube).



1572 Brahe discovers a nova (star explosion) in Cassiopeia, and from the small parallax he measured, he determined that its distance is much larger than the moon. That was in conflict with Aristo who claimed that all transient events in the sky happen in the atmosphere, and made his mind that celestial bodies are not static and are not eternal.

Brahe believed earth is stationary, since a stone falls down, and not westward – opposite earth relative motion to the stars.

1577 Brahe followed the motion of a comet, and determined it was far beyond the moon, and moves in a circle around the sun, but its speed varies.

Frederick II king of Denmark 1534-1588 donated Brahe an island Hven, to build his observatory, Uraniborg, in 1580 and another one, Stjerneborg, in 1584, that included advanced measurement equipment such as Sextants. From 1576 and for 20 years Brahe accumulated precise tables of the positions of planets which he kept without publication. He found that when Mars is closer to earth when moving against the sun motion. Since its orbit crosses the sun orbit he concluded that the sky is not a single crystal sphere, but is fluid. He still determined that the sun orbits around earth, but the stars rotate around the sun. Even after concluding his detailed measurements on the orbits of Mercury, Jupiter Venus and Saturn he still maintained that they, like the sun, all rotate around earth.

Christen Srensen Longberg, Longomontanus 1562-1647 was Brahe's assistant, that measured and discovered the anomaly in the motion of the moon.

1597 Due to dispute with the new Danish king, Christian IV 1577-1648, Brahe moves to Prague, and since 1599 became the court mathematician of Rudolph II the Czech Emperor. At 1598 Kepler, his assistant, published a catalogue of 1004 stars. He published all measurement note books in Prague only after Brahe's death.

Johannes Kepler 1571–1630

Kepler was born in Wurttemberg, Germany as a protestant, and studies in Tübingen where he was exposed to Copernicus work by Michael Mastlin 1550-1631. He moved to Graz, ran away from the Catholic reformation to Hungary, and returned without admitting to Catholics. At 1600 he moved to Prague to work for Tiho Brahe, and when his master died, he took all his notebooks. He moved to Linz when Rudolph II died. To earn money, he published the “Rudolphian tables” to honor Emperor Rudolf. They were published 1627. A year before, his mother was accused of witchcrafts, and fled to Ulm. 1630 he settled in Regensburg, where he died. He kept contact with Galilei.

Kepler wrote appendices to the important medieval optics scripts by **Witelo** 13th century, and **John Peckham 1232-1292**. For example: the illumination intensity fall with square distance from the source (1604), vision is due to image formed on the retina, and he explains the source of visual impairments of short and long sight due to eye lens failure of focusing on the retina. He describes light propagation in straight lines, and in 1611 explains rainbow formation by internal reflection in water droplets. He writes the laws of refraction at small angles, the basis of Geometrical optics.

Kepler had mathematical contributions as well: He used Cusa’s exhaustion method to prove that the volume of a sphere segmented into large number of pyramids with vertex at the sphere center and bases on its surface is a third of its surface area times the radius. He showed that of all cylinders inscribed in a sphere, the one with Ratio of diameter to height of $\sqrt{2}$ has the largest volume. He used The terms “indivisible” and “incremental”, the foundations of calculus.



Kepler was influenced by Copernicus, and proposed his own Heliocentric model. He believed that orbits of the five planet must match the five perfect geometrical shapes, but could not reconcile this model with his measurements. He therefore traveled to Tihobrahe, who had the most accurate and extensive measurements. He was accepted there warmly at 1600, but Brahe prevented access to his notebooks to Kepler, since he did not accept the heliocentric model which Kepler wanted to confirm. He finally let him see the Mars data of the last 20 years. Kepler estimated it will take him two months to fit a model to Mars orbit, but this job lasted seven years. Kepler used the logarithmic tables of Napier to carry his elaborate calculations, and devised an ingenious method to fit the Mars motion from Brahe's measurements every 687 days, the Mars year. Then he calculated the orbit of Mars around the sun. A deviation of 8' between Brahe's measurements and his calculations brought him to come up with **the first Kepler law**: that the orbits are not circular but elliptic, and the sun is in one of the ellipse foci. This idea was probably an influence of **Gilbert**, who believed that the force that the sun exerts on the planets is magnetic, and inversely proportional to the distance. The elliptic eccentricity, to Gilberts understanding, is due to the altering attraction and expulsion every $\frac{1}{4}$ of the orbit.

The second Kepler law: that resulted from fitting Brahe's measurements: Planet year length, T, when he completes one rotation, Depends on the long axis of the ellipse. The area, A, covered by the line between the planet and the sun per unit time is constant along the orbit (therefore speed is bigger close to the short axis)

The third Kepler law: $T^2 \sim A^3$. A graph shows that the closer to the sun, the faster the speed and shorter "year".



Kepler laws are the first laws in the history of physics that were determined from fitting to a long series of highly accurate measurements. It is worth emphasizing that Brahe's measurements were performed with a viewing tube, not a telescope. The telescope did not improve the accuracy of measurements of star positions, but later revealed the moons of Jupiter, an important observation of a moon system not centered around the sun (or earth). The measurements on the orbits of Jupiter moons were faster and easier to confirm Third Kepler's law.

Another point to appreciate is that the eccentricities Kepler extracted from Brahe's measurements were very small (see table below), e.g. 10% eccentricity of Mars cause few degrees deviations from circular orbit. Similarly, the evaluations of the long and short orbit axes was a great achievement in scientific computations, considering that measurements were relative to star positions, and required parallax corrections due to earth motion.

	ellipse half long axis	eccentricity	Year length
Mercury	0.3871	0.206	87.969 days
Venus	0.7233	0.0068	224.70 days
Earth	1.000	0.017	365.256 days
Mars	1.5236	0.093	686.98 days
Jupiter	5.2026	0.048	11.856 years
Saturn	9.5719	0.053	29.369 years

Kepler's 1st law

The planets orbit is elliptical, and the sun is in one of its foci

The law was proposed by Kepler based on orbit measurements, but proved by the Gravitational force formula of Newton.

$$\underline{F} = m\underline{a}$$

$$\underline{F} = \underline{u} * GMm/r^2$$

\underline{u} is a vector from the sun to the planet

\underline{a} is the projection of the acceleration on \underline{u}

$$ma = md^2r/dt^2 - mrw^2$$

$$w = \text{Angular speed} = L/mr^2$$

Mrw^2 Centripetal force

$$d^2r/dt^2 = GM/r^2 + L^2/mr^3$$

Substituting: $r = 1/s$ $t \Rightarrow q$ we get: $d/dt = Ls^2/m d/dq$ since: $L = mr^2w = m/s^2 dq/dt$

$$dr/dt = -1/s^2 ds/dt = L/m ds/dq$$

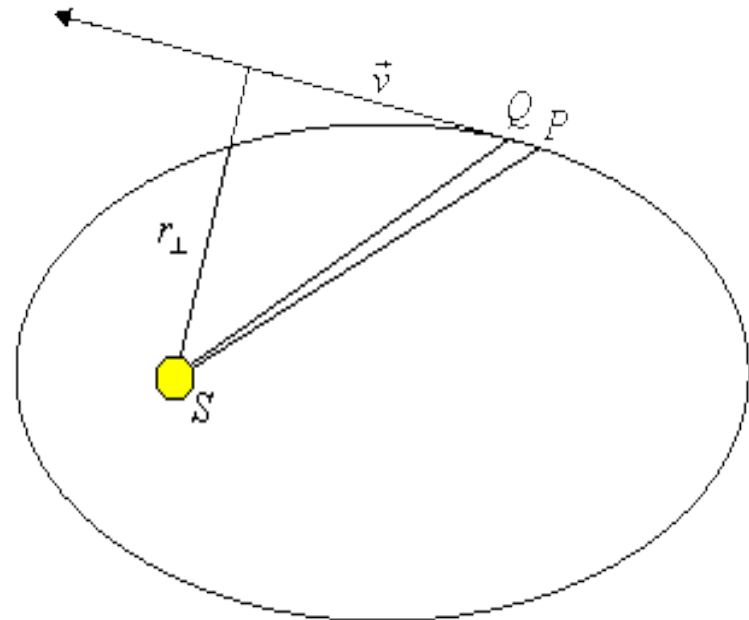
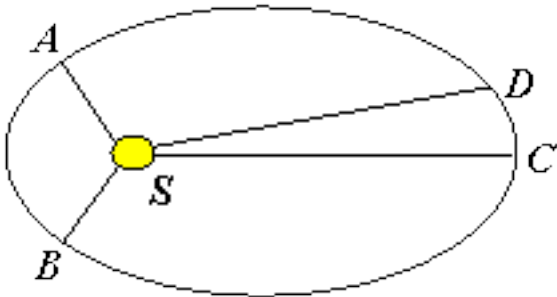
$$d^2r/dt^2 = L^2S^2/m^2 d^2s/dq^2$$

The equation of motion: $d^2s/dq^2 + s = GMm^2/L^2$

The solution: $s = 1/r = GMm^2/L^2 + E' \cos q$

The integration constant, E' is the ellipse eccentricity

$$A(1-E^2)/r = 1 + E \cos q \quad L^2/GMm^2 = A(1-E^2)$$



Kepler's 2nd law

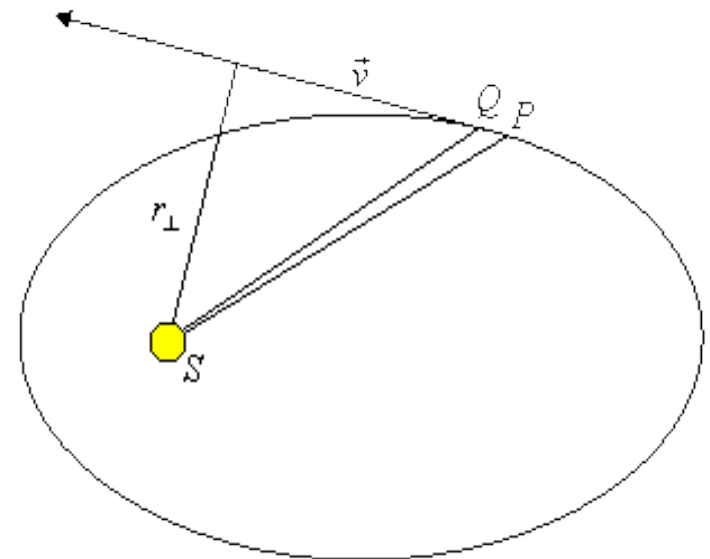
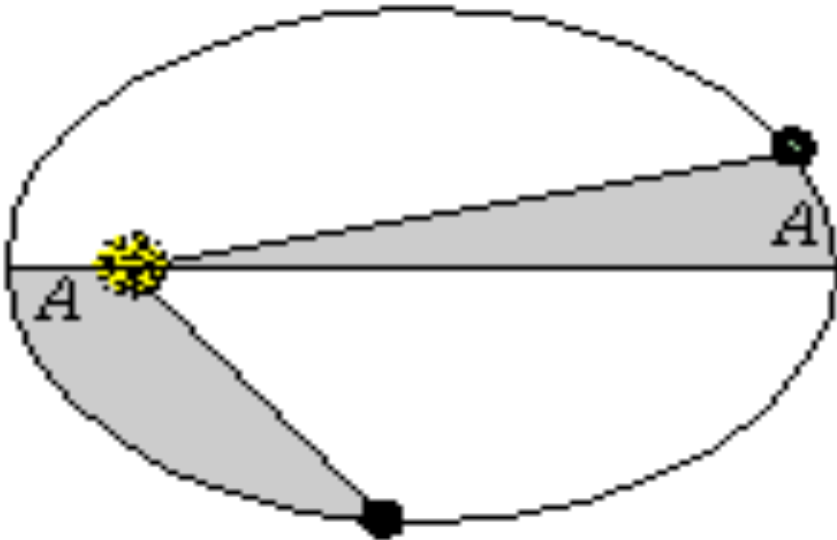
The area per unit time swept by the line between the sun and the planet is constant along the orbit, (and is proportional to the angular momentum, L)

$$r\dot{\phi} = \text{const}$$

$$dS/dt = r\dot{\phi} = \text{const}$$

$$L = mvr \quad dS/dt = 1/2rv = L/2m$$

The angular momentum of the planet does not change along the orbit, since the force between the sun and the planet acts along the axis joining them, therefore adds zero angular momentum.



Kepler's 3rd law

The time to complete a revolution of a planet around the sun, T, depends on the long axis of the ellipsoid orbit, but independent on the short axis.

$$T^2 \sim A^3$$

T Planet year ; A Long axis of the ellipse ; B Short axis ; E Ellipse eccentricity

$$B^2 = A^2 (1 - E^2)$$

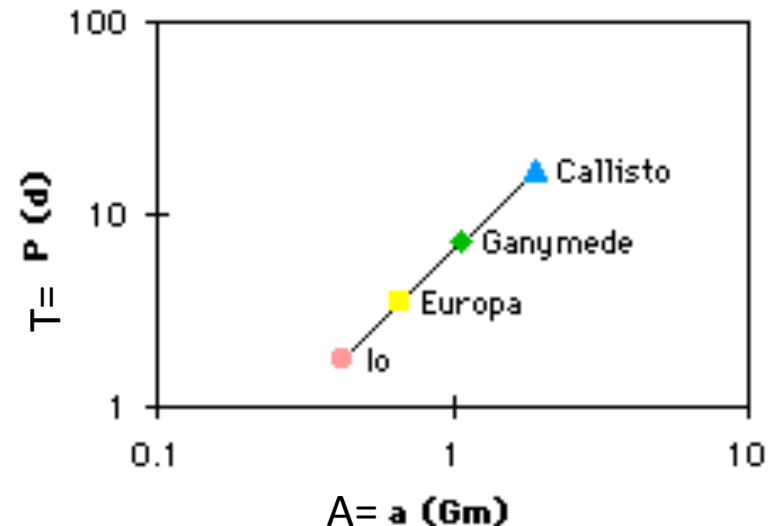
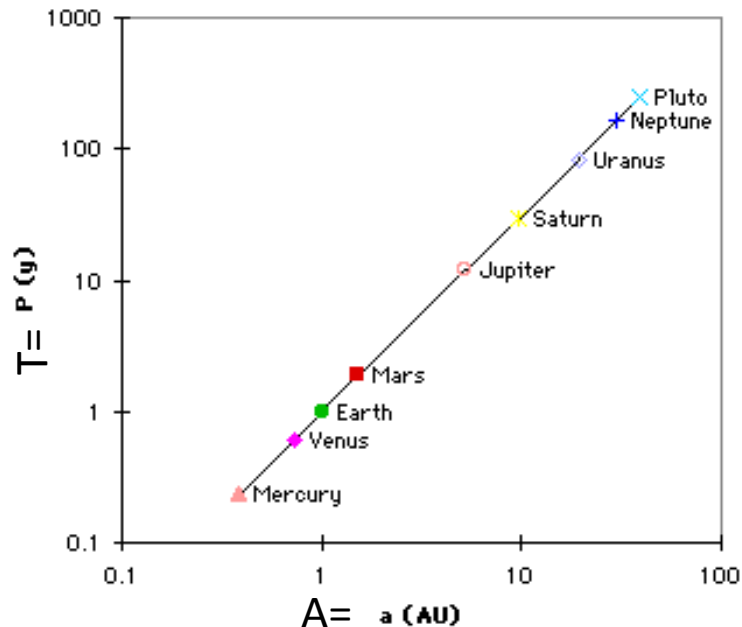
Area of the ellipse = πAB

$$dS/dt = L/2m \Rightarrow T = \pi AB / [L/2m];$$

$$T^2 = (2\pi m AB / L)^2 = (2\pi m AB)^2 / (GMm^2 A (1 - E^2)) = (2\pi m AB)^2 / (GMm^2 A (B^2 / A^2)) = 4\pi^2 A^3 / GM;$$

Independent on E

In logA vs. logT plot of the planets around the sun, as well as the moons of Jupiter that Kepler discovered through his telescope, the planets are on a straight line with slope 3/2



Sky coordinate system

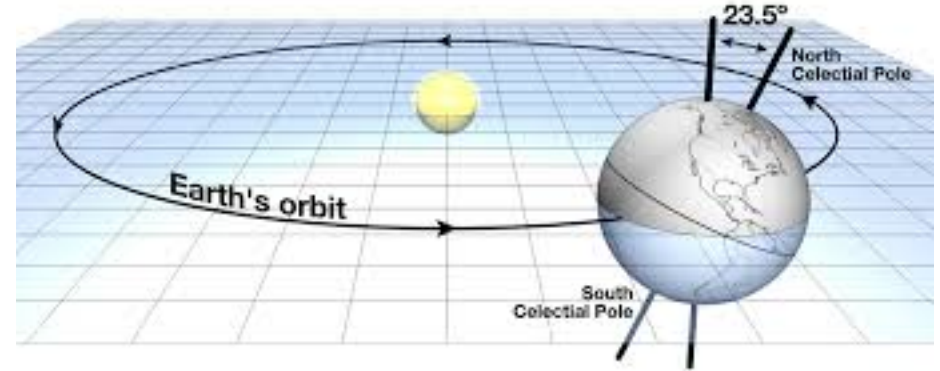
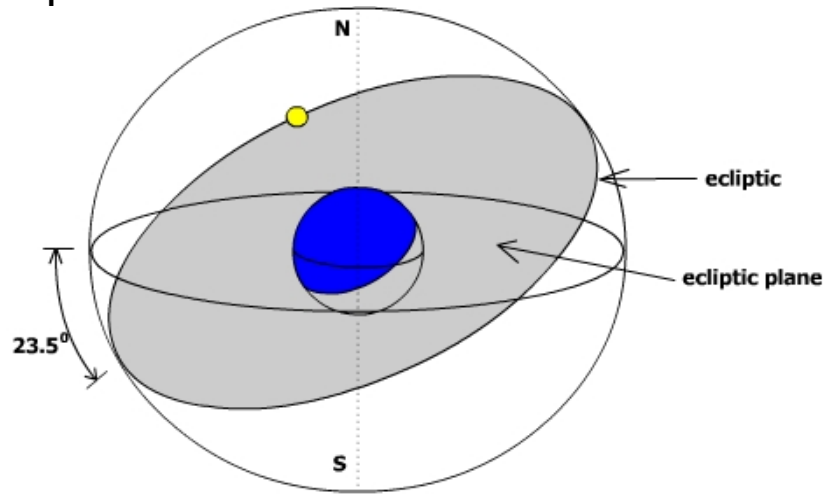
Based on the north pole, and the circle of the sun orbit at spring, March 21st (day and night lengths are equal, “equinox”). Height angle is the angle between the star and the equinox. The latitude angle is the angle with the “sky equator”, the plane perpendicular to the north star axis.

In a cycle of 26,000 years the equinox performs a precession in the “ecliptic” plane, the plane of rotation of earth and moon around the sun. This precession moved the north pole axis with respect to the stars, therefore shifts the sky coordinates. The correction is hard to perform even today.

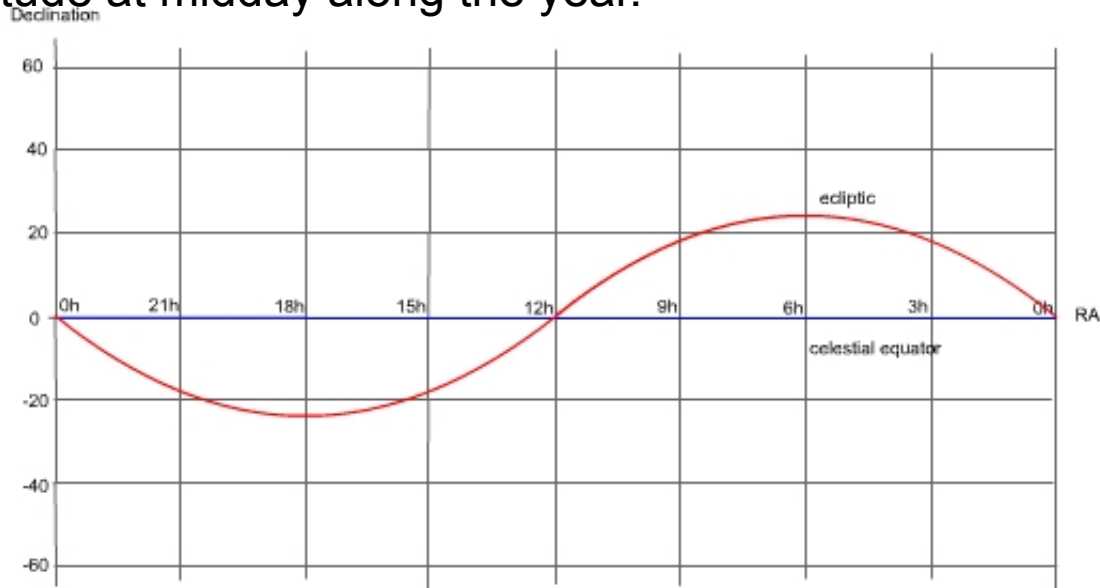
Ptolemy and other ancient world astronomers knew about the precession of the north pole, and preferred to use ecliptic coordinates, where the height angle is measured between the ecliptic meridian and the spring equinox. The ecliptic latitude is the angle between the star and the ecliptic plane, and only this angle changes by the precession, and can be corrected with years.

The Arabic astronomers used “altazimuth” angle between the horizon and the star. The “Azimuth” is the angle between the star orbit and the geographical north axis. “altitude” is the angle with the horizon. But the 24 hours sky rotation changes star angle with the horizon and requires a precise determination of the hour of measurement.

The equatorial plane, perpendicular to the axis of earth rotation, is tilted with respect to the plane of earth rotation around the sun.

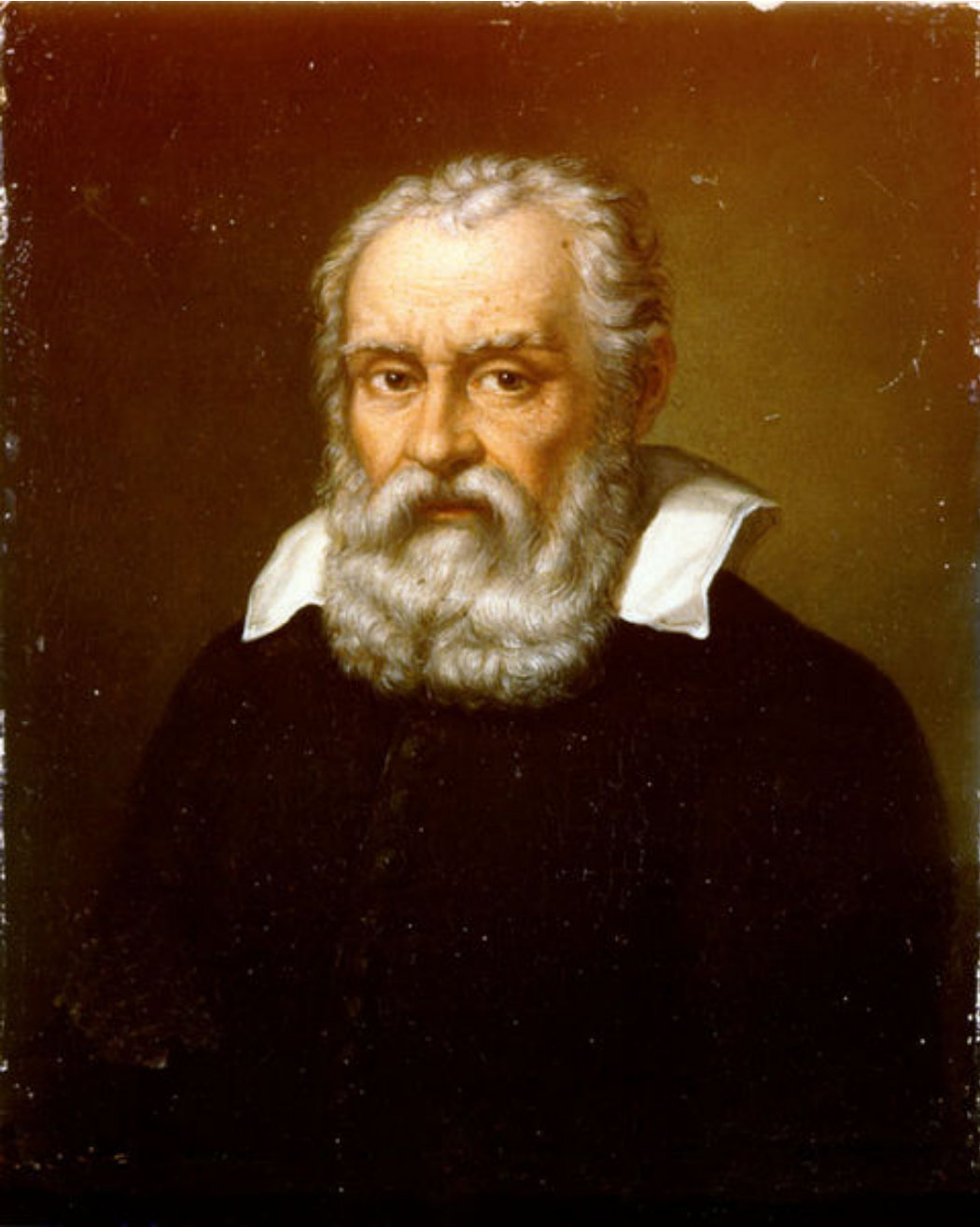


Plot of the sun altitude at midday along the year.



These complex rotational movements are due to asymmetry during the creation of the galaxies and solar systems, which source is not clear yet today.

Galileo Galilei 1564–1642



Galileo, table of dates

- 1543 - **Nicolaus Copernicus** publishes that earth rotates around the sun.
- 1563 - Galileo's parents, Vincenzo & Julia, marry. 1564 Galileo is born in Pisa.
- 1570 – **Thomas Digges** describes a telescope built 1540-1559 by his father Leonard
- 1573 – **Tycho Brahe** published “on the new star” denying Aristo’s crystal sky with aether around the moon.
- 1576 – **Giuseppe Moletti** preceded Galileo in the chair of mathematics in Padua.
Reports that bodies fall at constant speed, independent on their material (that is weight).
- 1581 – **Vincenzo Galilei** A dialog about ancient and new music – A musical theory.
Galileo starts medical school in University of Pisa.
- 1582 – Influenced by the mathematical lectures of Ostilio Ricci, Galileo decides to study sciences and math.
- 1585 – he leaves the university without a degree, and works as a teacher.
- 1586 – Invents hydrostatic balance, publishes: “the small balance”.
- 1586 – **Simon Stevin** published measurements about free fall of lead weights from 10m.
- 1588 – **Tycho Brahe** publishes study on comets with a different model of the universe.
- 1589 – Galileo appointed to the chair of mathematics in the university of Pisa
- 1590 – He partially completes a manuscript “on motion” a mechanical theory that depends on density instead of mass, which he never published.
- 1591 - **Vincenzo Galilei** died.
- 1592 - Appointed to the chair of mathematics in the university of Padua, where he served 18 years
- 1593 – Invents a thermometer (eventually depends on pressure too).

1595 – Develops improved ballistic computation, and a military compass. He later used his compass for civil land surveys, and earns money by selling his compass with instructions.

1597 – Galileo writes to Kepler, to support his planet model.

1600 – Publishes “mechanics”. His first daughter, Virginia, is born.

William Gilbert publishes “Magnets”: earth is a huge magnet, support Copernicus.

Giordano Bruno is burnt in Rome by the inquisition for supporting Copernicus.

1604 – Measures the distance of a supernova by parallax: not an atmospheric event, but further away from the moon.

1605 – Sued by his brothers in law for neglecting to pay dowry.

1606 – his son, Vincenzo, was born.

1606 – Publishes “calculating compass” a manual for use of his compass.

1608 - **Hans Lippershey** invents the two-lens telescope.

1609 – Galileo designs an improved telescope based on Lippershey’s.

1609 – **Kepler** publish “new astronomy” with his two first laws, enforcing Copernicus model and rejecting Ptolemy’s.

1609 - **Thomas Harriot** Draws a picture of the moon, 4 month before Galileo.

1610 – Galileo publish “ “Star messenger” about the moon mountains and craters, and the four Jupiter moons, which he named after the Grand duke of Tuscany, but are named today Io, Europa, Ganymede and Callisto after **Simon Mayr 1573-1624** who claimed he first saw them (the debate is due to date difference of 10 days between the Gregorian and Julian calendar)



- 1610 - **Martin Horky** publishes a manuscript against Galileo's theory.
- 1610 – Kepler** asks Galileo for a telescope or lenses, but Galileo answers he is too busy to build a new telescope, and he does not possess a spare.
- 1610 – Galileo accepts a life position as a mathematician in University of Padua, and a philosopher of the court of Cosimo II de Medici, the Grand Duke of Tuscany.
- 1611 – Galileo discovers the phases of Venus, meets the pope, and becomes a member of the **Licean academy** (established at Rome, 1603, by Federico Cesi 1585-1630 as a private research institute and science publication house). Galileo's manuscripts were funded by this academy.
- 1611 - – **David Fabricius** publishes “sun spots and their rotation with the sun”.
Preceded publications by **Christoph Scheiner** and Galileo.
- 1612 – Galileo proposes that the Jupiter moons will be used as a universal clock to determine latitude position on land and at sea.
- 1613 – **Francesco Sizzi** discovers yearly changes in the motion of sun spots.
Galileo sends a letter “on sunspots” to the Grand Duchess of Tuscany, Christina, that was published only at 1636.
- 1616 – Warned to deny Copernicus models. The Church adds his books to the list of prohibited books that need amendments.
- 1616 – In a private letter he discusses tides.
- 1617 – Moves to live near a monastery in Florence, reports viewing the star pair Mizr, in Ursa Major constellation.
- 1619 – Kepler publishes “Harmonic universe” that include his third law.
- 1619 – Galileo discusses comets.

1623 – **Maffeo Barberini** becomes Pope Urban VIII.

Galileo published “The Assayer”.

1624 – The pope praises Galileo during his visit. Galileo interprets this as a permit to publish a comparison of Copernicus to Ptolemy.

1625 – Galileo uses a compound microscope and publishes drawings of insects.

1630 – The Censor of the church approves publishing the dialogue about the two models of the universe, which was published 1632.

1632 – Galileo finds that force creates acceleration, not velocity.

1633 – Galileo was accused by the inquisition, and his sentence was eased to home arrest at Arcetri for violation of a warning from 1616. The dialogue is added to the forbidden list. For sure Galileo escaped the prosecution of Bruno 33 years before his inquisition trial, and seemed to reconcile with them.

Nevertheless he was told to say after his sentence:

“Eppur si muove”, and yet it moves ...

1638 – The “Dialogue on two scientific theories” is nevertheless published.

He defined “statics” and “dynamics”

1642 – Galileo dies in Arcetri.

1668 – Isaac Newton builds the mirror telescope.

1687 – Newton publishes “Principia” that derives Kepler’s laws from laws of motion and Gravitational forces.

one of the main ideas of the scientific revolution of the renaissance is that experimental science must be the basis for laws of nature. **Galileo Galilei** was the proponent and symbol of this revolution. He believed in mathematical description of nature, and in quantitative measurements as the only way to establish the validity of such laws. He supported Copernicus model that earth rotate around the sun, and not vice versa, which antagonized the church. Yet, for unclear reasons Galileo did not refer to the measurements of **Brahe** and **Kepler**. His clear contribution to dispute **Ptolemy's** model of the universe was his **telescope** development that allowed him to observe new stars at the years 1540 and 1604, and determine by parallax that their distance is much beyond the moon. This phenomena, that we call today supernova explosions, was a conclusive evidence against Aristo's crystal sky of stable stars. This use of the telescope to study the morphology of the sky was an important component of Galileo's work. He improved the telescope eyepiece, and reported several new discoveries:

- He described sunspots, moon craters and mountains, not ideal spheres.
- He observed the four Jupiter moons, a decisive proof that not all celestial bodies rotate around earth. He also observed, after compensating fro earth motion, that the moons velocity became smaller with distance.
- He detected the “ears” of Saturn (what he reported as “triple form”).

Only 100 years later **Christian Huygens 1659** proposed that they are a ring of moons rotating in the equatorial plane of Saturn.

- He determines that the moon and the sun are oblate ellipsoids, as the
- He observes the phases of Venus, interprets them and the phases of the moon as a proof of rotation around the sun, rejecting Ptolemy's epicycles.
- He claimed that the milky way is built of stars, and is not a dense region of the sky. He believed that we see only a small fraction of the stars in the universe.



Additional scientific contributions:

1581 – He describes pendulum period dependence on length, and independence on “narg” (amplitude) and mass. He was paid 30,000 Gulden from the Dutch merchant association for the pendulum, yet it was implemented in clocks only 50 years later by **John Harrison 1693-1776**. Galileo understood it was impossible to use pendulum for marine clocks due to ship jerks, and proposed to use the Jupiter moons as a clock. The implementation was too complex, and was never tried. Galileo built a compass with scales for angle and ruler distance on the arms. It was used as a drawing tool, survey measurement device, and computer for calculating interest, square and cube roots, and multiplier to calculate area and volume. He sold this compass and the manual that came with it for good profits. It was soon replaced by logarithmic tables.



1612 Galileo studies the laws on moments and levers, pulleys, capstans and cranks, and demonstrates that small force with fast speed can be transformed by such devices to large force at slow speed, recovering Archimedes laws.

1589-1607 – Galileo observes that large and small hail balls fall at the same time. He argued that if large balls fall faster, they must be formed higher in the clouds or the smaller balls started to fall earlier, both unreasonable assumptions. He was therefore driven to perform experiments on balls sliding down a slope, to show that the acceleration was independent on the ball mass. The public demonstration was the famous experiment of free fall of large and small balls from the top of the Campanile del Duomo di Pisa, reaching ground at the same time, and rejecting Aristo's theory. Some claim this experiment had never happened. Similar experiments were published earlier by **Benedetti Giambattista in 1553** and by the Dutch engineer **Simon Stevin in 1586**. Galileo found experimentally that the orbit of a projectile is parabola.

A FOOTNOTE:

The concept of friction, by air or when a body is dragged on a surface, was misinterpreted by the classicists, and was never rectified until 200 years later.

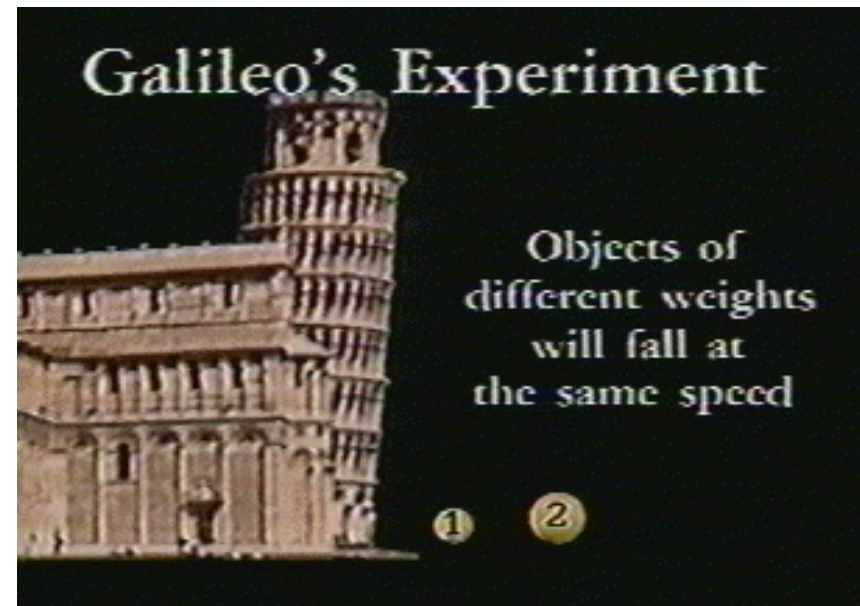
George Gabriel Stokes 1819–1903

wrote the formula for friction of a falling ball in water, after realizing that they reach a terminal velocity in free fall, therefore the frictional force on them by the water equals their weight:

$$\text{Weight} = mg \approx R^3 = \text{Friction} \approx R$$

Thus larger balls reach higher terminal velocity,

Since weight $\sim R^3$ and friction $\sim R$



Galileo's dynamics:

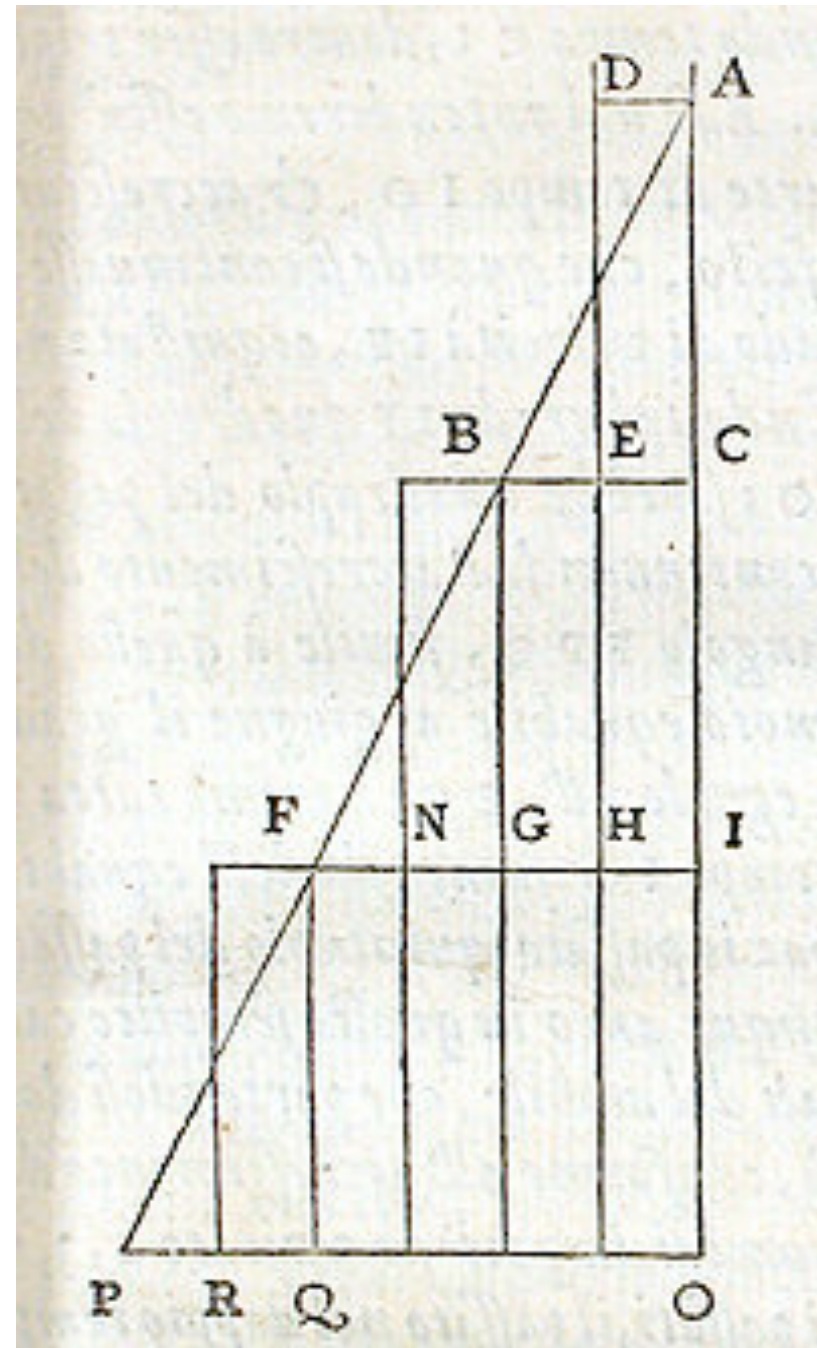
In his youth Galileo tried to express mathematically the mechanics of impetus, a common theory emerging from Aristo. However, the impetus concept was incompatible with Aristotle's physics, and was confusing and impossible to quantify by measurements. He therefore abandoned this theory, and replaced it by a new mechanics. While Aristotle claimed that bodies attempt to reach their natural position, Galileo proposed the principle of inertia – a body continues to move as long as no forces act on it. Thus for Aristotle motion requires a reason, that is driving force. For Galileo motion continues by itself, and force changes the motion. What requires a reason is the change of motion, not its continuation. A body thrown up from a moving boat continues to move horizontally at a speed equals that of the boat, and only its vertical motion changes. Therefore it falls at the same position he was thrown up from, just as it would be for a standing boat. Thus we cannot determine if the boat is moving or at arrest. This also applies to throwing bodies on the moving earth, an experiment that cannot display if earth is in motion. The equivalent behavior of bodies in constant speed or zero speed is a revolutionary concept rejecting Aristotle attributing a desire to moving bodies, that does not exist when they are at rest...

The important consequence of this theory is that we can accept **Copernicus** model that earth is moving at large speed, although it does not affect the experiments we perform.

The “equivalence” between systems moving in constant speed with respect to each other, meaning that mechanics cannot offer an experiment to determine movement at constant speed, is the principle that led **Einstein** to his Special Relativity.



Galileo graphical description of accumulated
Distance of travel at constant speed: $x=vt$



1609 Galileo developed “occholino” a small eye: a magnifying optical system built from two lenses: concave and convex. The occholino was displayed at 1624 to **Federico Cesi**, the founder of Accademia dei Lincei. The names Microscope and Telescope are attributed to **Giovanni Faber** a year later.

The web site of the Galileo Galilei virtual museum

<http://catalogue.museogalileo.it/index/IndexObjectsInAlphabeticalOrder.html>



1589 – Galileo develops hydrostatic balances, to measure density of b=solids (according to Archimedes law of floatation). He applied the study to build a thermometer, (see picture). He found that water pumps in mines did not work from depth above 10 meters. He encouraged his student **Evangelista Torricelli 1608-1647** to research this property.

1592 – Torricelli builds a thermometer (“thermoscope”) from an air bulb with water pipe.

1643 – Torricelli invents the mercury barometer, of a tube of mercury column and vacuum above. He demonstrated atmospheric pressure and vacuum. Mercury, has low freezingn high boiling temperatures and low vapor pressure unlike water, has negative affinity to the glass walls, reducing capillarity effect, has higher density, therefore shorter mercury column height below the vacuum, and does not require color to be seen in thin capillary.

1612 – **Santorio Sanctorius** uses thermometers for medical diagnostics.
Studies tides (maybe without reading Galileo’s work).

Torricelli's
barometer



Hydrostatic thermometer

"Philosophy (natural sciences) is written in a great book which ever lies before our eyes – I mean the universe – but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written. This book is written in the mathematical language ... without which one wanders in vain through a dark labyrinth."

Galileo Galilei

Indeed, today we have no other ways to describe the laws of nature. Yet one cannot answer the question if a mathematical models are strictly equivalent to nature, or just approximates its behavior with precision that depends on the accuracy of the measurements.

The Beauty of Algebra
Philosophy is written in that great book which ever lies before our eyes — I mean the universe — but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written. This book is written in the mathematical language... without which one wanders in vain through a dark labyrinth."

-Galileo Galilei

30% off \$20

$$\text{Discount} = \boxed{30\%} \times \$20 = 0.30 \cdot \$20$$

Let x be ^{nondiscount} price of product \$15

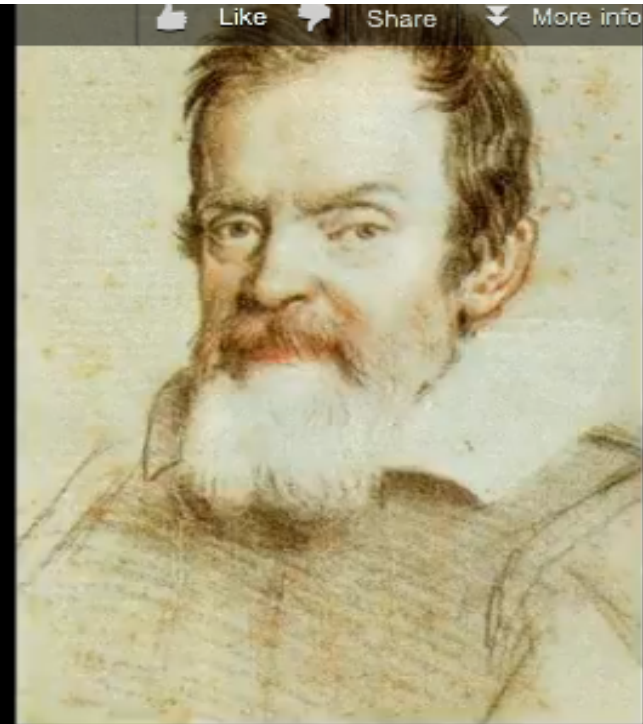
$$\text{Discount} = \boxed{30\%} \cdot x = 0.30 \cdot x$$

Let p = percentage off

let y = discount

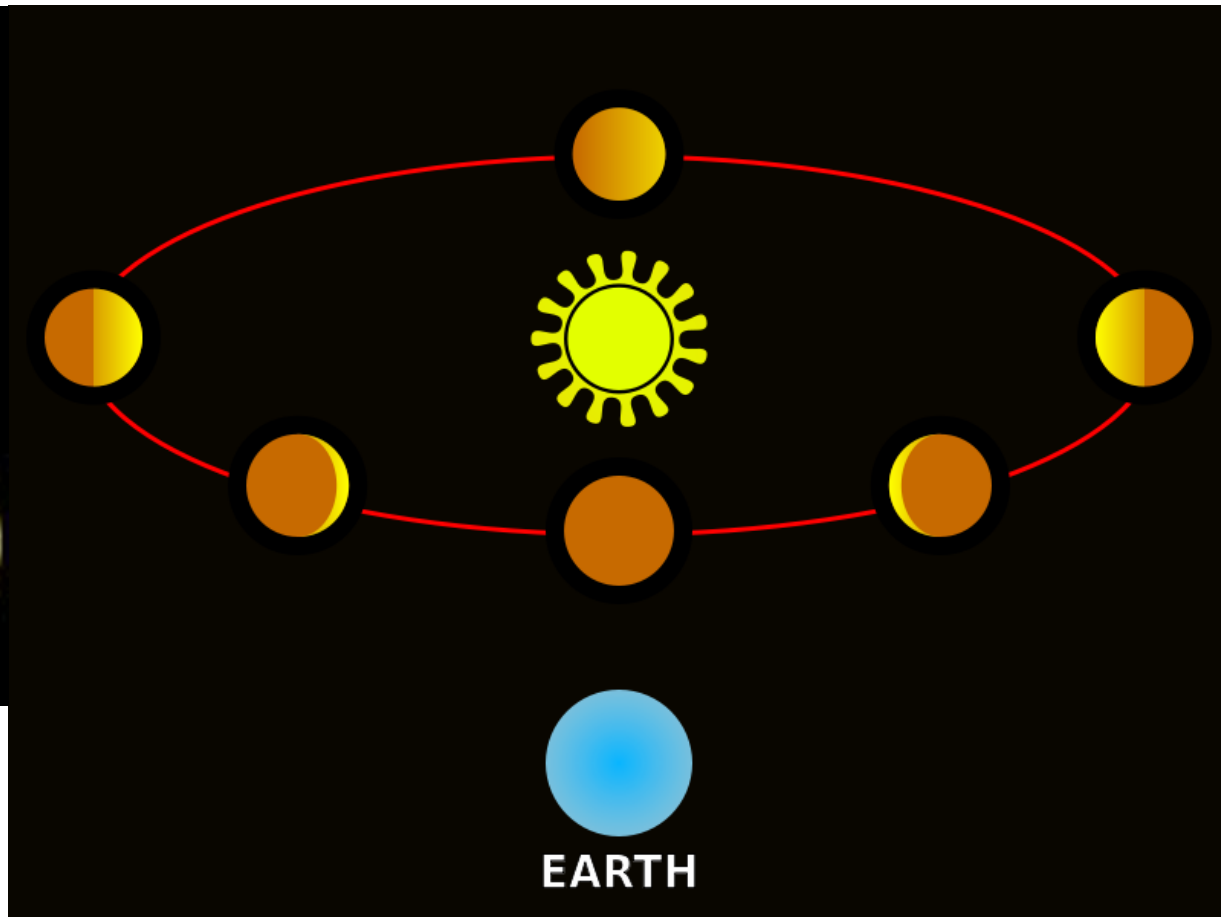
$$\text{Discount} = p \cdot x$$

$$y = p \cdot x$$

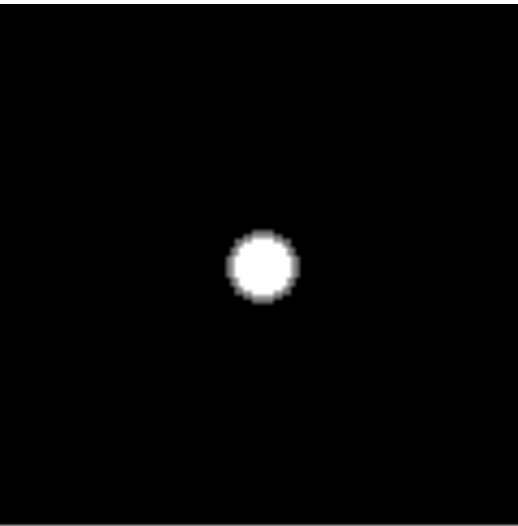


The phases of Venus

According to Ptolemy, the center of Venus orbit is on a line between earth and the sun, and being under the sun should be seen always as a thin sickle. By the Copernicus model, Venus should display all phases, from fully illuminated to dark, like the moon. Only the use of telescopes provided a detailed view to resolve Venus phases.



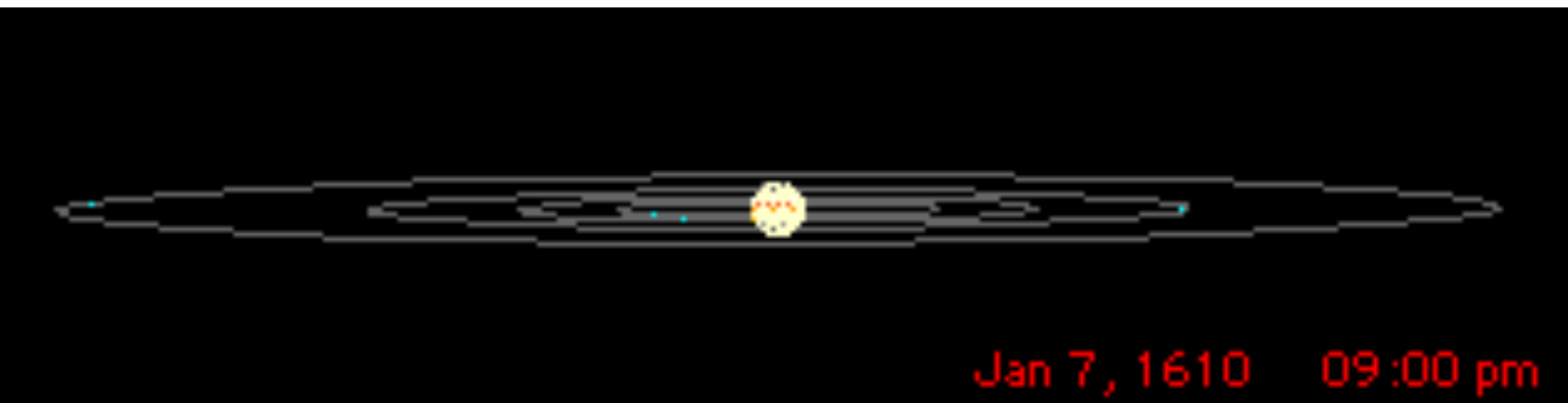
Venus (red star, top right) orbit , and its phases (top left) as Galileo saw in his telescope (top movies),
and the four moons of Jupiter (bottom movie)



Telescope View



Unaided View



OPTICS

Optics – telescopes and microscopes

Nicholas of Cusa 1451 development of lens polishing and better tools to measure angles. Unknown scientists at the 16th century invented the telescope. Later many scientists and developers attempted to gain priority for the invention.

Hans & Zacharias Janssen 1590 was a Dutch lens manufacturer for eyeglasses. He and his son built the compound microscope with two convex lenses. The objective lens with a short focal length generates a magnified and inverted image of the object, and the ocular, or eyepiece, that accepts the objective image and creates an imaginary image for the eye.

Probably at the same time, and independent on **Lippershey**, they assembled telescopes.

Thomas Harriot 1560-1621 A few months later, in Oxford, he built a field monocular with magnification X6 to inspect the moon surface.

Leonard Digges and his son, **Thomas** built 30 years before Lippershey, a reflecting telescope.

Christopher Scheiner 1573-1650 was the first to study sunspots using camera obscura.

1608 Hans Lippershey the Dutch artisan, builds the first telescope. He wrote a patent for a magnifying tube, a device assembled by his sons playing with extra eyeglasses he discarded in his production.

1609 - Galileo Galilei saw in Venice copies of Lippershey's telescope, and rushed to build such devices himself, mainly for their commercial value as viewing tubes for the Venetian sailors. He tried convex with concave lens combinations, and formed straight up image at X20 magnification, but with strong spherical and chromatic aberrations, and a small field of view (smaller than the viewing angle of the moon). Yet, Galileo was the first astronomer to watch the stars with telescopes. He draws his observed moon surface features on a screen using **camera lucida** : seeing both stars and drawing screen at the same time. He developed a micrometer to measure angles between stars, by looking at the star with one eye, and reading the micrometer with the other. He also used the Camera Lucida for microscope drawings of insects he inspected.

1611 – Galileo presents his “spy glass”. The Greek mathematician **John Demisiani** called it “telescope”.

The telescope led immediately to a series of new observations: Mapping the craters on the moon, discovering the moons of Jupiter, the ears of Saturn, and movement of sunspots.

Johannes Kepler 1571 – 1630 was short sighted, and did not see well in Galileo's telescope. He therefore built “Kepler telescope” using two concave lenses. The image is inverted, therefore it is not useful as a field monocular, but the field of view is much larger than for Galileo's telescope. The tube length is shorter, but becomes longer when applied for field glasses with inversion optics relay (erector system **Paolo Belletti** 1640s).

1611 —Kepler Johanes discovered internal reflection, writes the laws of refraction for small angles, and the optical laws for thin lenses (geometrical optics)

$$N f = n' f'$$

1619 - Cornelius Drebbel 1572 – 1633 presents in London a microscope with two convex lenses

1621 Willebrord van Roijen Snell publishes the law of light refraction that carries his name, that implies the speed of light in transparent materials is proportional to the speed of light in it:

$$N \sin(f) = n' \sin(f')$$

Additional telescope developers:

Francesco Fontana, Evangelista Torricelli, Eustachio Divini, Giuseppe Campani

Jeremiah Horrocks 1618-41, and William Crabtree 1610-44

1639 – the two astronomers observed the passage of Venus in front of the sun.

James Gregory 1638-1675

1663 – designs a telescope with a reflective mirror that replaces the objective, that solved the chromatic aberration problem. 1668 Newton introduced him to the Royal Society.

Laurent Cassegrain 1629 –1693

1672 – Cassegrain presents megaphone, and two-mirrors telescope.

Giovanni Domenico Cassini 1625-1712

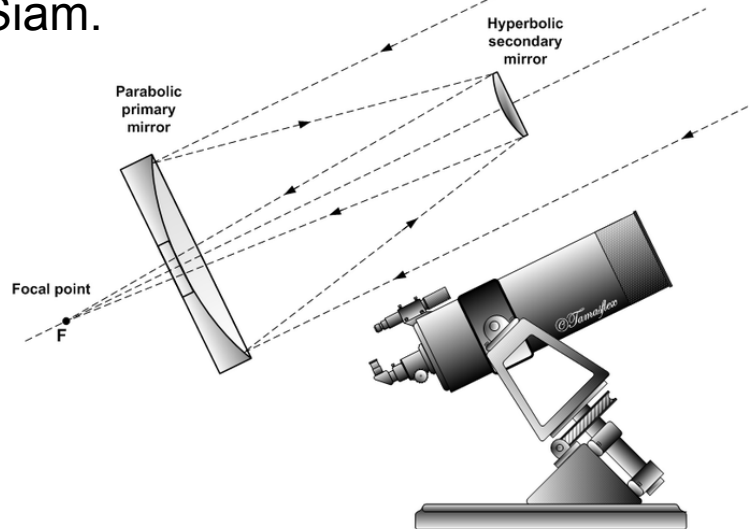
1671 - Cassini is invited to head the newly built telescope in Paris.

1675 – The Royal telescope at Greenwich.

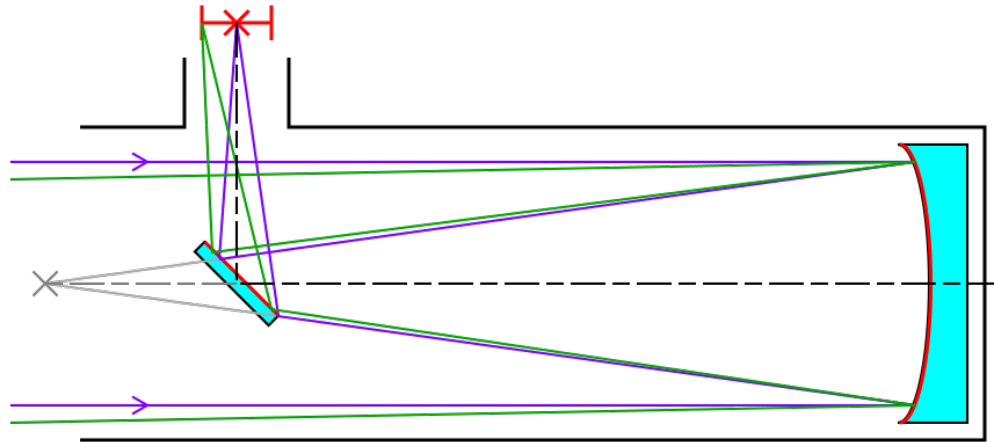
Olé (Olaus) Christensen Römer 1644-1710

1672 - joined Cassini in Paris, where he was introduced to the possibility to measure the speed of light from the cycle times of Io's, Jupiter's moon.

End of the 17th century – Jesuits brought and built telescopes in Japan, China and Siam.



Right: Schmidt-Cassegrain telescope below: :Newtonian telescope



1660 Public telescope competitions

Development of optics at the 16th century, and the application of the prospering eyeglasses industry to telescopes and microscopes charmed not only scientists and science amateurs, but also Emperors, Nobility and the Rich. The telescope was most attractive since its field capabilities could be easily demonstrated.

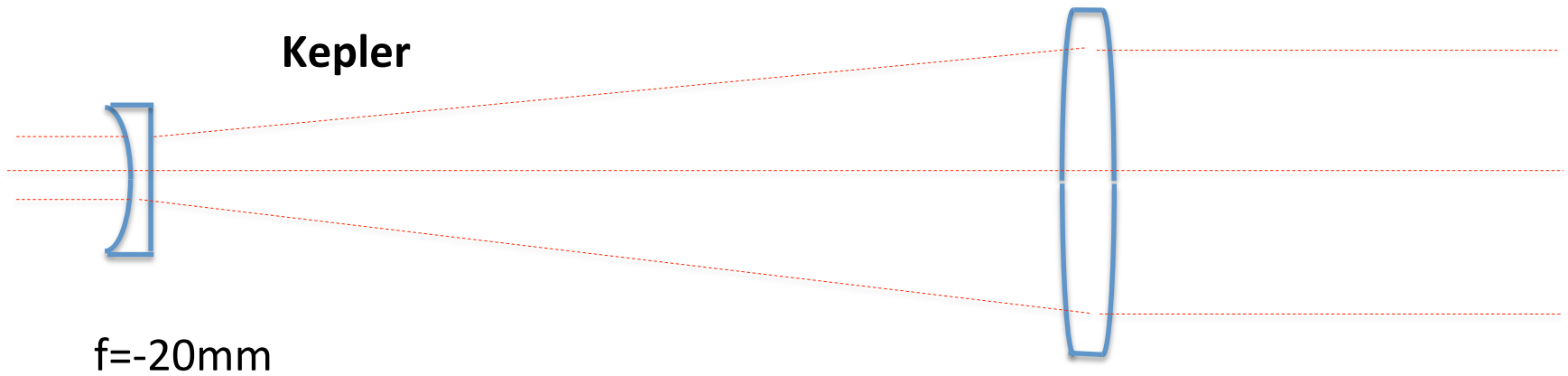
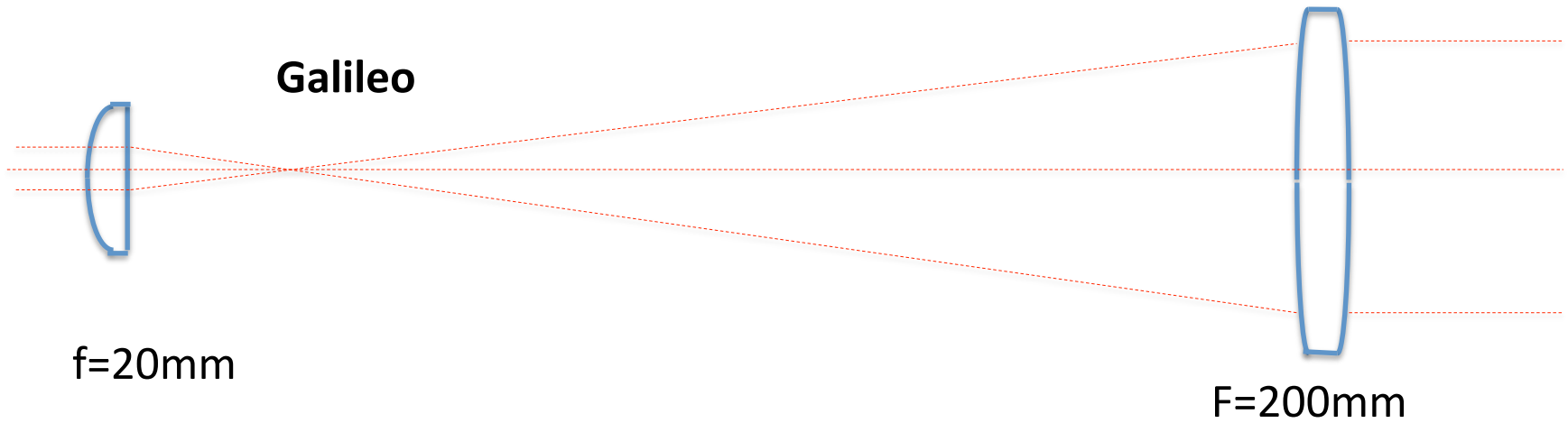
At the 17th century at Rome, a competition was announced. The competitors provided telescopes, that were tested by reading from increasing distances mixed-order words from texts by Petrarca, Dante and Aristo. They also used resolving tests pattern prepared by del Cimento academy in Florence, similar to the ones opticians use today fitting eyeglasses.

Telescopes built by the famous instrument builder **Eustachio Divini** were tested against the telescopes of the young clockmaker **Giuseppe Campani**. Although no winner was decided, Campani gained fame and publicity.

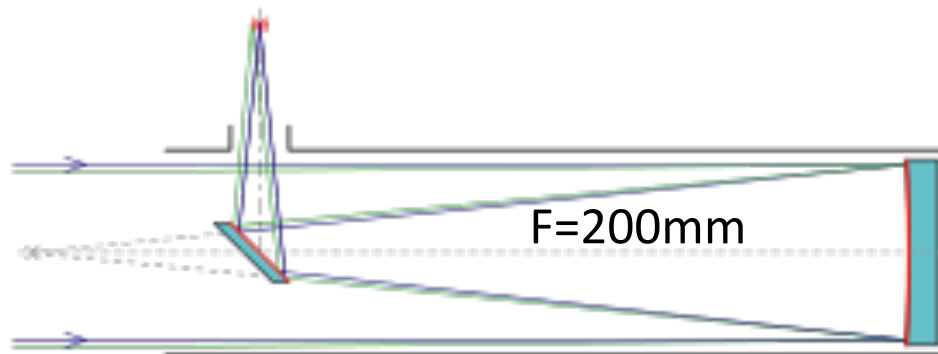
The two telescopes were bought for the Medici collection, and are on display today at the Science Museum in Florence.

The following slides include a few optical diagrams:

Kepler, Galileo and Newton telescopes, all with x10 magnifications:

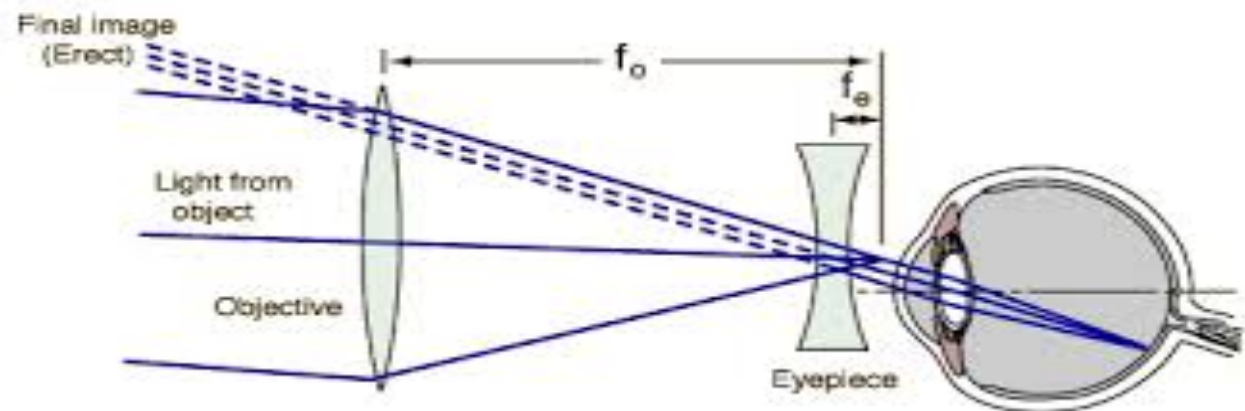
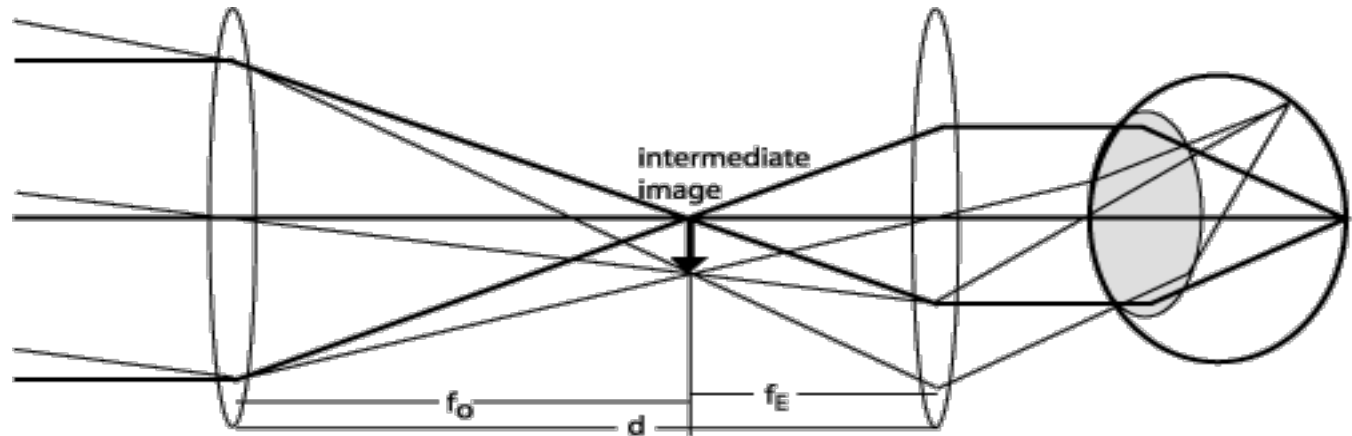


Newton



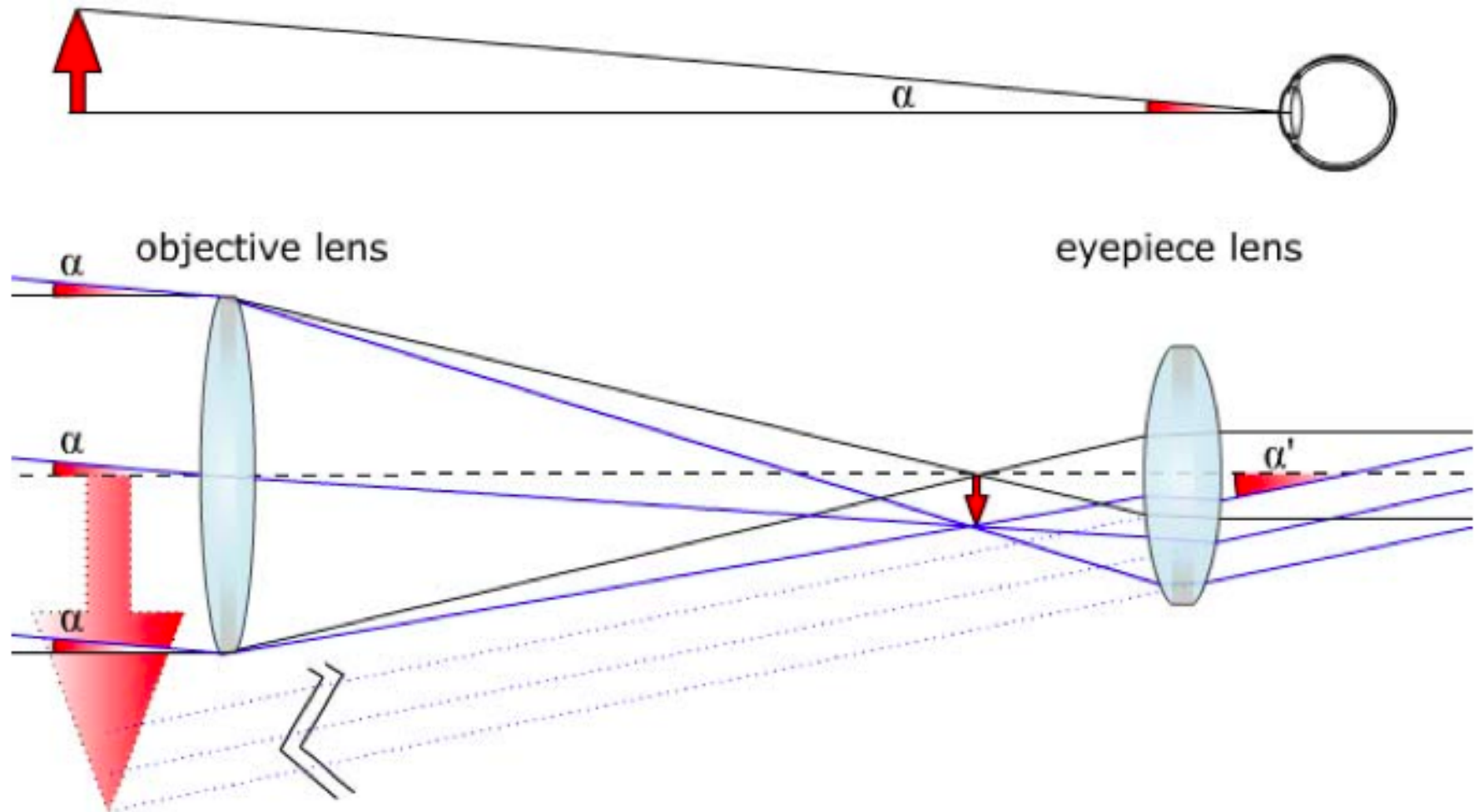
The image in Galilei's telescope is right up.

Kepler's telescope has negative-lens eyepiece, inverted image, larger field of view (see off-axis rays), and better brightness compared to Galileo's. It replaced Galileo's within a short time.



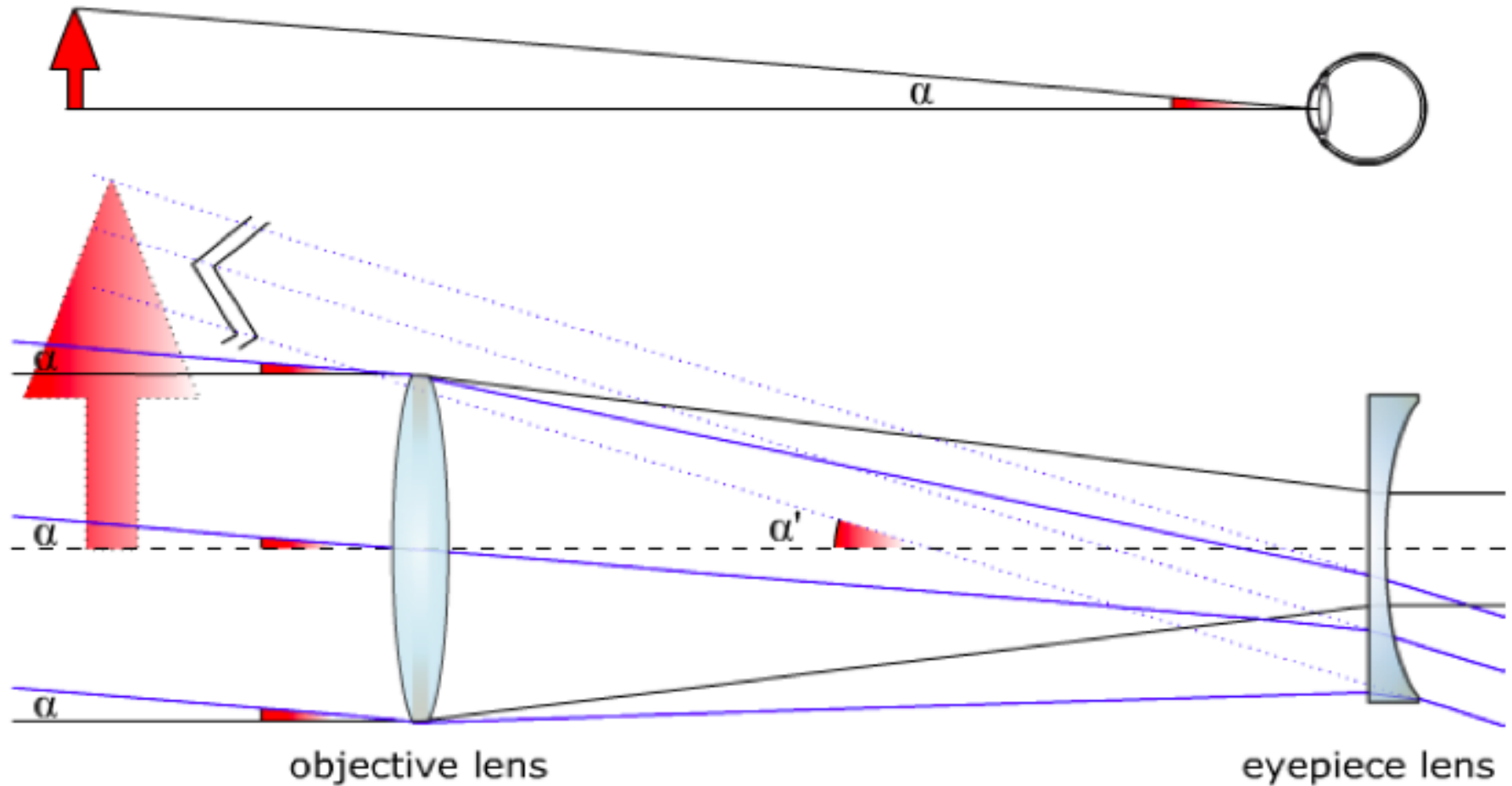
Galileo's telescope with biconvex eyepiece.

Drawings show angular magnification: bare eyes, α , versus magnified image, α' .



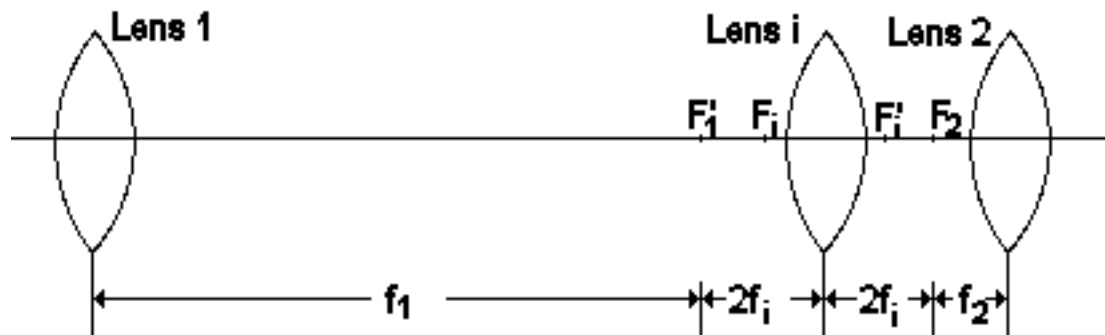
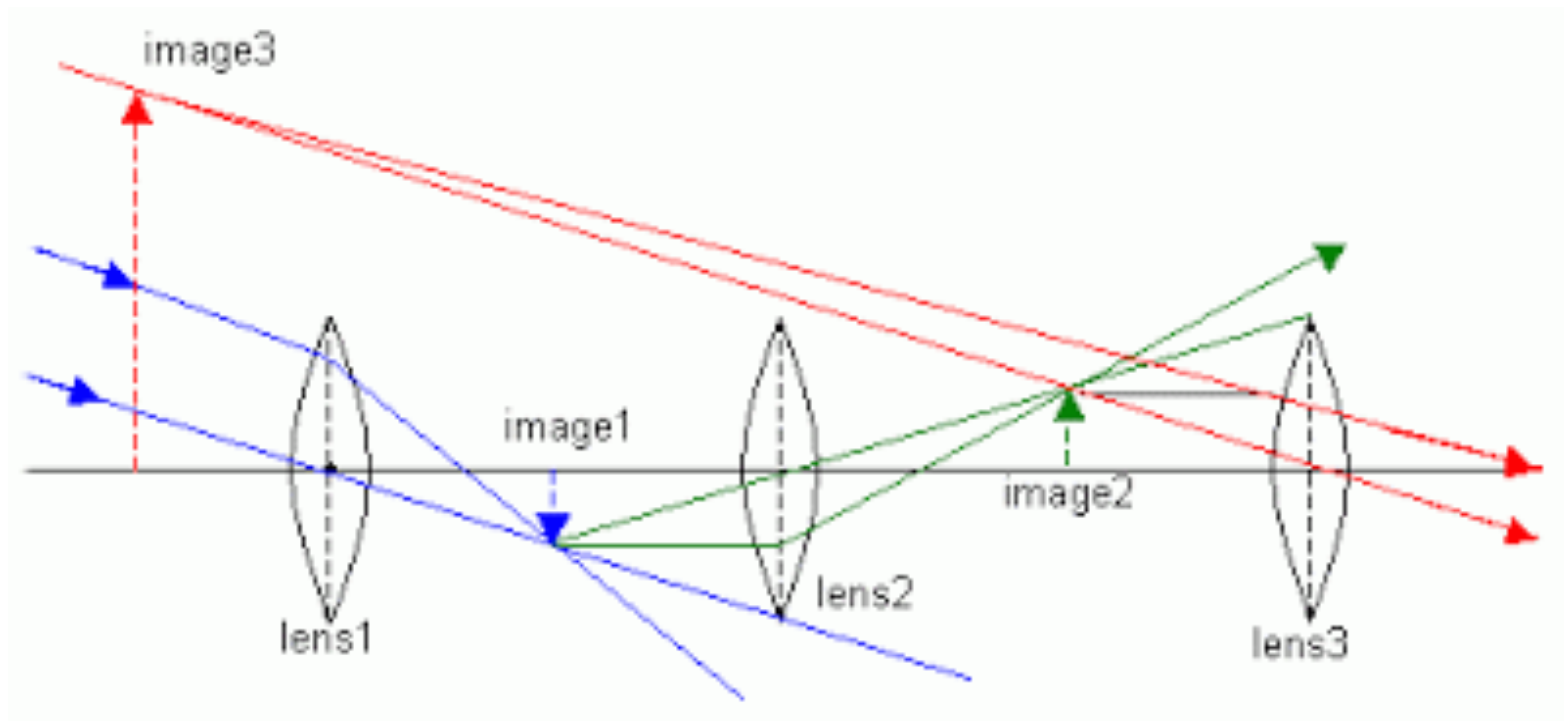
Kepler's telescope with biconcave eyepiece.

Drawings show angular magnification: bare eyes, α , versus magnified image, α' .



erector system **Paolo Belletti** 1640s

Combines two lenses to invert the image in Kepler's telescope for use as field glasses.



The difference between telescopes and microscopes

MICROSCOPE

Objective: small diameter
& short focus

$$\text{Mag} \sim 1/f_{\text{ob}} * 1/f_{\text{oc}}$$

Object close to objective

Large Aperture (collection angle)

TELESCOPE

Objective: large diameter
& long focus

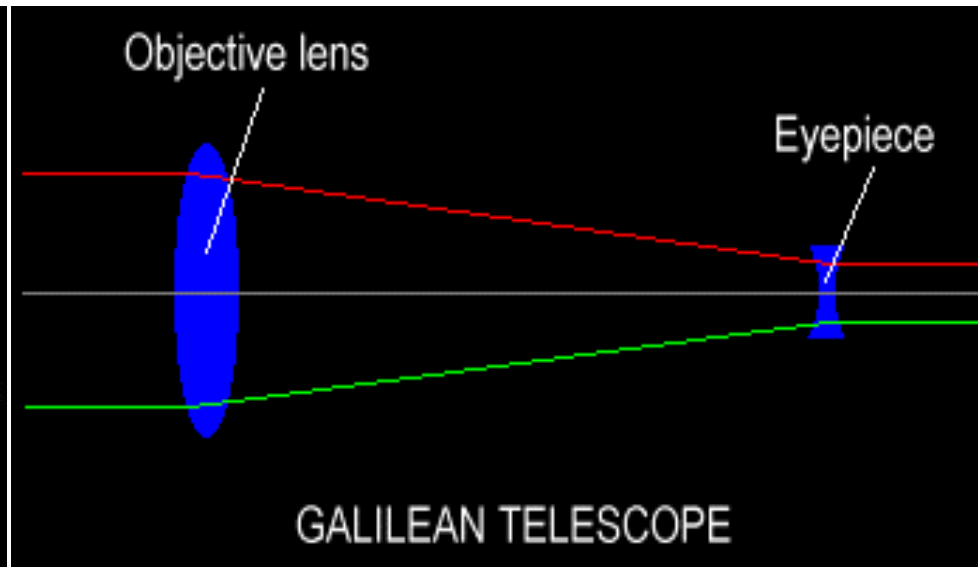
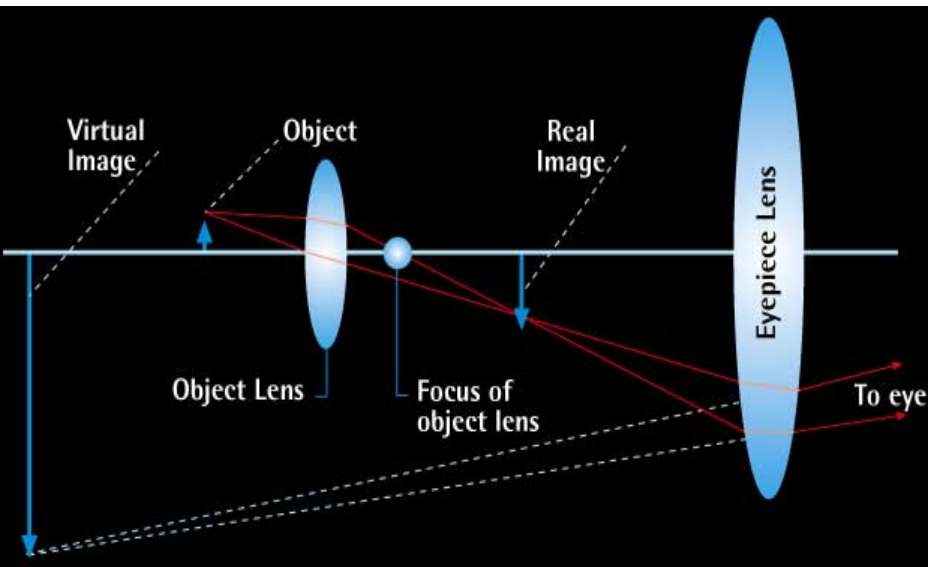
$$\text{Mag} = f_{\text{ob}}/f_{\text{oc}}$$

Object at infinity

Small Aperture

When you look through a binocular from the objective side, the field seen is smaller, but looking into a microscope objective noting can be seen. Why?

Hint: the eye can see only images positioned between 25cm and infinity.



Birth of modern optics

The efforts to improve telescopes induced research of the principles of their action. The basis of modern optics is Kepler's book: "Astronomiae Pars Optica" the optical part of astronomy.

The back and forth flips of the physics of light

Christiaan Huygens 1629-1695 Believed that light is a wave, propagating in a finite speed (as proved by Ole Rømer). He used the concept of wavefront to explain refraction, light angle change when transferred between two media. He also explained the double refraction in Calcite crystals.

Isaac Newton 1642-1726 propagated a particle theory of light, which explains well reflection of light in mirrors, but not refraction. Newton's theory was commonly accepted, until **Thomas Young 1773-1829** made his two slit interference experiments, proving that light is a wave.

~150 years later **Albert Einstein 1879-1955** explained the photoelectric effect showing that light, as well as all matter, can only be described by dual properties of both particles and waves.

(See details in the "Age of Reason" presentation).

TECHNOLOGY

AVIATION

1496 Giambattista Danti della Porta the Italian mathematician floats from a tower in a glider. Published a theory and drawings for kite building.

1500 Hieronymus Bosch paints in the Triptych of the Temptation of St. Anthony flying vessels over a burning city.



1638 John Wilkins, Bishop of Chester proposes what will become a missile in his book "Discovery of the world of the moon".

1638 Evliya Çelebi (Derviş Mehmed Zillî) 1611-1682 the Ottoman explorer, tells in his travel stories about his floatation with artificial wings of **Hezarfen Ahmet** from Galanta tower in Istanbul over the Bosphorus, landing in Dogancılar square in Üsküdar.

All these records may or may not be exaggerated or even true, however, they emphasize the persisting desire of mankind to fly.

GEOGRAPHY

1475 Jacopo d'Angelo publishes a Latin translation of maps from Ptolemy's "Geographia". Information from the encyclopedia of Pliny the Elder contradict these maps. World maps are incomplete both east and west – inaccurate description of Asia came from few land convoy reports. Africa is sparsely known from sailing along its western coats. And due to lack of navigation means the oceans are unexplored.

1492 Cristóbal Colón makes a short estimate of the perimeter of earth, and his boat fails to reach China.

Gerardus Mercator 1518-1594 Dutch Cartographer and Geographer, that draw maps of earth using a projection of a sphere to a plane that carries his name. The cover of his maps volume carries the picture of Titan Atlas carrying a sphere. Although in the mythology Atlas was condemned to hold the celestial heavens for eternity, Mercator's use of the Atlas icon stamped the meaning of the sphere as earth.

Bartholomeu Diaz 1451-1500 the Portuguese sails around the cape of good hope, the south tip of Africa.



CLOCKS AND TIME

At the end of the 15th century clockmakers in Nuremberg replaced pendulums with springs. The inventive development provided smaller carry-on watches, as well as marine chronometers, essential for determining latitudes and longitudes during mid-ocean sails.

1335 A mechanical clock in Milan

1581 Galileo studies pendulum cycle times, its dependence on length but independence on mass. He reports to the Dutch authorities. This information must have reached **Huygens**.

1659 Jacopo di Michelangelo Viviani 1622-1703 draws a clock by the instructions given by Galileo to his son **Vincenzo**. The Florentine clockmaker **Eustachio Porcellotti** use these drawing and built “time measurer“

1656 Christian Huygens builds a pendulum clock. All Parisian clocks are synchronized.

MILITARY TECHNOLOGY

“Greek fire” was used by the Byzantines as projectiles. Consisted of a mixture of Sulfur, Kerosene, Bitumen (asphalt), and Resins, and was thrown by Catapults.

1527 Matteo Bresan the supervisor of the Venetian arsenal, is responsible for the construction of galleons, multi-mast sail boats, with cannon windows.

Ports have floating docks built for large boats service.

1546 Pedro Nunes 1502-1578 issue instructions manual for sailing in a full circle (including against the wind).

Invented the Nonius (used by Tycho Brahe), in order to increase accuracy of nautical star measurements, and was later improved by **Vernier**.

Astrolabes for measuring latitude at sea.

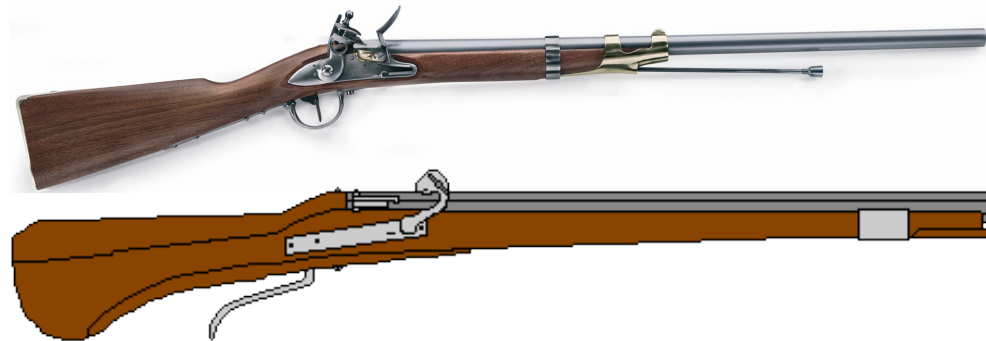


End of the 15th century:

Use of gunpowder on land and on boats.

Musket and Arquebus “hand cannons”.

Improving furnaces for iron casting.



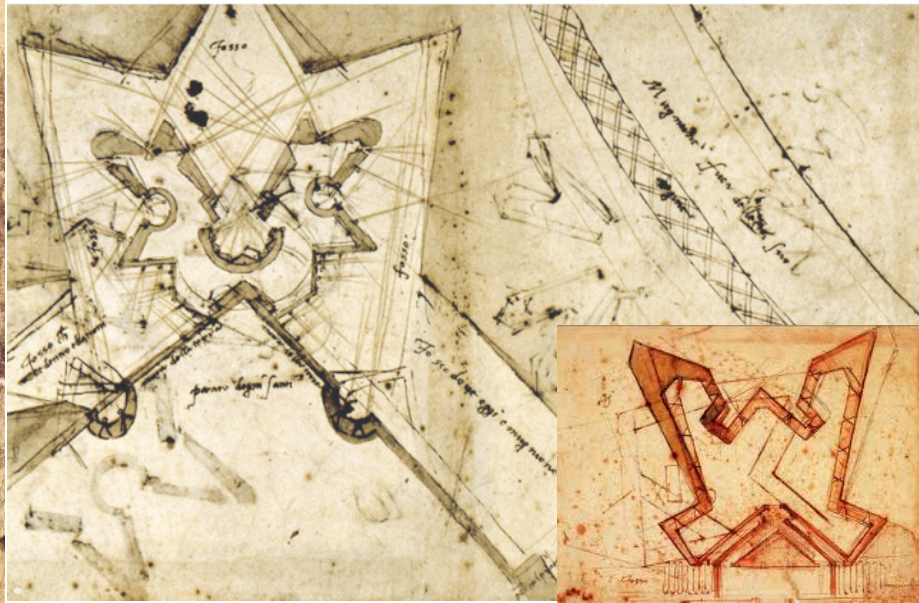
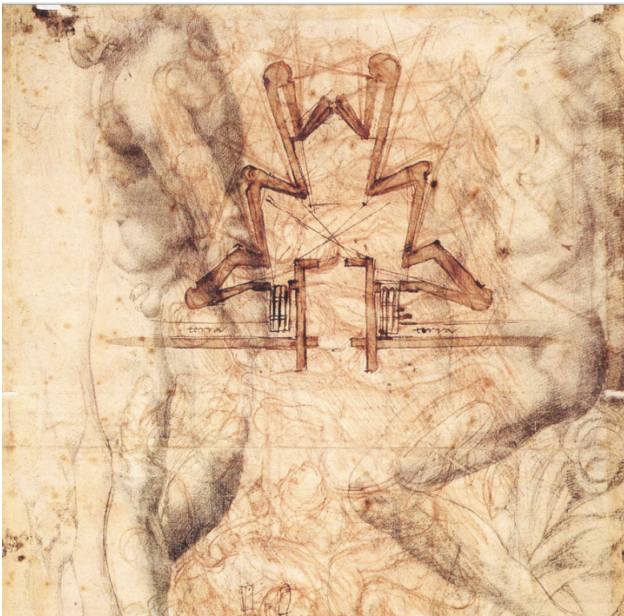
Winches, Shakes, Wood mills operated by water streams.

Parachutes, gliders, kites.

City fortifications designed by military engineers.

Michelangelo di Lodovico Buonarroti Simoni 1475-1565 was such an engineer:

1527 Karl V, Emperor of the Holy Roman Empire (Germany and Spain) imprisons Pope Clemens VII of the Medici family at a fortress in Rome. The Florentines exiled the hated Medici family that controlled Florence. The Pope signs a surrender agreement with a promise to retrieve Medicis to power in Florence. A wave of patriotism against Medici swept the youth of Florence, and Michelangelo, aged 52 then, joined and accepted to design the fortifications. He was by then an admired artist ("the godly") who was supported by Lorenzo de Medici, but sculptured and positioned "David" as a provocation to "Goliath" the Medici, and left for Rome, where he sculptured "Moses" and painted the ceiling of theistine chapel at the order by Pope Borgia. He traveled to Pisa and Ferrara (the Duke Alfonso de'Ara was famed fortifications expert). 20 drawings notebook detailing walls, towers and artillery positions are on display today at the Casa Buonarroti in Florence. The wall stood up for a year, but the siege caused hunger and diseases, and Florence surrendered. The Pope later forgave Michelangelo, and continued to employ him. Professionally, there were accusations against Michelangelo's design of the Star-like fortifications had sharp angles, and too narrow to host cannons, and too many firing slots that weakened the walls.



BIOLOGY and MEDICINE

The end of Galenism: 15th century Italy

The body is a machine

The art of the renaissance is linked to an anatomical breakthrough:

Leonardo da Vinci studied human anatomy for his paintings, but was stopped by the church. Maybe due to his fear of their authority, he did not publish his experiments of drilling holes in heart to study blood cycle.

Michelangelo also performed body autopsies secretly.

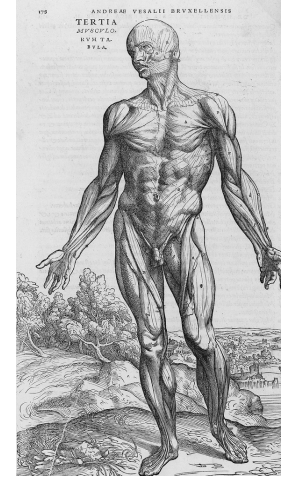
Despite the religious restrictions, physicians accumulated information about internal organs, their normal functions, as well as involvement in diseases: Kidneys, Urine bladder, Uterus and Ovaries, Muscles and tendon links to bones, Blood cycle and relation to lungs, Nerves, and even beginning of connection between diseases and proliferation of bacteria in the body.

Autopsies improved the diagnostics of diseases, although did not contribute to treatments. Very few effective medications were known, oil creams were applied to all wounds, and many folkloristic treatments based on poisons and metallic compounds were used.

Andreas Vesalius (1514-1564)

Belgian physician. Composed “De humani corporis fabrica” which is illustrated by the best artists of his time, who took part in his lectures, and created fine wood plates for the Swiss printer. It was published 1543, and included 7 volumes:

- 1: The Bones and Cartilages
- 2: The Ligaments and Muscles
- 3: The Veins and Arteries,
- 4: The Nerves
- 5: The Organs of Nutrition and Generation
- 6: The Heart and Associated Organs,
- 7: The Brain



Vesalius publications conflicted Galen, whose anatomy was based on dissections of animals, and had no microscopes or magnifying glasses. for example Galen claims that the great blood vessels originated from the liver. Vesalius found that blood cycle result from the heart pump action, However, he also made wrong statements, e.g. with respect to the blood cycle, believing that different kind of blood runs in veins and arteries, as reports based on autopsies could not study dynamic life features.

He assembled human skeletons from bones he collected from corpses. Such displayed skeletons became a landmark in medical schools, and his anatomical books replaced those of Galen.

1540 Valerius Cordus

Applied Ether during surgeries.

1530 Girolamo Fracastoro

Studies and names syphilis. Identifies Typhus and proposes that contagious diseases are infecting by small, sperm-like bodies that multiply in the sick body and can be transferred by contact or also in air to healthy people. He ingeniously attributes the burst of a plague to modifications in the toxicity of the small bodies. This is exactly how we understand today the spread of a flu due to mutations in the flu viruses.

1556 George Bauer (Georgius Agricola)

The German physician, studies Saxon miner diseases. His book “About metals” is a milestone in mineralogy, and was translated to English by Herbert Hoover, to become American president.

1562 Gabriel Fallopio

Follows Vesalius, and some say he was the more innovative. Describes the ear and blood vessels associated with many organs in our body.

Theophrastus Bombastus von Hohenheim,
who called himself **Philippus Aureolus Paracelsus**
1493-1541



Alchemist by education. Studied human anatomy. Burnt The books of Ibn Sina, Galen and Hippocrates. But he replaced Galens 4 humors with 3 chemical properties: Sulfuric (flaming) Mercuric (liquid variability) and Salt (solid stability), another humorism. But the emphasis on a special drug for each disease is his important contribution. He concentrated in analyzing miner diseases.

Hieronymus Fabricius 1533-1619

His book “Surgeries” is mainly based on Celsus, Paul of Aegina and Abulcasis, and describes clogging of arteries, birth, and newborn anatomy and physiognomy.

Berengario da Carpi 1460-1530

Observes that kidneys are not a sieve, and that the urine bladder has only one opening to the urine tube. Described the vermiform appendix, and the thymus lymph. His book "Isagoge breves" published in 1522 was most influential study in anatomy before Vesalius.

Ambrose Paré 1510-1590

1537 Paré stitches cuts and wounds, and develops artificial limbs – agrees to amputations only in extreme cases.
Renews the use of bandages for shooting wounds, and
Use of ointments for burns. Performed surgeries of Hernia.
Manipulate the new born position to avoid Breech birth.

Giambattista Canano 1515-1579

1541 Publishes a list of all muscles and connection to bones.

Additional physicians and surgeons of the 16th century,
all describe treatments according to Paré:

Caspar Stromayr or Stromayer ~1520 - 1566

Pierre Franco 1500 -1561

Bartholomeo Maggi at Bologna, 1477-1552

Felix Wurtz of Zurich, 1514-1574

Léonard Botal in Paris, 1519-1587

Thomas Gale (English surgeon) 1507-1586

William Clowes 1540-1604 Military Surgeon



1518 Established The Royal College of Physicians, with specialist associations.

Andreas Vesalius 1514 –1564

Anatomist, corrects mistaken “facts” in Galen’s manuscripts.

The historical hierarchy of great anatomists:

Hippocrates => Galen => Avicenna => Vesalius



Michael Servetus (1511-1553)+ Realdo Colombo (1515– 1559)

First Europeans to describe the Pulmonary blood circulation (through the lungs).

Servetus was born in Spain. Both worked at the University of Padua, and became rivals after Colombo revealed mistakes of Servetus...

Miguel Servet y Reves or Michael Servetus

Servetus was sentenced to be burnt at the stack due to heresy.



Guillaume Rondelet 1507-1566

1555 Classification of fishes. Compares breathing and blood cycle through lungs and gills. Studies Dolphins and Sharks. Mistakenly believed that the floatation sac is a kind of lungs.

Julius Caesar Arantius 1529-1589

1564 Discovered that although both mother and fetus blood systems come in contact with the placenta, there is no blood intermixing.

Andrea Cesalpino 1519-1603

1583 publishes “De Plantis” sorting plants by classes according to number of leaves, location and shape of fruit parts.

William Harvey 1578–1657

Blood cycle. See “Age of Reason” presentation.

Valerius Cordus 1515-1554

German botanist that cataloged herbs and pharmacological plants.
Synthesized ether

Pierre Fauchard 1678-1761

Father of dentistry.



Charles Estienne 1504-1564

1545 Publishes drawings of Veins, Arteries and Nerves.

Peter Lowe 1550-1612

Surgeon, founder of the Royal College of surgeons in Glasgow.

Amato Lusitano 1511-1568

1556 describes valves in veins. By dissecting the (spinal) Azigos vein he proposed an alternative path of blood to the heart. At his time common belief was that blood in veins and arteries do not mix, since capillary arteries could not be observed without a microscope.

Garcia de Orta ~1501-1568

1563 Starts Tropical medicine. Wrote a composition about Indian diseases, working from Goa, the Portuguese colony.

Li Shizhen 1518-1593

A Chinese acupuncturist.

1596 He published a list of a large collection of medical herbs: “Běncǎo Gāngmù ”

BOTANY

Leonhart Fuchs 1501-1566, Otto Brunfels 1489-1534

Hieronymus Bock (Tragus) 1498-1554

The three fathers of Botany: Classification and sorting of plants, mainly done in Germany.

Konrad Gesner 1561-1565

1552 Swiss, creates the basis of **Carl Linnaeus** taxonomy, published 200 years later. Same names to skeletal bones in birds and mammals.

His classification of animals and plants separately sorts genus and species.

He studies from collecting and sorting (but also draws imaginary creatures...).

Pierre Belon 1517-1564

1553 Dolphins and whales are mammals and breath air to their lungs (no gills as fish have). Wrote a book about bird eggs.

Started comparative anatomy, mainly of skeletal bones.

Cocoa, Coffee and Tea

Cocoa: Arnan Cortez 1485-1547

1519 Cortez drinks Cocoa at the court of Montezuma II, the Aztec king, and imports the drink to Europe.

1657 A Cocoa house in London and North America.

1765 Chocolate produced in Massachusetts.

Coffee:

1000AD Arabs bring coffee from Ethiopia.

1475 Ottomans open coffee house in Constantinople.

1600 Coffee brought to Venice by traders and spreads fast in Europe. Coffee plantations in North America then in Brazil. Coffee houses all over the world.

Tea:

3000 BC Chinese use tea, and from 300 AC cultivate tea plantations.

Before the 12th century tea became common in Japan.

1610 The Dutch East India company starts tea import from China. At the same time Mongols spread Tea in Russia. By the end of the 17th century tea trading from China to Europe expands. Tea houses everywhere. The British impose high taxes on tea sales, which eventually caused the Boston tea Party and American independence.

1830 British “steel” tea plants from China and start extensive tea production in India.

MATHEMATICS

Lionardo de Pisa Fibonacci 1175-1250

Italian mathematician, theory of numbers: Fibonacci's numbers, whole square numbers. See medieval mathematics.



Bonaventura Francesco Cavalieri 1598-1647

Cavalieri's principle: two bodies that have equal area of sections at all heights have equal volumes. Similarly: if have equal perimeter at all section heights, have also equal surface area.

This is the beginning of the development of concepts of differential and integral calculus.



Niccolò Fontana Tartaglia 1500-1557

Was a mathematician, map surveyor and fortification engineer for Venice, who was wounded during the Holy League wars in the taking over of Bregia by the French, and had difficulty talking (Stutterer). He translated to Italian Archimedes and Euclid books, studies projectile tracks (Galileo confirmed his findings).

1560 He published a manual for land survey and calculations using compass and measuring tape.

Tartaglia's solution for a third order equation $ax^3 + bx + c = 0$

Was published by Cardano. See Appendix.



The formula for the volume of a tetrahedron given his three edges as vectors, a,b,c:

$$V = \frac{|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|}{6},$$

Tartaglia’s equation based on the distances between the four vertices, d_{ij}:

$$V^2 = \frac{1}{288} \det \begin{bmatrix} 0 & d_{12}^2 & d_{13}^2 & d_{14}^2 & 1 \\ d_{21}^2 & 0 & d_{23}^2 & d_{24}^2 & 1 \\ d_{31}^2 & d_{32}^2 & 0 & d_{34}^2 & 1 \\ d_{41}^2 & d_{42}^2 & d_{43}^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\text{volume} = \frac{\sqrt{(-a + b + c + d) (a - b + c + d) (a + b - c + d) (a + b + c - d)}}{192 u \, v \, w}$$

$$a = \sqrt{xYZ}$$

$$b = \sqrt{yZX}$$

$$c = \sqrt{zXY}$$

$$d = \sqrt{xyz}$$

$$X = (w - U + v) (U + v + w)$$

$$x = (U - v + w) (v - w + U)$$

$$Y = (u - V + w) (V + w + u)$$

$$y = (V - w + u) (w - u + V)$$

$$Z = (v - W + u) (W + u + v)$$

$$z = (W - u + v) (u - v + W).$$

Where uvw are the edges of the basis,
and UVW are the basis vertices distances to the top vertex

Tartaglia's formula is an extension of Heron and Brahmagupta's area of a triangle given its three edges, a,b,c:

$$T = \frac{1}{4}\sqrt{(a + (b + c))(c - (a - b))(c + (a - b))(a + (b - c))}.$$

$$\begin{aligned} &=1/4\{a^2+2ab+b^2-c^2\}\{c^2-a^2+2ab-b^2\}^{1/2}=1/4\{-(c^2-a^2-b^2)^2+4a^2b^2\}^{1/2}= \\ &=1/4\{2a^2c^2+2b^2c^2+2a^2b^2-a^4-b^4-c^4\}^{1/2} \end{aligned}$$

$$T = \frac{1}{4}\sqrt{-\begin{vmatrix} 0 & a^2 & b^2 & 1 \\ a^2 & 0 & c^2 & 1 \\ b^2 & c^2 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}}$$

$$=1/4[a^2\{b^2+c^2-a^2\}-b^2\{b^2-c^2-a^2\}+\{a^2c^2+b^2c^2-c^4\}]^{1/2}=1/4\{-a^4-b^4-c^4+2a^2b^2+2a^2c^2+2b^2c^2\}^{1/2}$$

Scipione del Ferro 1465-1526

A mathematician from Bologna. Wrote in his notebook the solutions to 3rd and 4th order equations, but never published them. His son in law, who held his position in University of Bologna probably showed this notebook to Cardano during his visit of Bologna at 1543.



Similar to 2nd order equation:
that has a solution:

$$x^2 = (2\sqrt{a^2 - b})x^0 + 2a$$

$$x = \sqrt{a + \sqrt{b}} + \sqrt{a - \sqrt{b}}$$

Del Ferro assumed that a 3rd order equation:
has a solution:

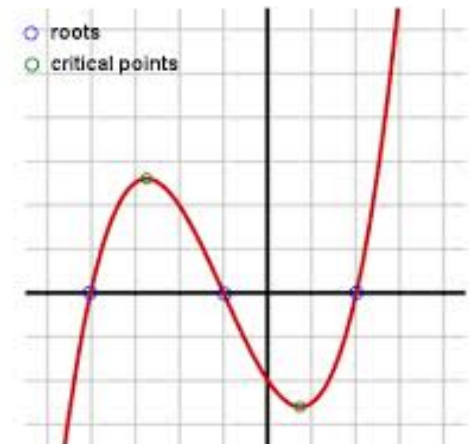
$$x^3 = (3\sqrt[3]{a^2 - b})x + 2a$$

$$x = \sqrt[3]{a + \sqrt{b}} + \sqrt[3]{a - \sqrt{b}}$$

Therefore the equation:
has a solution:

$$x^3 = px + q$$

$$\sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$



François Viète 1579-1616

French. Contributions in Algebra. 1593 Viète's formula for approximating the value of π . Publicized by **Euler** 1737. ~1760 **Johann Lambert** proves that π is irrational.



$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2 + \sqrt{2}}}{2} \cdot \frac{\sqrt{2 + \sqrt{2 + \sqrt{2}}}}{2} \dots$$
$$\frac{223}{71} < \pi < \frac{22}{7}.$$

1591 Viète use letters as variables and unknowns in equations. Name “coefficient” and “ Π ” for product of series.

Solved 3rd order equations:

$$x^3 + px + q = 0,$$

Using “Viète’s substitution”:

$$x = w - \frac{p}{3w}$$

The 3rd order equation in x

$$w^3 + q - \frac{p^3}{27w^3} = 0.$$

becomes 2nd order in ω^3

$$w^6 + qw^3 - \frac{p^3}{27} = 0$$

See Appendix for the complete solution

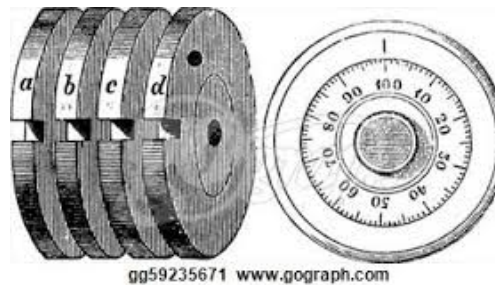
Girolamo Cardano 1501-1576

1545 Published Tartaglia's solution of 3rd order equations, claimed as the first achievement in Algebra since the Babylonians. The paper also presents Tartaglia and Ferarri's solution to 4th order equations, by reduction to 3rd order.

$$ax^3 + bx + c = 0$$

1562 Describes systematic calculation of probabilities (maybe since tried to supplement his bad finances by gambling...). For example, the probability to get an even number equals the ratio between even and all numbers. The paper was published 1663, and were used by De Moivre at 1711.

Made Mechanical inventions: Cardan shaft, Gimbal (employed in Gyroscopes), and combination locks.



Lodvico Ferrari 1522-1565

Cardano's student. Birth of modern Algebra



Is there a closed solution to 5th order equations?

Employed mathematicians in future centuries.

Paolo Ruffini 1765-1822

1799 Presented an incomplete solution.

Niels Henrik Abel 1802-1829

Describes a full proof.

Évariste Galois 1811-1832

Wrote an independent proof based on Galois group, using his Group Theory, showing that there is no close Solution to equations of any order above 4.

Bring-Jerard transform the general 5th order equation:

$$x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$$

To the “Normal” form:

$$x^5 - x + 1 = 0$$

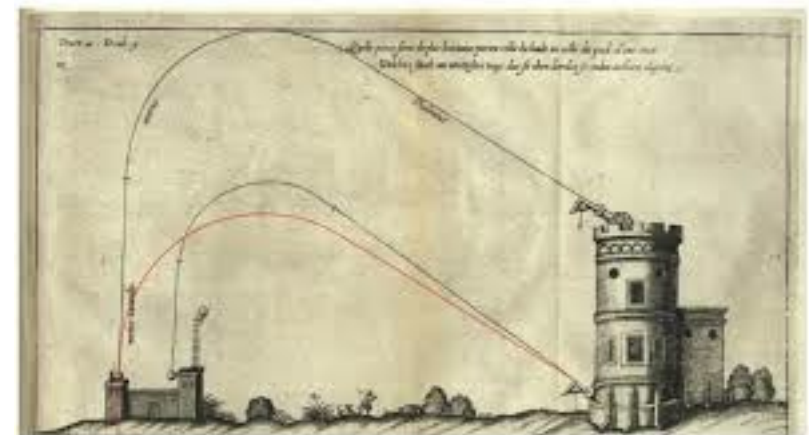
An example of a special form of equations that have solution:

$$x^5 - x^4 - x + 1 = 0$$

$$(x - 1)(x - 1)(x + 1)(x + i)(x - i) = 0$$



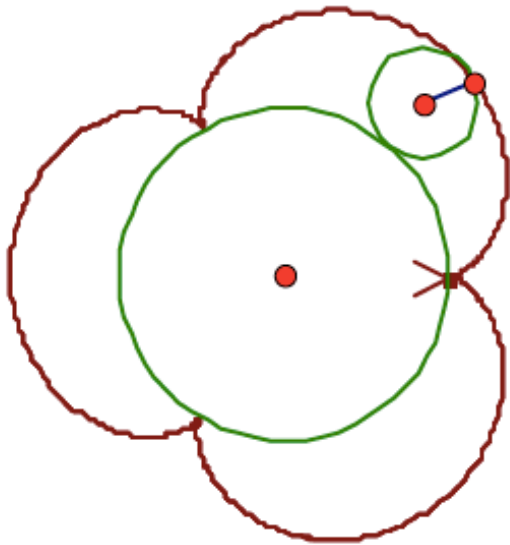
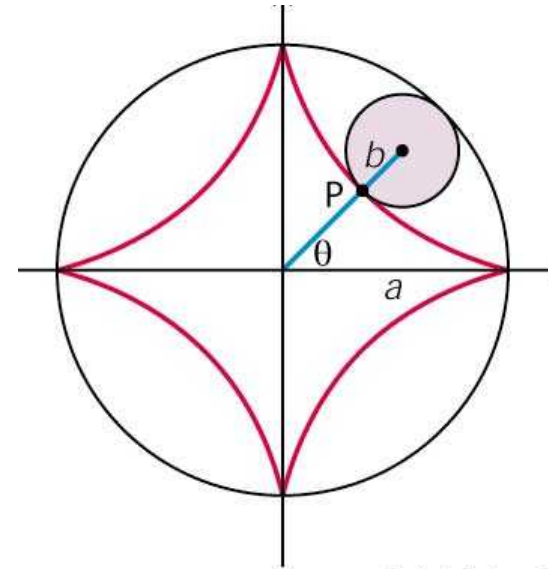
What is the motivation to find a close form to the solution of high order equations ? Maybe it is the tracks of cannon shells. Without air friction, the track is a parabola (2nd order equation). Therefore corrections due to air friction and winds can be approximated by 3rd and 4th equations, and close solutions would allow to calculate the location of the shell hitting as a function of the cannon angle and other parameters.



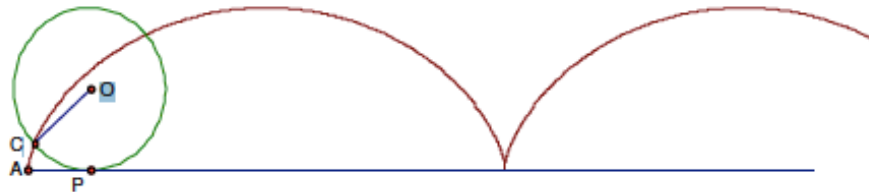
Epicycloids and Hypocycloids:

Where of interest in Astronomy, to describe the orbits of the planets.

A Hypocycloid



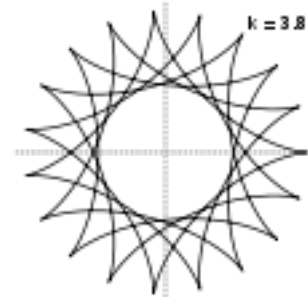
An epicycloid



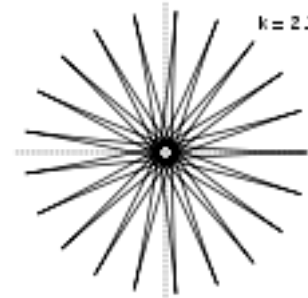
A cycloid

Hypercycloids

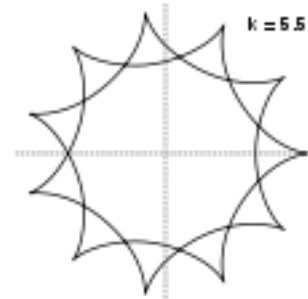
← $k=3$



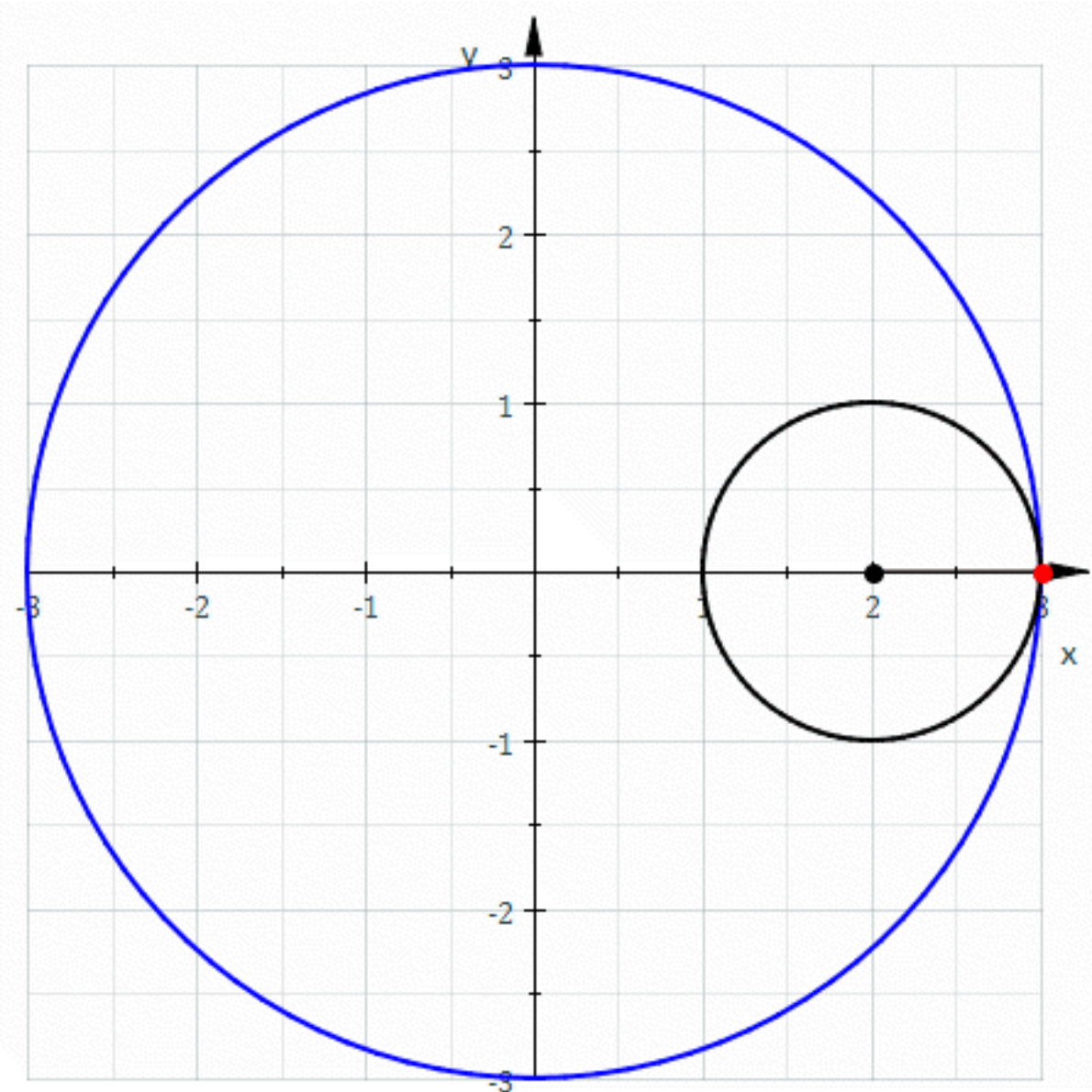
$K=3.8$



$K=2.1$

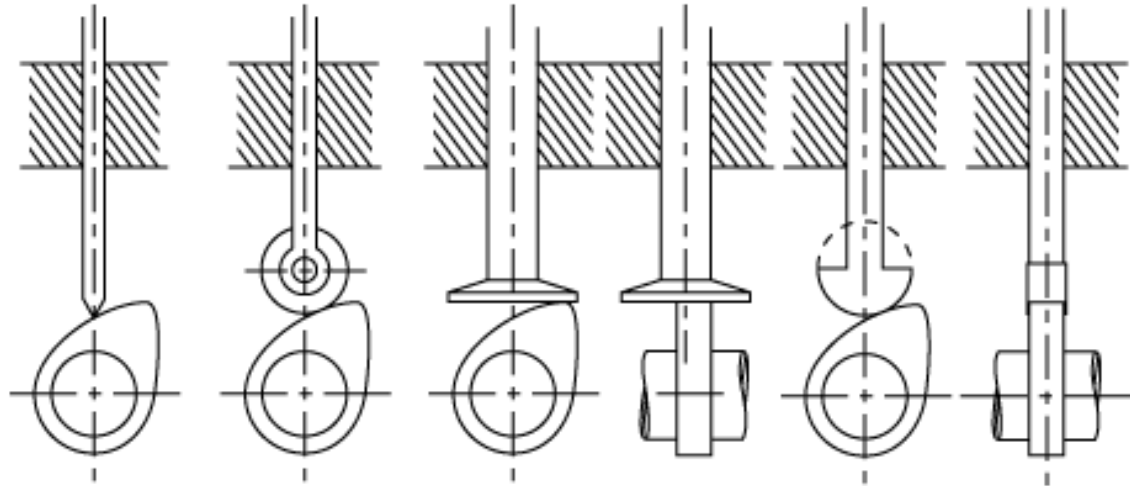


$K=5.5$

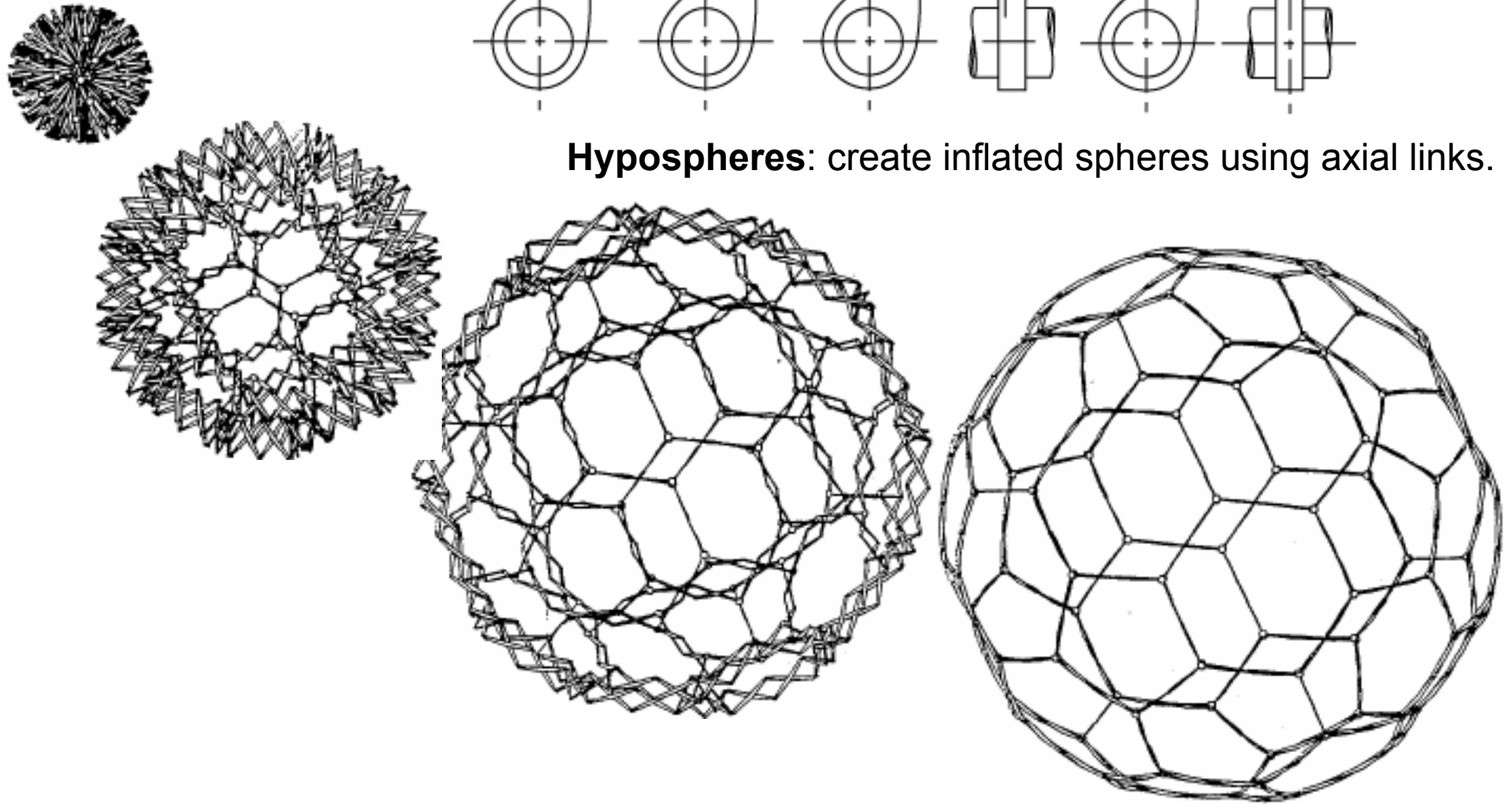


$$\begin{aligned} x(\theta) &= r(k-1)\cos\theta + r\cos((k-1)\theta) \\ y(\theta) &= r(k-1)\sin\theta - r\sin((k-1)\theta). \end{aligned}$$

Hypercycloids are in use when rotational movement is transduced to linear movement.

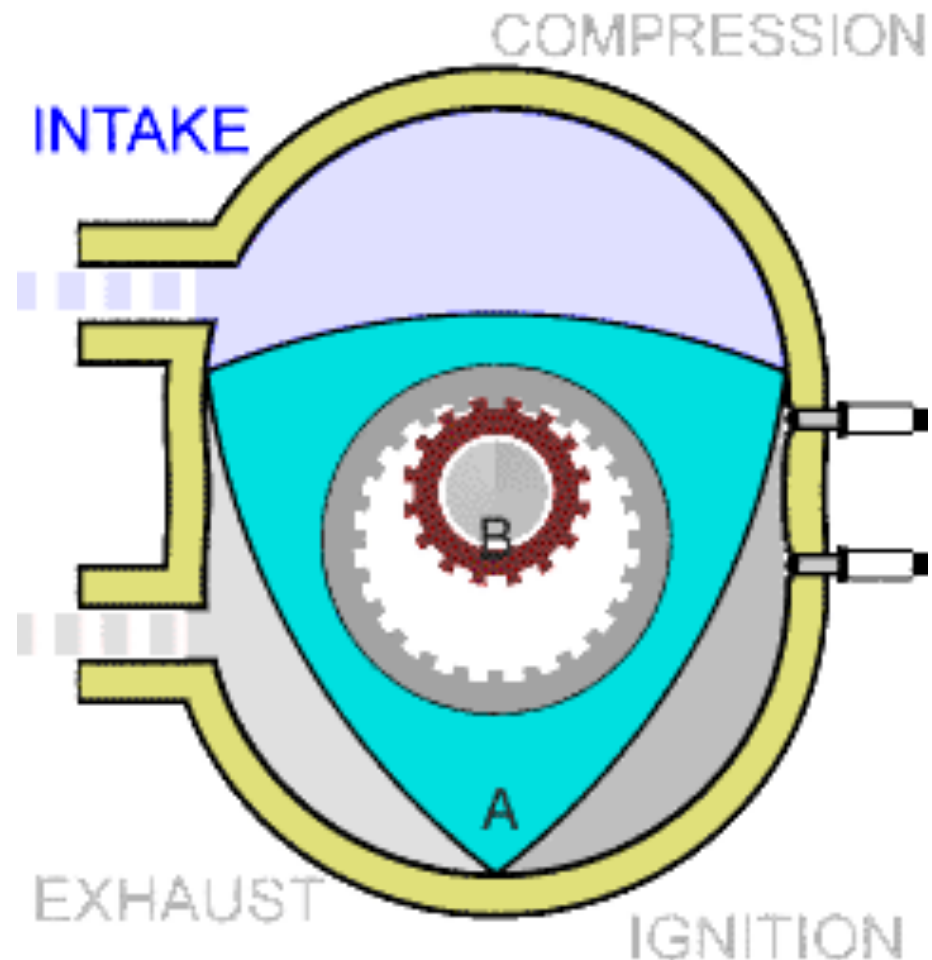


Hypospheres: create inflated spheres using axial links.



Wankel engine

Mazda mounted this very simple engine directly on their car wheels, avoiding transmissions and axels. However, the problem of sealing the three segments formed between the asymmetric piston and the walls of the ignition chamber made the engine inefficient, and required often replacement due to abrasion.

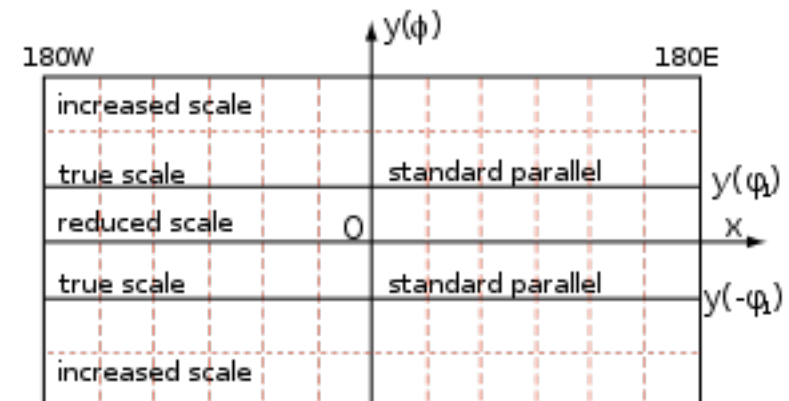
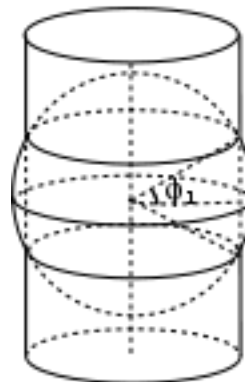
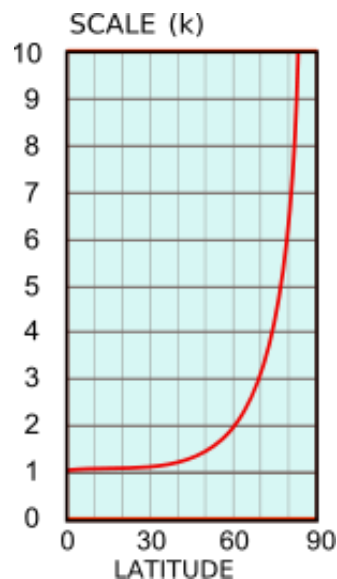
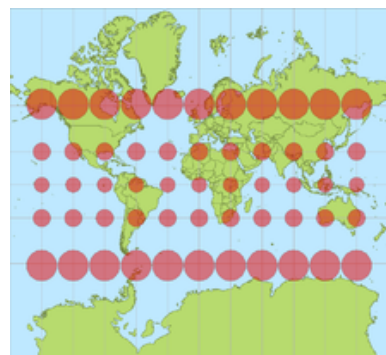


Gerardus Mercator 1512-1594

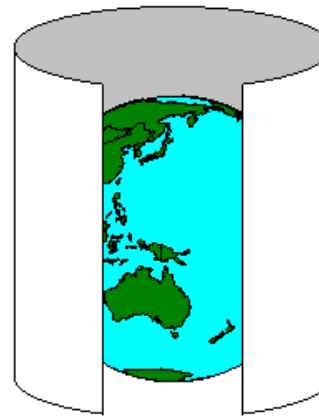
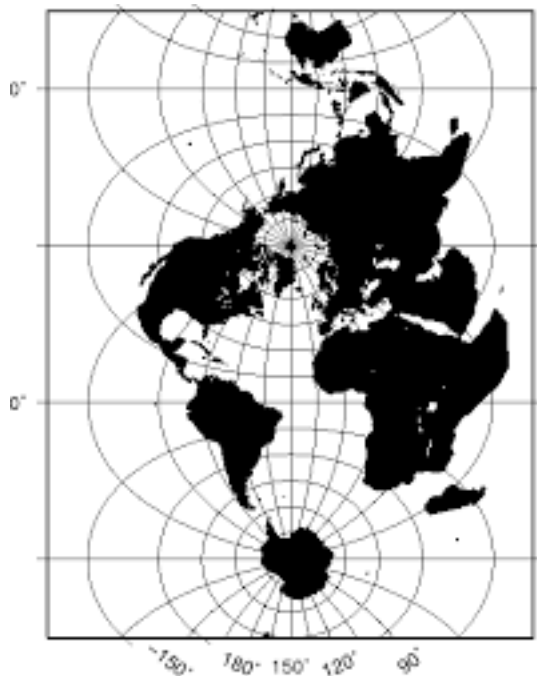


1569 Methods of projecting the spherical surface of earth on a plat map. The Flemish geographer invented this projection. The advantageous property preserving angles made it easy to navigators. It enables to measure azimuth Between the start and end points of a journey, and sail along this constant azimuth. Although it is not the shortest route.

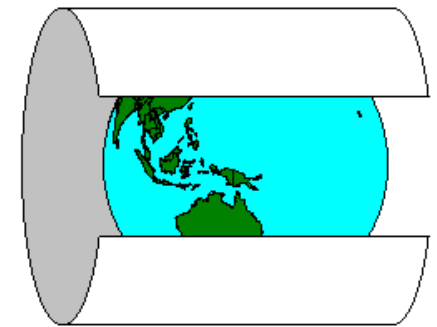
The projection is carried by “wrapping” a cylindrical map around earth sphere, and radially projecting every point on earth surface to the cylinder. Longitudinal and Altitudinal lines remain perpendicular, but a longitudinal distance distortions increase as we move away from the equator towards the poles. For example, Greenland looks as big as Africa, although its area is 1/13 of Africa’s area.



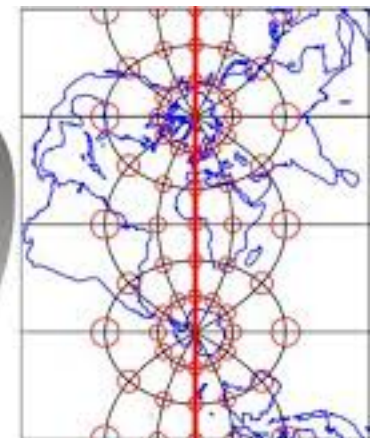
1722 Mercator presents another map, using the same projection, but “rolling” the cylinder at a perpendicular direction: “Transverse Mercator”. This map displays regions spreading longitudinally with less distortions.
UTM- **Universal Transverse Mercator** is based on 60 such strips, 6° each, between 80° south and 84° north.



Mercator projection



Transverse Mercator projection



Rafael Bombelli 1526-1572

Describes negative and imaginary numbers resulting in solutions of algebraic equations, and the rules of their multiplication (e.g. negative times negative is positive).





John Napier of Merchiston 1550 –1617

A Scottish land owner, who invented Logarithms at his free time, and introduced the digital point in decimal number presentation.

The Logarithm game: the basis for the calculating ruler.

$$1 \times 1 = 1 \quad \log(1) + \log(1) = \log(1)$$

$$2 \times 3 = 6 \quad \log(2) + \log(3) = \log(6)$$



CHEMISTRY

The transition from Alchemy to modern Chemistry

The start of understanding of the composition and disassembly of elements and compounds. The realization that elements cannot be trans-mutated. Middle age chemists concentrated on Sulfur and Mercury, maybe because their compounds were easy to assemble and disassemble, and created beautiful colors.

Paracelsus 1493-1541

Added Zink to the list. He understood that high concentration of many compounds makes them toxic.

Georg Bauer or Georgius Agricola 1494-1555

1556 Sorted minerals. Established physical geography.

Vannoccio Biringuccio, or Vannocio Biringuccio 1480 –1539

Composes a book about processing of metals. Isolated from Arsen Tin (Sn) and Antimony (Sb). Became useful in the printing industry to mold plates, since expands upon cooling.

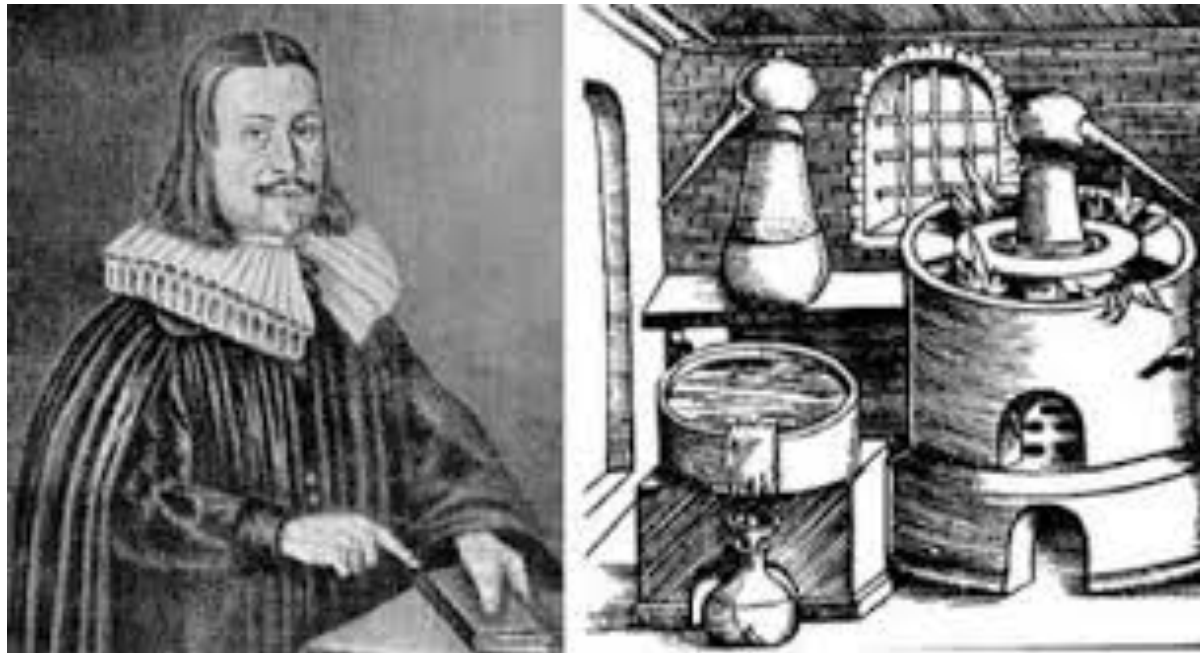
I.A																		VIIA					
1	H 1																					He 2	
2	Li 3	Be 4																B 5	C 6	N 7	O 8	F 9	Ne 10
3	Na 11	Mg 12																Al 13	Si 14	P 15	S 16	Cl 17	Ar 18
4	K 19	Ca 20	Sc 21	Ti 22	V 23	Cr 24	Mn 25	Fe 26	Co 27	Ni 28	Cu 29	Zn 30	Ga 31	Ge 32	As 33	Se 34	Br 35	Kr 36					
5	Rb 37	Sr 38	Y 39	Zr 40	Nb 41	Mo 42	Tc 43	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50	Sb 51	Te 52	I 53	Xe 54					
6	Cs 55	Ba 56	La 57	Hf 72	Ta 73	W 74	Re 75	Os 76	Ir 77	Pt 78	Au 79	Hg 80	Tl 81	Pb 82	Bi 83	Po 84	At 85	Rn 86					
7	Fr 87	Ra 88	Ac 89	Rf 104	Db 105	Sg 106	Bh 107	Hs 108	Mt 109	Uun 110	Uuu 111	Uub 112	Uut 113	Uuq 114	Uup 115	Uuh 116	Uus 117	Uuo 118					

6	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
7	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr
	90	91	92	93	94	95	96	97	98	99	100	101	102	103



Andreas Libavius 1560-1616

Writes a practical book “Alchemistry”, maybe the first laboratory manual of precise protocols, unlike Paracelsus’ blurred descriptions. Its aim was to extract materials from compounds for metallurgy, medicine and every day uses.



APPENDIX: List of Renaissance scientists

Red: India

Black: Europe

Leonardo 1486 – 1513

Widmann, Johannes 1460 -1498 German mathematician, introduced + and -

Ferro **Scipione del Ferro** 1465 -1526 Italian mathematician first solved $x^3+px=q$

Werner

Maior

La Roche

Dürer 1471–1528 German painter

Copernicus 1543

Tunstall Cuthbert 1474 –1559 Bishop of London ???

Ortega

Lax

Stifel

Ries

Maurolico

Fine

Rudolff

gglia (1500, Brescia –1557, Venice)

Jyesthadeva 1500 –1575 Indian math and astronomer

Cardan **Gerolamo Cardano** 1501 –1576 probability algebra cardan joint

Nunez

Gemma Frisius 1508 –1555 Dutch maps and navigation instruments. Teacher of Mercator

Recorde Robert 1512–1558 Welsh invented the = sign used preinvented + sign

Gerardus Mercator 1512-1594 Belgium cartographer



Rheticus Georg Joachim 1514-1574 trig tables, Copernicus' student,
published his *"Revolutions of the Heavenly Spheres"*

Ferarri

Bombelli Rafael 1526-1572 algebra

Benedetti Giovanni Battista 1585 mechanics

Porta **Giambattista della עפיפונים**

Danti Aligiery

Barocius

Calvius

Van Ceulen

Viète Franciscus 1540-1603 French math

Monte

Brahe Tycho 1546-1601

Digges Thomas

Cataldi

Steén

Savile

Lavanha

Napier John 1550 –1617 Scottish merchant – log tables

Bürgi

Valerio

Ricci Mateo

Harriot Thoma English astronomer

Fincke

Lansberge

Pitiscus

Roomen

Briggs

Galileo

Kepler

Oughtred

Guldin Paul 1577-1643 Swiss math [in 17-cent]

Faulhaber

Hérigone

Snell (Willebrord van Roijen Snellius) 1580–1626 Dutch astronomer

Bachet

Gunter

Jean-Baptiste Morin 1583– 1656 French math Opposed Galileo

Vernier Pierre 1580-1637 French math

Saint-Vincent Gregoire de 1584–1667 Jesuit mathematician

Mydorge Claude 1585 – 1647 French geometry & physics

Turner Joseph Mallord William 1775–1851 English painter ???

Jungius Joachim 1587 –1657 German math Logics



Gassendi Pierre 1592 –1655 French astronomer (transit of Mercury over the sun) speed of sound indep. On pitch
Mersenne Marine 1588 –1648 French father of acoustics: freq. of stretched string= $f = \frac{1}{2L} \sqrt{\frac{F}{\mu}}$,
Christopher Clavius 1538 –1612 astronomer
Giovanni Battista Riccioli 1598-1671 Italian astronomy

APPENDIX

Solution of
Cubic and Quadratic
equations

Solution of 3rd order equation

$0 > \Delta$: Three different real solutions

$0 = \Delta$: Real double solutions

$0 < \Delta$: One real and two imaginary solutions

$$ax^3 + bx^2 + cx + d = 0 \quad (1)$$

$$\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2.$$

$$x_1 = -\frac{b}{3a}$$

$$-\frac{1}{3a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$-\frac{1}{3a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$x_2 = -\frac{b}{3a}$$

$$+ \frac{1 + i\sqrt{3}}{6a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$+ \frac{1 - i\sqrt{3}}{6a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$x_3 = -\frac{b}{3a}$$

$$+ \frac{1 - i\sqrt{3}}{6a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d + \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

$$+ \frac{1 + i\sqrt{3}}{6a} \sqrt[3]{\frac{1}{2} \left[2b^3 - 9abc + 27a^2d - \sqrt{(2b^3 - 9abc + 27a^2d)^2 - 4(b^2 - 3ac)^3} \right]}$$

Tartaglia-Cardano method to solve 3rd order equation

The solution is attributed to **Scipione del Ferro** and **Tartaglia**
and published by **Gerolamo Cardano** at **1545**

In the general 3rd order equation: $ax^3 + bx^2 + cx + d = 0$ (1)

Substitute: $x = t - b/3a$ $t^3 + pt + q = 0$. (2)

$$p = \frac{3ac - b^2}{3a^2}$$

$$q = \frac{2b^3 - 9abc + 27a^2d}{27a^3}.$$

$$u + v = t$$

Substitute in (2) yields: $u^3 + v^3 + (3uv + p)(u + v) + q = 0$ (3)

$$3uv + p = 0$$

u,v $u^3 + v^3 = -q$ $u^3 v^3 = -p^3/27$

Therefore: $z^2 + qz - \frac{p^3}{27} = 0$.

Then u^3 and v^3 are roots of the quadratic equation $(z - u^3)(z - v^3) = 0$

With sum and product: $u^3 + v^3 = -q$ $u^3 v^3 = -p^3/27$

Therefore:

$$u^3 = -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \quad v^3 = -\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

Since Cardano did not know complex numbers, he assumed:

$$\frac{q^2}{4} + \frac{p^3}{27} > 0.$$

$$t_1 = u + v = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

Viète's substitution in $x^3 + px + q = 0$,

Substitute $x = w - \frac{p}{3w}$

Multiply by w^3 $w^3 + q - \frac{p^3}{27w^3} = 0$.

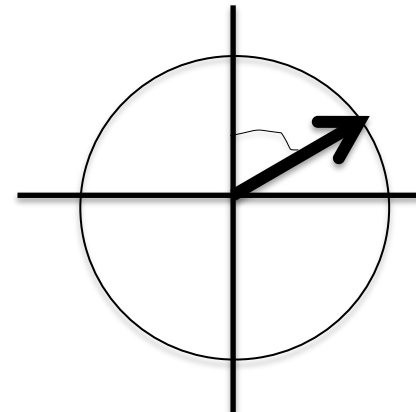
Yields second order equation in w^3 $w^6 + qw^3 - \frac{p^3}{27} = 0$

With the solution: $W^3 = -q \pm \sqrt{q^2 - 4p^3/27}$

If w_1, w_2, w_3 are the cubic roots of $w^3 = [R \cdot \exp(i\theta)]^3 = R^3 \cdot \exp(3i\theta)$

$$x_1 = w_1 - \frac{p}{3w_1}, \quad x_2 = w_2 - \frac{p}{3w_2} \quad \text{and} \quad x_3 = w_3 - \frac{p}{3w_3}.$$

$$\sqrt[3]{8^3} = \begin{cases} 8 \\ -4 + 4\sqrt{3}i \\ -4 - 4\sqrt{3}i. \end{cases} \quad \sqrt[3]{-27i} = \begin{cases} 3i \\ \frac{3\sqrt{3}}{2} - \frac{3}{2}i \\ -\frac{3\sqrt{3}}{2} - \frac{3}{2}i. \end{cases}$$



Let:

$$p_1 = 2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace$$

$$p_2 = p_1 + \sqrt{-4(c^2 - 3bd + 12ae)^3 + p_1^2}$$

$$p_3 = \frac{c^2 - 3bd + 12ae}{3a\sqrt[3]{\frac{p_2}{2}}} + \frac{\sqrt[3]{\frac{p_2}{2}}}{3a}$$

$$p_4 = \sqrt{\frac{b^2}{4a^2} - \frac{2c}{3a} + p_3}$$

$$p_5 = \frac{b^2}{2a^2} - \frac{4c}{3a} - p_3$$

$$p_6 = \frac{-\frac{b^3}{a^3} + \frac{4bc}{a^2} - \frac{8d}{a}}{4p_4}$$

Then:

$$x = -\frac{b}{4a} - \frac{p_4}{2} - \frac{\sqrt{p_5 - p_6}}{2}$$

$$\text{or } x = -\frac{b}{4a} - \frac{p_4}{2} + \frac{\sqrt{p_5 - p_6}}{2}$$

$$\text{or } x = -\frac{b}{4a} + \frac{p_4}{2} - \frac{\sqrt{p_5 + p_6}}{2}$$

$$\text{or } x = -\frac{b}{4a} + \frac{p_4}{2} + \frac{\sqrt{p_5 + p_6}}{2}$$

Ludovico Ferrari 1540

Solution of 4th order equation

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

Published with the 3rd order
solution by his teacher

Gerolamo Cardano 1545

Solution of 4th order equation

$$\alpha x^4 + \beta x^3 + \chi x^2 + \delta x + \varepsilon = 0$$

$$x = y - \beta/4\alpha$$

$$y^4 + Ay^2 + By + C = 0$$

$$A = \frac{\gamma}{\alpha} - \frac{3\beta^2}{8\alpha^2},$$

$$B = \frac{\delta}{\alpha} - \frac{\beta\gamma}{2\alpha^2} + \frac{\beta}{8\alpha},$$

$$C = \frac{\varepsilon}{\alpha} - \frac{\beta\delta}{4\alpha^2} + \frac{\beta^2\gamma}{16\alpha^3} - \frac{3\beta^4}{256\alpha^4}.$$

Add and subtract

$$2sy^2 + s^2$$

$$(y^2+s)^2 - [(2s-A)y^2 - By + s^2 - C] = 0$$

Then we factor the quadratic polynomial

$$(2s - A)y^2 - By + s^2 - C = (2s - A)(y - y_+)(y - y_-)$$

and make $y_+ = y_-$, which will impose a constraint on s (equation (4)). We will get:

$$\left(y^2 + s + \sqrt{2s - A}y - \frac{B}{2\sqrt{2s - A}} \right) \left(y^2 + s + \sqrt{2s - A}y + \frac{B}{2\sqrt{2s - A}} \right) = 0,$$

where s satisfies the *resolvent cubic equation*

$$8s^3 - 4As^2 - 8Cs + (4Ac - B^2) = 0.$$

The four solutions of (2) are the solutions of (3):

$$y_1 = -\frac{1}{2}\sqrt{2s - A} + \frac{1}{2}\sqrt{-2s - A + \frac{2B}{\sqrt{2s - A}}},$$

$$y_2 = -\frac{1}{2}\sqrt{2s - A} - \frac{1}{2}\sqrt{-2s - A + \frac{2B}{\sqrt{2s - A}}},$$

$$y_3 = -\frac{1}{2}\sqrt{2s - A} + \frac{1}{2}\sqrt{-2s - A - \frac{2B}{\sqrt{2s - A}}},$$

$$y_4 = -\frac{1}{2}\sqrt{2s - A} - \frac{1}{2}\sqrt{-2s - A - \frac{2B}{\sqrt{2s - A}}}.$$

Thus, the original equation (1) has the solutions

$$x_k = y_k - \frac{\beta}{4\alpha}, \quad k = 1, 2, 3, 4$$

