

Best seller

THOMAS PAINE
THE AGE
OF REASON



The age of REASON

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The 17th century, The age of Reason

The most “studies” century in high school curriculum.
Started the scientific revolution, which continued at the 18th century.

Newton - followed the renaissance scientists Kepler and Galileo in mechanics and astronomy.

Torricelli, Pascal and Boyle – created physical chemistry, studying physical properties of solids (stress, strain) liquids (hydrostatics, hydrodynamics), and gasses (absolute temperature, thermodynamics, energy). This formed the theoretical and practical basis for steam engines.

Gilbert and Guericke – studies electric charges and magnets: electrostatics.

Expanded table of elements and compounds (both inorganic and organic).

Pascal, Fermat and Taylor with Leibnitz and Newton – formalized and applied differential and integral calculus.

Napier, Laplace and Fourier – Logarithms and calculating machines.

Hooke, Leuwenhoek and Harvey – microscopes, cell and microbiology and the blood cycle.

Haley, Rømer, Huygens and Cassini – optics and astronomy.

Descartes, Bacon and Locke – Science philosophy.

Steno and Hooke - connect fossils with the geological history of earth.

In this century the struggle between the religious establishment and scientists continued. The scientists themselves were religious, and searched for ways to reconcile the explanations of their inventions and scientific findings with the holy scripts and religious beliefs.

Galileo, the renaissance man, believed that mathematics can truly describe nature. Francis Bacon, Thomas Hobbes and Isaac Newton in puritan England, Robert Boyle in Catholic Scotland, Blaise Pascal and René Descartes in Catholic France and many others wondered between the attempts to understand “how” to describe a world under set laws and the question “why” answered by the religious belief. They all were careful not to replace or even confront god with science.

Yet many discrepancies between the accepted religious dogma and the scientific findings caused bitter struggles, starting with Copernicus, and continued through the 17th century and on to Darwin at the age of enlightenment, 18th century. It can be claimed even today, that although science “won” over religious dogma, all religions did not give up proposing answers (often quite convoluted) to reconcile between interpretation of the holy scripts and the present scientific theories and laws.

In one aspect though scientists gave up: our world is understood to be highly complex, and the limitations of scientists to describe it accurately are well accepted. Scientists experienced reformulation of the laws of nature during the 20th century, after three centuries of rising security of scientists to be able to come up with a “true” description of nature. Today “scientific truth” is no longer claimed, and is replaced with scientific models best fitting the present data we measured. This process is exemplified by the extension of Newton laws with Quantum mechanics and Einstein’s Relativity, the missing universal theory that combines them all, and difficulty in modeling the big bang.

17th century highlights:

Johannes Kepler-1609

German, the three astronomical laws for planet orbits .

Galileo Galilei – 1610

Italian, Telescopic observations, mechanics of gravitational forces.

John Napier – 1614

Scottish, Logarithms help fast computations.

William Harvey - 1628

English, blood cycle.

Giovanni Batista Baliani - 1630

Italian, tells Galileo that a siphon cannot pump water to a hill higher than 10.3 meters. Galileo explains that vacuum cannot hold higher water column, and induces Gasparo Berti to build a pipe to show that the water column height is always 10.3 meters, independent on the height of the vacuum above it.

Evangelista Torricelli – 1643

Italian, understands that not the vacuum but the atmospheric pressure balances the water column, and builds Mercury barometer.

Robert Boyle – 1662

Irish, Gas laws, relating pressure and volume.

Philosophical Transactions of the Royal Society- 1665

The first scientific journal with reviewing procedure prior to publication.

Francesco Redi – 1668

Italian, demonstrated that spontaneous generation of life is impossible.

Nicholas Steno – 1669

Danish, Fossils are remains of ancient life that were trapped in sediment – a way to date geological times.

Jan Swammerdam – 1669

Dutch, showed that larva and pupa are embryonic life forms of insects. Employed a microscope for dissections, and described contraction of muscles and blood cells.

Leibniz, Newton – 1675

German and English, infinitesimal (differential and integral) calculus.

Anton van Leeuwenhoek – 1675

Dutch, observed microorganisms in his single-lens microscope.

Ole Rømer – 1676

Danish, measured the speed of light from motion of planets.

Isaac Newton – 1687

English, three laws of motion, laws of gravity. Classical physics.

Patent laws

3rd century, Phylarchus, writes that in Greek towns in Italy royalties were given for one year, rewarding inventors that brought convenience or special culinary contributions.

1331, England, the king donated royal monopoly to a company to manufacture a product they developed.

1449, Henry VI donates 20 year royal monopoly to a Dutch inventor and manufacturer.

1450 and on, Patents are registered in Venice. Open publication is linked to inventor's rights against infringements for 10 years. Most patents registered were for the glass industry. Emigration of professionals from Venice spread the protection of inventions all over Europe.

1555 the French Henry II implements protection of inventions for their public disclosure. The patent is registered by the royalty or the Parisian parliament, after novelty is checked by the French academy of sciences. Yet, disclosure of patents was highly supervised.

In England, intellectual property laws preceded the industrial revolution, and were a seminal factor in its propagation. A “document presenting development” was registered and publicized in Latin, and was different from a personal letter. After the king donated such documents for payment, independent on novelty (for example distribution of salt...), the parliament issued patent laws that limited registration only for inventions and for a set number of years.

1665-1714 during the ruling of **Queen Ann**, patent applications were required to include detailed description of the invention. After the debate between the inventors of the steam engine of **James Watt**, improvements of prior patents of practical value were allowed. Soon the patent laws became problematic because owners of patents prevented competition and established monopolies, the exact opposite of the original intent of public disclosure. This caused continued evolution of patent laws, and created differences between countries. Patents are valid only in the country of registration, imposing multiple registration in many countries.

In the USA patent laws require presentation of a functioning model of the invention (embodiment), however, this requirement was eased today.

The question of patent rights for cloned genes and gene sequences altered by directed mutations is still debated. On one hand genes are naturally occurring entities and not an invention. But their natural functions and their modified functions upon alterations is a knowledge acquired by devoted scientists and is of great importance for medicine, and the public has interest in rewarding tedious research work that discloses such gene properties and their potential uses for treatment of diseases.

MATHEMATICS

John Pell (1611-1685)

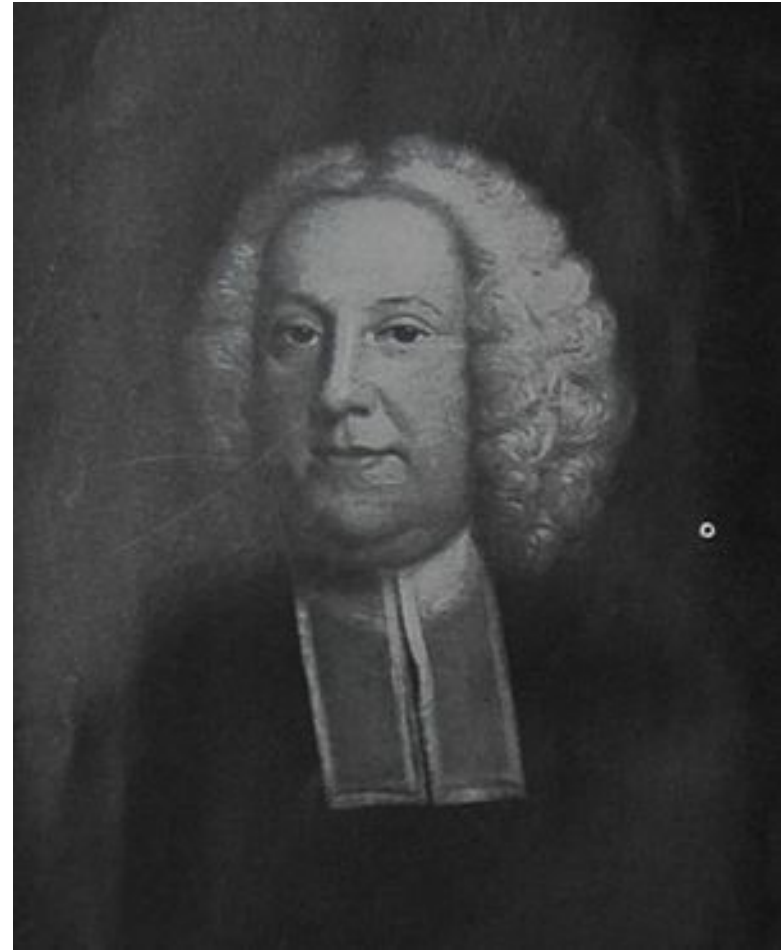
Pell equation, Diophantic equation, $x^2 - ny^2 = 1$

http://en.wikipedia.org/wiki/Pell%27s_equation

For n that is not a square number, the equation has an infinite number of solutions, creating rational approximations to \sqrt{n}

Pell was sent by Oliver Cromwell to Switzerland in order to unify all protestant churches under England, a failed mission.

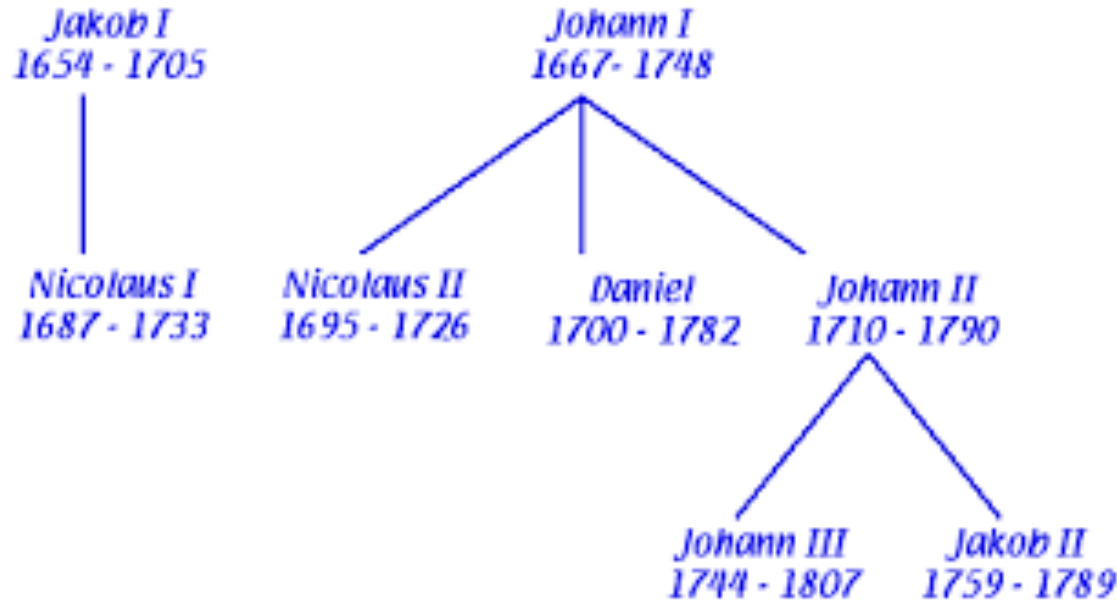
Pell together with Johann Heinrich Rahn stamped the sign for division \div and edited his book "Algebra".



James & Johann Bernoulli

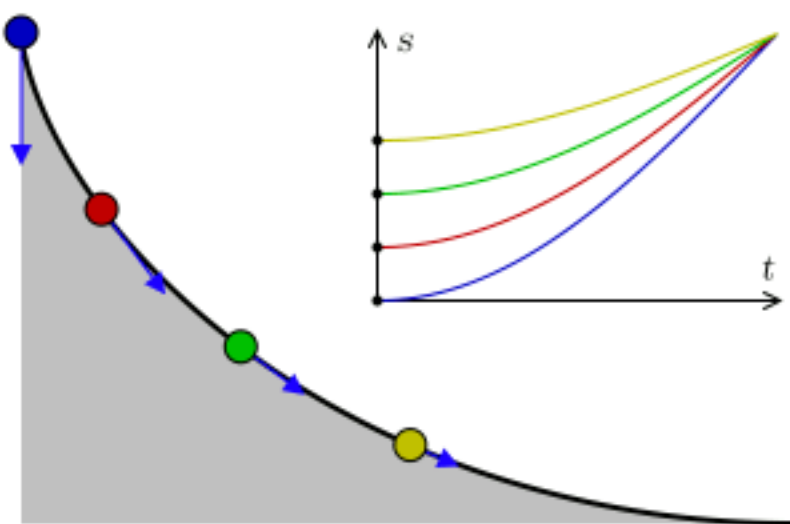
A Dutch-originated family, that fled the harassment towards Huguenot (protestants) to Frankfurt and Switzerland.

The grandfather was a physician, the son, Jacob, was a spice trader who moved to Basel.



The problems engaged by Johan-James and Jacob Bernoulli were in implementing infinitesimal calculus recently developed by Leibnitz. Jacob Bernoulli's nonlinear differential equation: $y' + P(x)y = Q(x)y^n$ the solution is the Bernoulli probability distribution for coin tossing $f(k; p) = p^k(1-p)^{1-k}$ for $k \in \{0, 1\}$.

Other contributions: Bernoulli experiment, Bernoulli's process, Bernoulli's numbers, Taylor's coefficients for Tangent function (the first algorithm implemented in Babbage computing machine), Bernoulli's polynomial, and Bernoulli's principle in hydrodynamics.



James (Jacob) Bernoulli 1655–1705

1690 shows that cycloid is the solution for the isochrone curve (or tautochrone)
The curve that a ball reaches its bottom in time independent on its starting point.



1691 Johann Bernoulli shows that a hanging chain between two points creates the curve:

$$y = a \cosh\left(\frac{x}{a}\right) = \frac{a \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)}{2}$$

1691 James Bernoulli shows that the solution for the chain yields the lowest center of mass.

1696 Johann Bernoulli shows that the cycloid is also the curve yielding the fastest falling time of the ball.

These proofs preceded in 50 years **Leibnitz, Maupertuis and Euler** minimal action principle.

Johann Bernoulli stood up for Leibnitz against Newton over priority on calculus. He also mistakenly defended Descartes' vortex theory against Newton's gravity, and caused a delay accepting the correct theory in Europe.

Guillaume de l'Hôpital (1661-1704)

A French noble and army man, that due to extreme short sight became a mathematician, and the president of the French Academy of Science. He hires Johann Bernoulli as his math teacher, and gets the rights for his inventions.

1696 l'Hopital publishes a textbook for infi students that contains Bernoulli's work and acknowledges the Bernoulli brothers and Leibnitz. It includes "l'Hopital's rule" for the limit at $x \rightarrow c$ of a ratio of functions both having diminishing or undefined values at $x=c$: $0/0$ or ∞/∞ .

If functions $f(x)$ and $g(x)$ are two functions with derivatives $f'(x)$ and $g'(x)$, and if:

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x) = 0 \text{ or } \pm \infty \quad g'(x) \neq 0$$

Then:

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$$

For example:

$$\begin{aligned} \lim_{x \rightarrow 0} \text{sinc}(x) &= \lim_{x \rightarrow 0} \frac{\sin \pi x}{\pi x} \\ &= \lim_{y \rightarrow 0} \frac{\sin y}{y} \\ &= \lim_{y \rightarrow 0} \frac{\cos y}{1} \\ &= 1. \end{aligned}$$



Paul (Habakkuk) Guldin (1577–1643)

A Swiss Jesuit mathematician and astronomer, that served as a professor at Graz and Vienna. He rediscovered “Pappus-Guldin law” for calculating the surface area and volume of rotational bodies.

[Pappus from Alexandria 290-350]

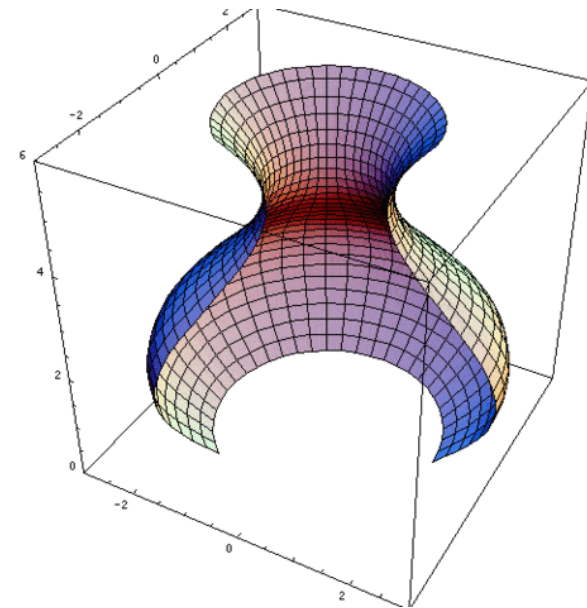
Was a friend of Kepler.

1658 Guldin writes “Paolo Casati”: imaginary dialog between Galileo, Mersenne and Guldin about cosmology, astronomy, geography and geodesy.

Surface area and Volume of rotation bodies:

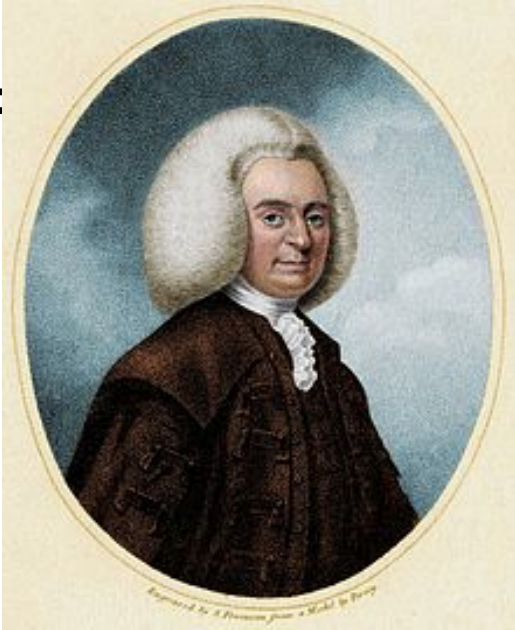
A planar curve of length L rotates around an axis that does not cross the curve. The surface area of the rotating body is $S=2\pi RL$ where R is the distance of the curve center of mass from the axis.

A closed planar curve with area A rotates around an axis that does not cross the curve (can touch it though). The volume of the rotating body is $V=2\pi RA$ where R is the distance of the closed curve center of mass from the axis.



Brook Taylor (1685-1731) Taylor-Maclaurin series expansion:

$$f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \dots$$



Colin Maclaurin (1698-1746) Scotland

1735 Euler-Maclaurin formula, the relation between sum and integral:

$$I = \int_m^n f(x) dx \qquad S = f(m + 1) + \dots + f(n - 1) + f(n)$$

And the relation between the integral and the derivative at the border of integration:

$$\sum_{i=m+1}^n f(i) = \int_m^n f(x) dx + B_1 (f(n) - f(m)) + \sum_{k=1}^p \frac{B_{2k}}{(2k)!} \left(f^{(2k-1)}(n) - f^{(2k-1)}(m) \right) + R.$$

R is the error term, B is Bernouli's numbers.

Pierre de Fermat 1601-1665

Fermat lived in Toulouse, and made his income as a lawyer and city parliament member. Being far from Paris, his work in mathematics was not exposed to other mathematicians. Fermat was especially keen of **Diophantus “Arithmetica”**, that a Latin translation of it he owned, and added notes in its margins, of many theorems without proofs. Later editions of the book embedded Fermat’s notes in the print, (Fermat’s copy was lost), and most of them were proven to be true (one he claimed as “might be true” was shown to be false).

1625 Fermat is recognized as an independent creator of analytical geometry: $F(x,y)=0$ is a curve in xy plane, its extrema are found from the points where the tangent angle is zero. He develops a method to find minimum, maximum and slope of curves prior to calculus.

1640 The small Fermat theorem:

If p is a prime, $a^p - a$ divides by p .

1643 calculates the center of mass of parabolic sections using differential calculus (not integration)



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Francis Bacon 1561 -1626

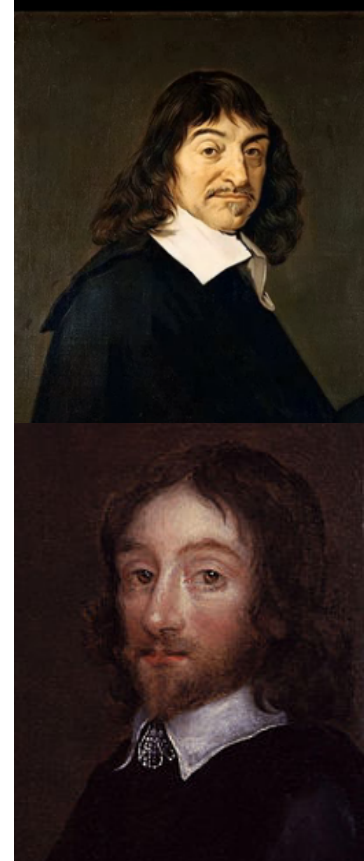
The scientific method – experiments, observations, skeptic criticism and logical analysis for a scientific explanation.

Wrote a paper about heat originating from motion of particles.



René Descartes 1596-1650

Scientific truth does not come from trial and error, but from rational logics. Link between the real world and human intellect brings back the discussions between Plato and Aristo.



Sir Thomas Browne 1605 –1682

Researched science and medicine, and using the scientific method to study nature.

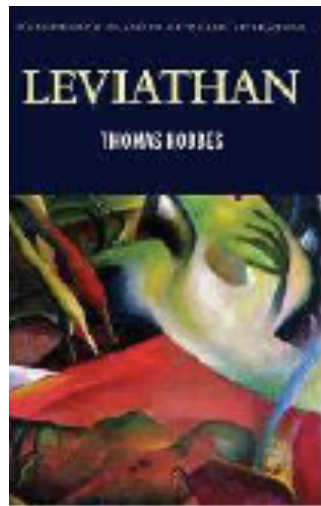


Thomas Hobbes 1588-1679

Founder of political philosophy.

1651 during the English civil war (Oliver Cromwell) of the Royalists against the Parliamentarians, wrote “Leviathan”, to support strong government provided it will be law abiding and preserve civic freedom.

1647 was hired to teach math to young Charles prince of Wales. Published mathematical works, some controversial (e.g. squaring the circle).



Hobbes' Leviathan:
a best seller
till today.

John Wallis (1616-1703).

English. Infi, Analytical Geometry
Cryptographic algorithms (for encryption).
Taught deaf mute people.

Wallis introduced the product series for π :

$$\prod_{n=1}^{\infty} \left(\frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \right) = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \frac{8}{7} \cdot \frac{8}{9} \cdots = \frac{\pi}{2}$$

1656 moves from geometry of lines to arithmetic of numbers,
and stamps the sign ∞ for infinity.



Christiaan Huygens, 1629 1695

1646 - The curve of a hanging chain is not a parabola
(see Bernoulli).

1647-8 works on squaring the circle and the ellipse.

String vibrations – well tempered scale. Used logarithms.

1657 – first works about game theory, probabilities, and life expectancy tables.

1656 – meets Pierre Fermat, 1660 – Baruch Spinoza,
Antony van Leuwenhoek and Newton.

His most significant contribution is wave optics – see following



Blaise Pascal 1623-1662

A “wunderkind” of French science. Contributed in projection geometry (see following), probability (with Fermat, chances of winning in gambling), Hydrodynamics (built hydraulic press), clocks (built hand watches), and designed public transportation that became operative in Paris **1662**. At the age of **18** he built a calculating machine for addition of 5 digits based on lifting and dropping of weights. **1659** he used infi to calculate sums. **1670**, 8 years after his death, his manuscript about his deep Catholic religious beliefs was published. Was Descartes’ opponent: “heart has virtues that reason does not recognize”. Proof of gods existence based on thousands of Jews existence. God helps the gambler he wants to salvage, and cause the gambler he he wants to punish to loose...

Pascal’s identity:

$$(n+1)^{k+1} - 1 = \sum_{m=1}^n ((m+1)^{k+1} - m^{k+1}) = \sum_{p=0}^k \binom{k+1}{p} (1^p + 2^p + \dots + n^p)$$

Pasdcal’s triangle:

The binomial coefficients

$A+B$

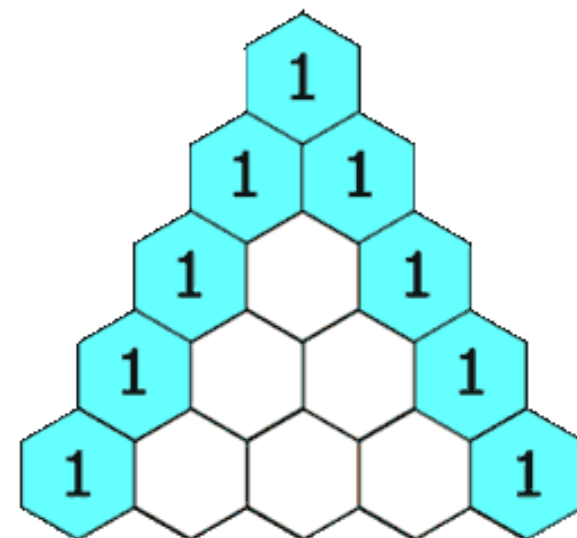
$$(A+B)^2 = A^2 + 2AB + B^2$$

$$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$$

$$(A+B)^4 = A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$$



Pascal’s calculator.



Giovanni / Jean-Dominique Cassini 1625-1712

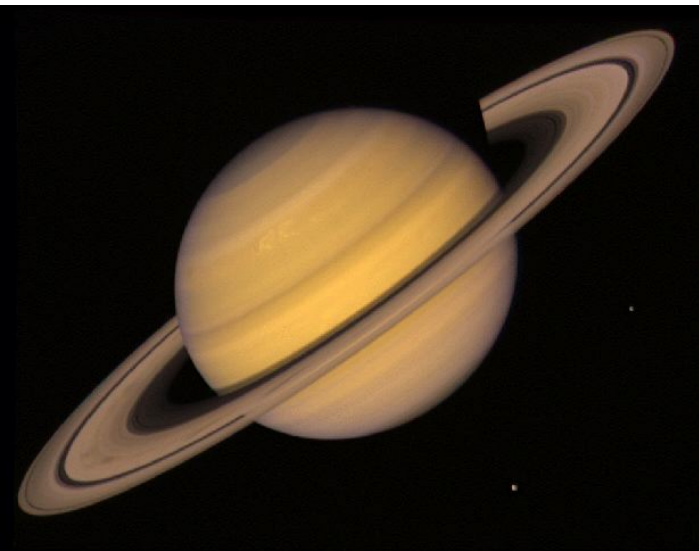
An Italian astronomer, that moved from Bologna to Paris to head Luis XIV observatory (still exist near the Luxemburg Gardens). Discovered four satellites of Saturn, and the segmentation of its rings. Edited topographical maps of France.

In number theory: a recursive formula for Fibonacci's numbers:

Cassini's equation –
Catalan's extension –
Vajda's extension -

$$\begin{aligned}F_{n-1}F_{n+1} - F_n^2 &= (-1)^n. \\F_n^2 - F_{n-r}F_{n+r} &= (-1)^{n-r}F_r^2. \\F_{n+i}F_{n+j} - F_nF_{n+i+j} &= (-1)^nF_iF_j.\end{aligned}$$

1672 – Cassini meets Leibnitz, the diplomat who plans a calculator.

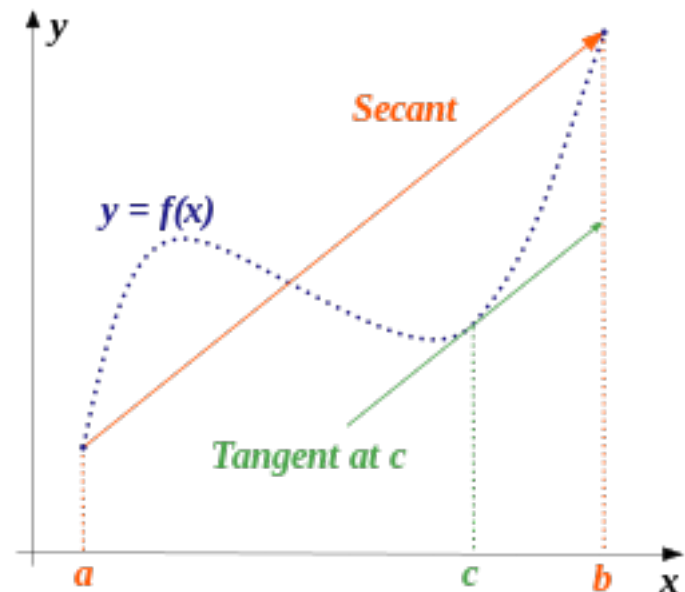
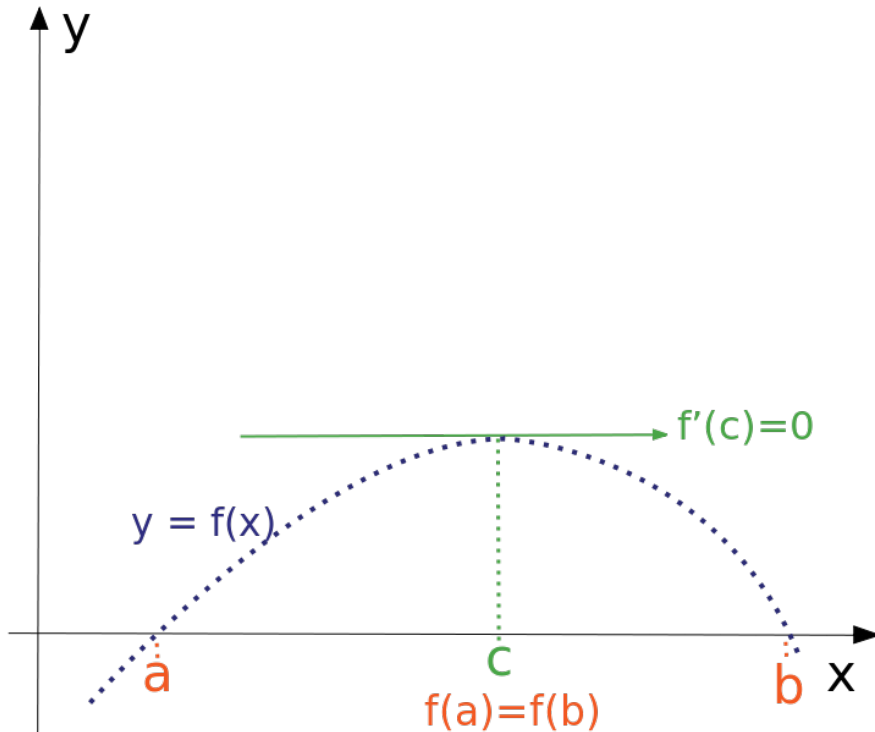


Michelle Rolle 1652-1719



A French mathematician, known for Rolle's theorem:
if a function f is continuous on the closed interval $[a, b]$ and
differentiable on the open interval (a, b) such that $f(a) = f(b)$,
then $f'(x) = 0$ for some x with $a \leq x \leq b$.

This theorem is essential for many proofs in the theory of functions
and infinitesimal calculus. E.g. expression of a function as a Taylor series, and the
mean value theorem: For a continuous and differentiable function in the interval $[a, b]$
there is a midpoint where the slope is parallel to the cord connecting $f(a)$ and $f(b)$.

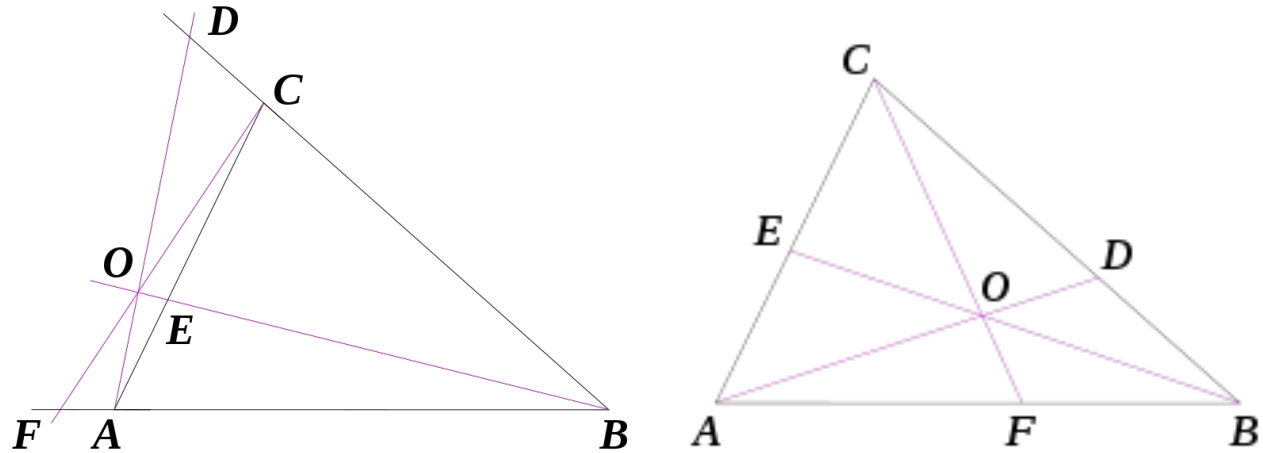


Giovanni Ceva 1647-1734

An Italian geometrician.

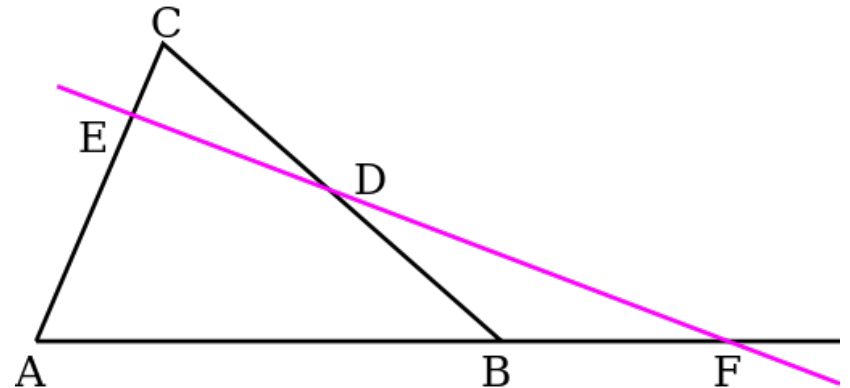
Ceva theorem: If we draw three lines from the vertices of a triangle to the opposite edges, they will meet at a point if and only if:

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1.$$



It is similar to **Menelaus' theorem 70-140** in Alexandria:

$$\frac{AF}{FB} \times \frac{BD}{DC} \times \frac{CE}{EA} = -1.$$



Abraham De Moivre 1667-1754

A French mathematician, a colleague of Newton, Haley and Sterling. Known for De Moivre formula for the powers of complex Numbers, making their presentation in polar coordinates convenient:

$$(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx),$$

$$\sin x = \frac{1}{2}(\sin(nx) + i \cos(nx))^{1/n} + \frac{1}{2}(\sin(nx) - i \cos(nx))^{1/n}$$

Wrote a manuscript about probabilities after **Cardano** where he describes De Moivre law: assuming death events are distributed evenly in time between 0 and a maximal age ω , the probability of a person to live x years is:

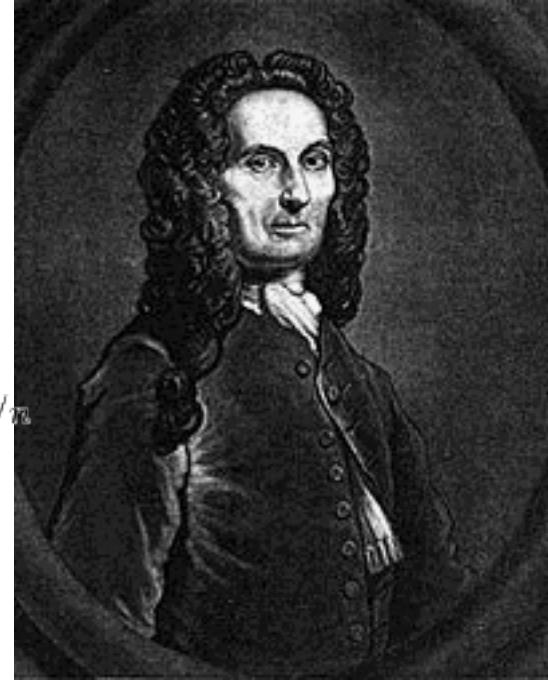
$$S(x) = 1 - \frac{x}{\omega}, \quad 0 \leq x < \omega.$$

The probability of a baby who lived x years to live t more years is:

$${}_t p_x = \frac{S(x+t)}{S(x)} = \frac{\omega - (x+t)}{\omega - x}, \quad 0 \leq t < \omega - x,$$

If death events cause exponential decay of the population, or $S(x) = \exp(-\mu x)$ then:

$${}_t p_x = S(x+t)/S(x) = \exp(-\mu t)$$



GEOMETRY

The development of analytical geometry at the 17th and 18th centuries had seminal influence of the mathematical formulation of classical mechanics at the 18th and 19th centuries, and formed the basis for the mathematical formulation of quantum mechanics at the 20th century, the conservation laws of quantum numbers, and the prediction of new elementary particles based on the symmetry in the space of the quantum numbers.

The non-Euclid geometries and coordinate transformations in multi-dimensional spaces were the mathematical basis of the special and general relativity theories of Einstein.

Almost 2000 years after Euclid, this field of mathematics revived, still established on axiomatic perfect logics, and demonstrated again the unexpected link between basic and applied sciences.

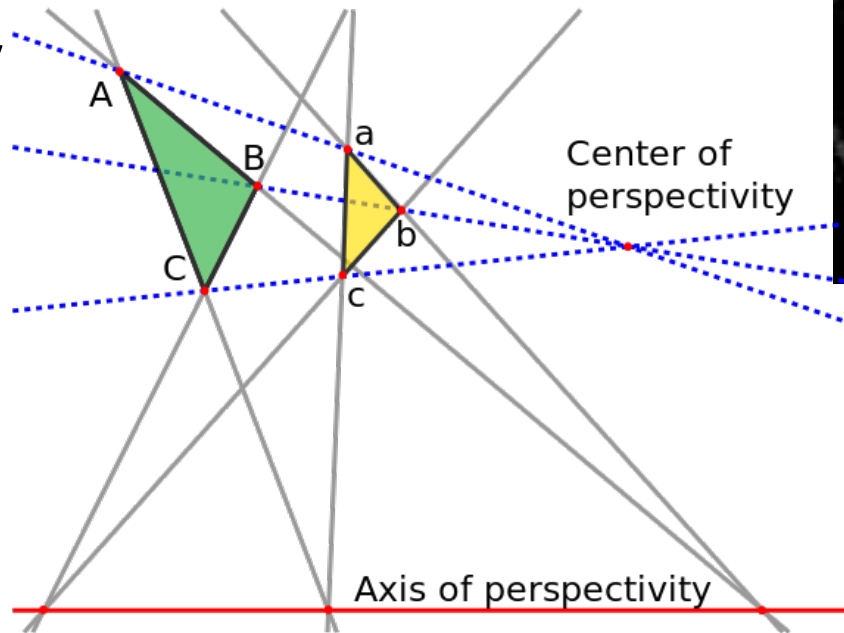
Here are some key mathematicians exemplifying the contributions in 17th century geometry.

Girard Desargues 1591–1661

A French mathematician and engineer. One of the founders of projective geometry and perspective, based on the work of Elhazen, Kepler, Brunelleschi, and the classicists conic sections.

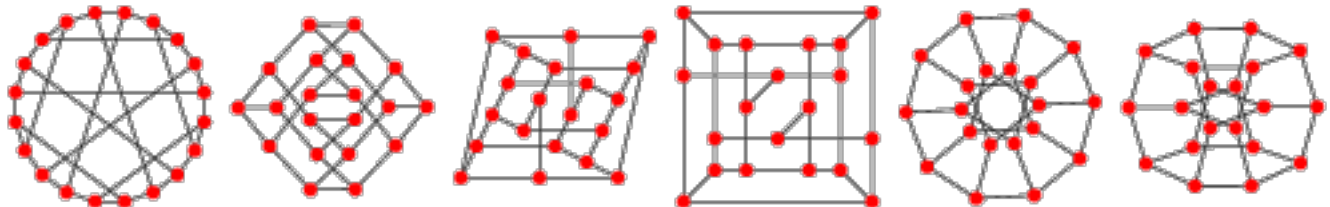
Desargues' theorem:

Two triangles are in perspective if and only if the continuation of their edges meet on One line (red)



Desargues graphs:

Regular graphs, in use by chemistry where every atom has a set coordination number.



1596-1650 René Descartes

A French philosopher and mathematician who claimed at 1641: cogito ergo sum
“I think therefore I am” (in French: “je pense donc je suis”).
Introduced Cartesian coordinate system, applied Algebra for Geometry (e.g. the equation for a straight line $y=ax+b$). Used exponent function, and roots of negative numbers.



René Descartes 1596-1650

✱ "I think; therefore I am."

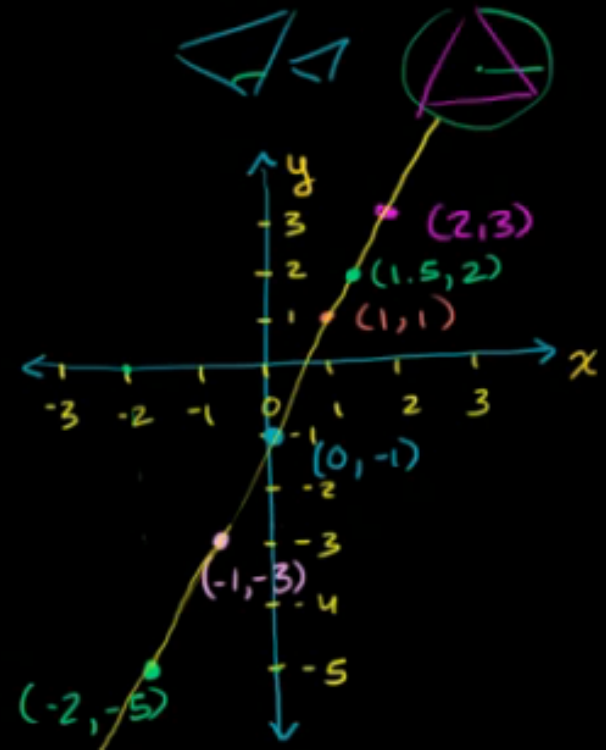
✱ "You just keep pushing. You just keep pushing. I made every mistake that could be made. But I just kept pushing."

Algebra

$$y = 2x - 1$$

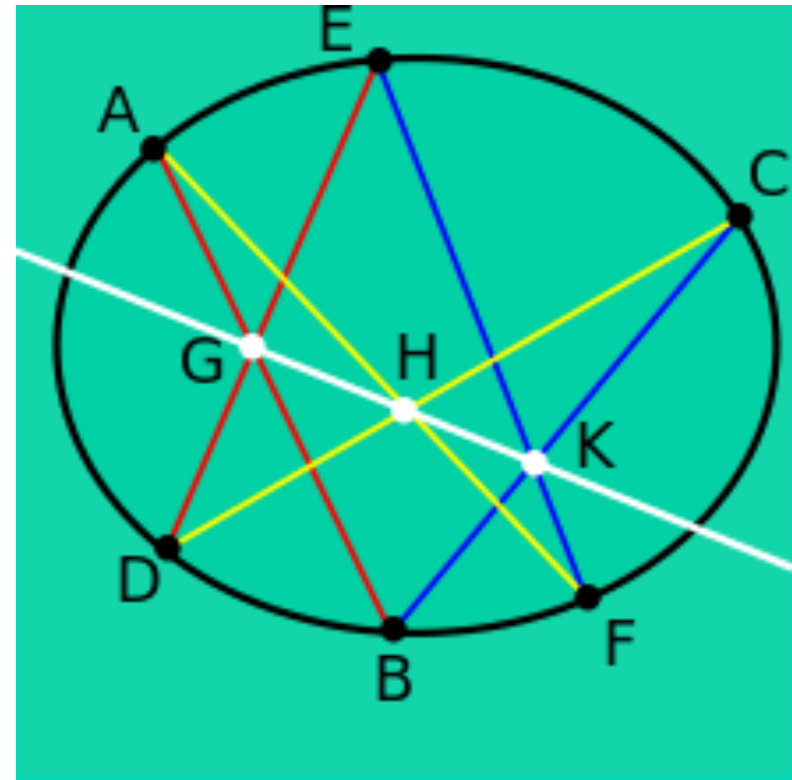
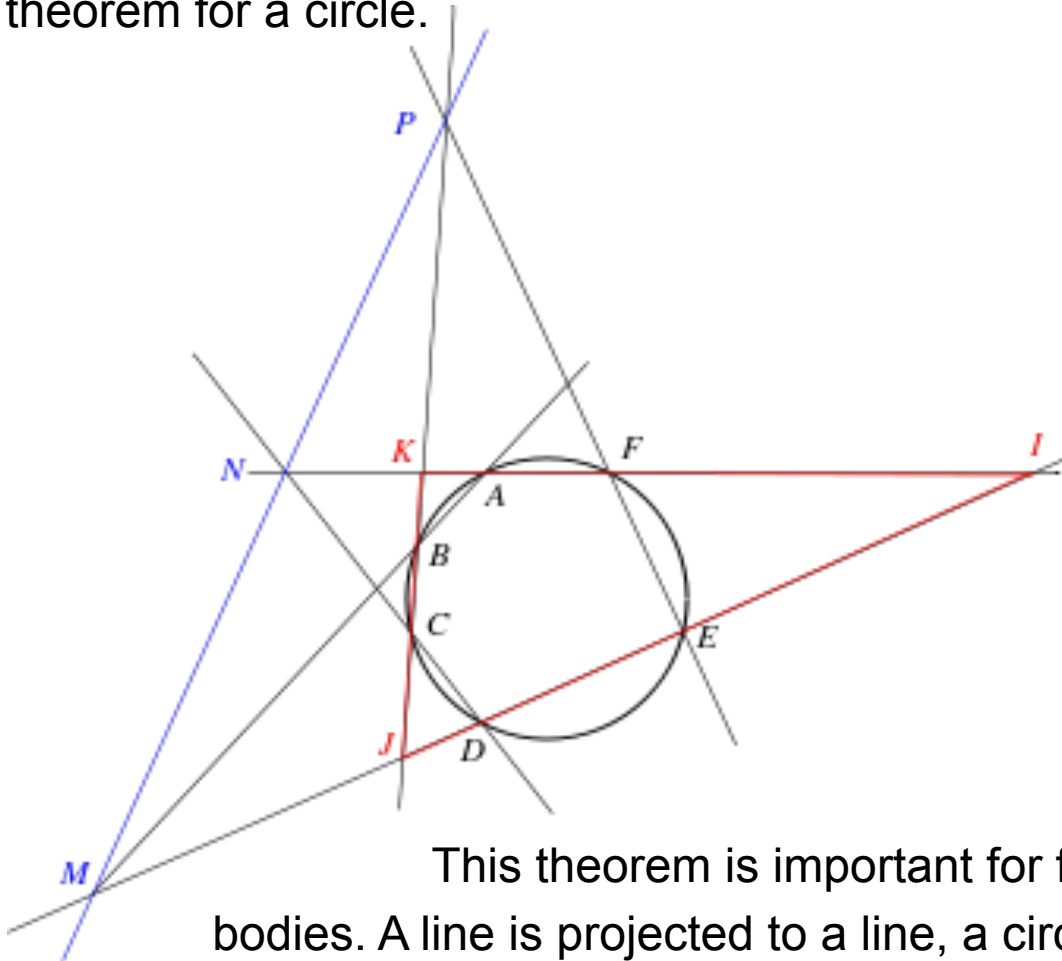
x	y
-2	$2(-2) - 1 = -5$
-1	$2(-1) - 1 = -3$
0	$2(0) - 1 = -1$
1	$2(1) - 1 = 1$
2	$2(2) - 1 = 3$

Geometry



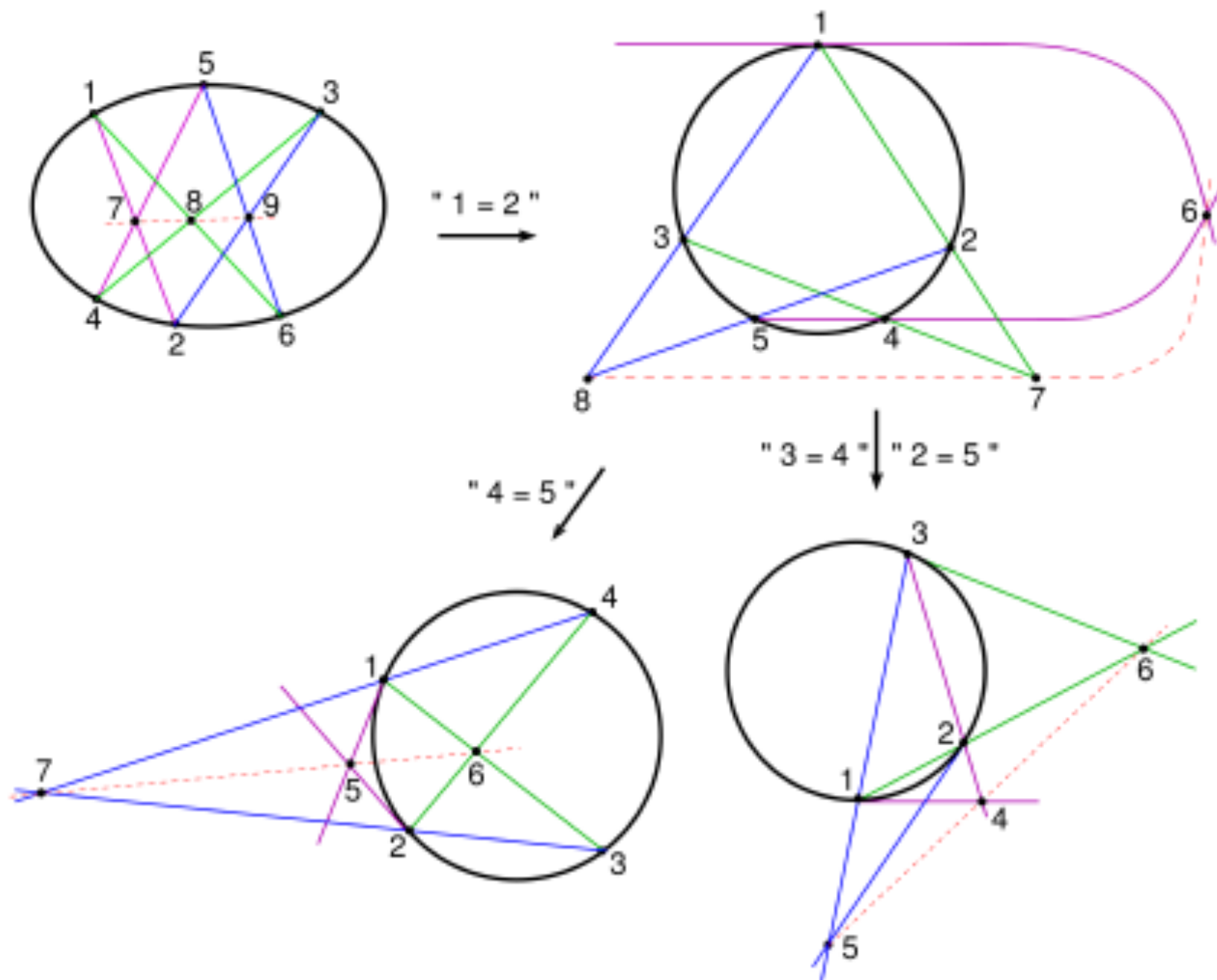
Blaise Pascal 1623-1662

In his essay about conic sections he became a co-inventor of projective geometry, and proved many theorems, the best known is Pascal's theorem 1639: 6 arbitrary points on a conic section joined by edges form an hexagon. The lines between pairs of opposite vertices cross at 3 points on a single line. This is a generalization of Pappus theorem for a circle.



This theorem is important for faithful projection of three-dimensional bodies. A line is projected to a line, a circle, depending on the projected plane, may be projected to a conic section: either a circle, ellipse, parabola or hyperbola. In general, a conic section is projected to a conic section.

Degenerated cases for Pascal's theorem (parallel lines):



CALCULUS

It is amazing how the infinitesimal limit already engaged the Greeks. The concepts of infinity (as well as zero) were foreign to them, therefore the race between Zeno and the Turtle could not be mathematically resolved, although the practical solution was clear (see Antiquity “Numbers” presentation).

Although the link between Geometry and Infinitesimal calculus is not obvious, the field emerged with Newton's need to describe the orbit of moving bodies under forces, and Leibnitz' desire to describe geometry analytically.

Only following 16th and 17th centuries extended use of functions, their derivatives and integrals, the 18th century mathematicians were urged to establish the rigorous basis to infinitesimal calculus. It was first limited to smooth, continuous functions, but was extended to singular functions (with undefined or infinite values or derivative at some points, or along lines). “Complex functions analysis” became a powerful formulation for field theory, where the electrical field near a charged point diverges (as well as the electrical forces derived from the field function). Yet, integration over field functions around singularities can be defined, and “miraculously” recover the charge enclosed inside a closed integration surface.

Landmarks in Infinitesimal (Differential and Integral) calculus

Egypt: Moscow papyrus 1820 BC Numerical solutions to calculations of areas and volumes.

India: Sulba Sutras contain “derivatives” of trigonometric functions.

Babylon: Integration on measurements for Jupiter orbit using the Trapeze law

Greece: Democritus concept of “void”

The paradox of Zeno and the turtle.

Antiphon and Eudoxus method of exhaustion

Archimedes approximated π from the perimeter of polygons, and calculated surface area and volumes of geometrical and rotating bodies

China: Liu Hiu discovered the exhaustion method at the 3rd century.

Zu Chongzhi 5th century, discusses integrals.

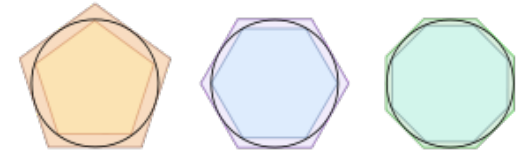
Cavalieri: The concept of integrals. Method of indivisibles.

16th and 17th centuries: Descartes, Fermat, Pascal, Wallis, Isaac Barrow (Newton's mentor): Functions derivatives, slopes of graphs.

Newton and Leibnitz: Modern infinitesimal calculus.

18th century: Cauchy, Bolzano – rigorous laws in the theory of functions (ϵ - δ)

19th century: Weierstrass – continuity of functions. Calculus of Variations.



Bonaventura Francesco Cavalieri 1598-1647

1635 Cavalieri's theorem: If two bodies have equal section area and perimeter at all heights, their volume and surface areas are equal too.

The picture schematically demonstrates this theorem for a pile of coins.



Gottfried Wilhelm von Leibniz 1646-1716

Leibnitz was a German mathematician and a diplomat, who was well connected not only in the German towns, but also in Paris and London.

He built a calculating machine with pinwheels, capable of adding, subtracting, multiplying and dividing. He discussed binary presentation of numbers.

1684 Uses “functions” and publishes differential calculus method, developed in parallel by newton. Today, our notations for derivatives follow Leibnitz, not Newton’s.

Leibnitz developed a logical syntax similar to AND, OR, NOT functions of modern logical operations.

He stated the law of kinetic energy conservation $E_k = mv^2$

Leibnitz used 7 philosophical principles in his scientific work (next slide), indicating his will to follow Euclid in establishing solid rules for science research, as well as his deep religious belief (he was a Protestant, but appreciated his patrons Catholic faith, and although he admired Spinoza’s intellect he resented his conclusions that were inconsistent with his religious beliefs).



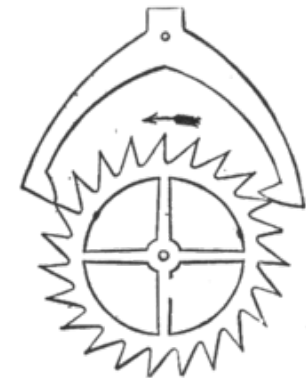
Leibniz variously invoked one or another of 7 fundamental philosophical Principles:

1. Identity/contradiction. If a proposition is true, then its negation is false and vice versa.
2. Identity of indiscernibles. Two distinct things cannot have all their properties in common. If every predicate possessed by x is also possessed by y and vice versa, then entities x and y are identical; to suppose two things indiscernible is to suppose the same thing under two names. Frequently invoked in modern logic and philosophy, the "identity of indiscernibles" is often referred to as Leibniz's Law. It has attracted the most controversy and criticism, especially from corpuscular philosophy and quantum mechanics.
3. Sufficient reason. "There must be a sufficient reason for anything to exist, for any event to occur, for any truth to obtain."
4. Pre-established harmony. The appropriate nature of each substance brings it about that what happens to one corresponds to what happens to all the others, without, however, their acting upon one another directly. A dropped glass shatters because it "knows" it has hit the ground, and not because the impact with the ground "compels" the glass to split.
5. Law of Continuity. Nature does not make jumps.
6. Optimism. God assuredly always chooses the best.
7. Plenitude. Leibniz believed that the best of all possible worlds would actualize every genuine possibility, and argued in *Théodicée* that this best of all possible worlds will contain all possibilities, with our finite experience of eternity giving no reason to dispute nature's perfection.

PHYSICS

Robert Hooke 1635-1703

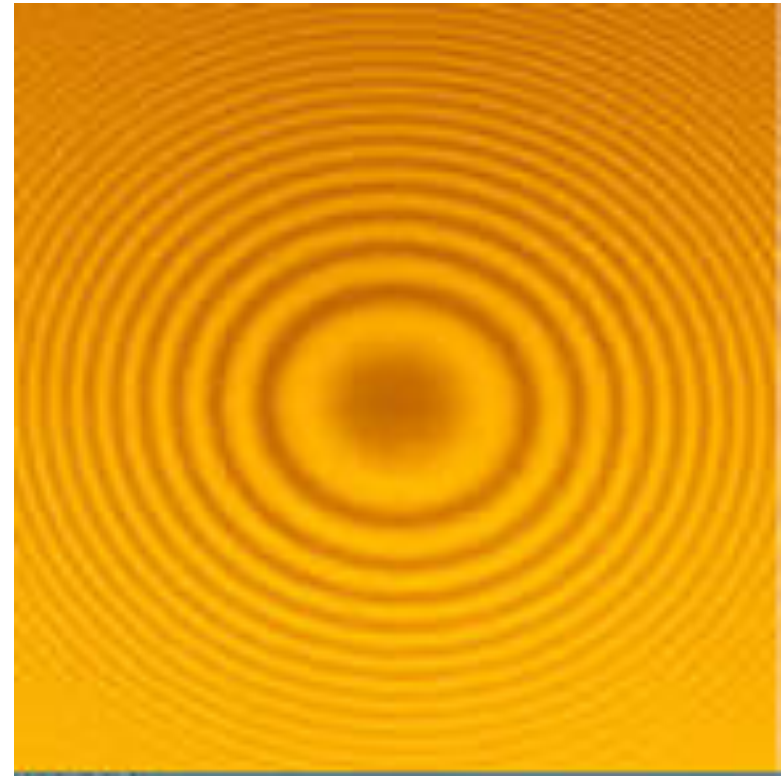
An English naturalist, who designed and built a two-lens microscope. Using sunlight or alcohol lamp for illumination, he described cellular structure of plants. He also investigated fossils and credited by the paleontologist Charles Lyell. He wrote the law for a force exerted by springs $F=kx$, and advanced the development of spring watches with an anchor on a ratchet that releases one tooth at each cycle.



Hooke Defined heat as the motion of body components. He was also engaged with astronomy, and recognized gravity without knowing its distance dependence. He described the law of forceless motion at straight line and speed before Newton.

He discovered the “black point” in soap bubbles (related later to light interference). Proposes (independent from Grimaldi) that light emerges from fast and short cyclic motion, that propagates in straight lines through homogeneous medium. **1667** reports light interference rings between two glass slides (see image). This was first noted by Boyle, but named “Newton rings”. The rings are explained by refraction and interference of waves moving through ether. Hooke proposes (wrongly) that white light is the simple wave perturbation, and colors are complex distortion of white light. This was only disputed after the prism dispersion of white light into colors by Newton.

Hooke was employed by Christopher Wren 1632-1723, the great Architect of London, as a surveyor after the big fire at 1666.



William Gilbert 1644-1703

Was the court physician of Queen Elizabeth. Many medical people believed at the 16th century (and some even today...) in the healing power of magnets. Gilbert discovered that earth is a huge magnet and explains the operation of the compass. He also studies electric charges, and builds a compass with a non-magnetic needle that was displaced in a magnetic field, similar to an electroscope. He called the objects that moved his needle “Electrical”, probably the first to use this term taken from the Greek “electrum” the name for Amber since by rubbing it became charged and attracted feather fractions. He did not discriminate between “magnetic pole” and “electric forces”.



Sir Francis Bacon 1561-1626

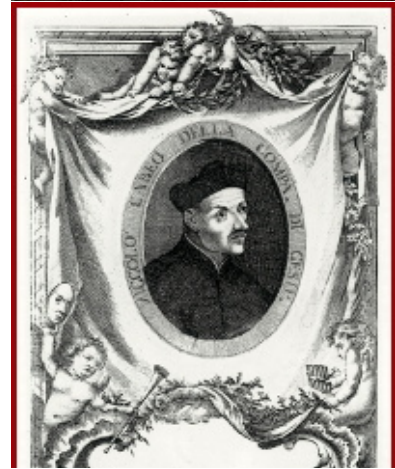
1605 publishes a textbook “the scientific method of research”.

1620 Thorough study about heat concludes that it is linked to motion.



Niccolò Cabeo 1586-1650

16329 An Italian. Discovers that electric charges come in two types, and can attract or repel each other.



1660 Public telescope competitions

Development of optics at the 16th century, and the application of the prospering eyeglasses industry to telescopes and microscopes charmed not only scientists and science amateurs, but also Emperors, Nobility and the Rich. The telescope was most attractive since its field capabilities could be easily demonstrated.

At the 17th century at Rome, a competition was announced. The competitors provided telescopes, that were tested by reading from increasing distances mixed-order words from texts by Petrarch, Dante and Aristo. They also used resolving tests pattern prepared by del Cimento academy in Florence, similar to the ones opticians use today fitting eyeglasses.

Telescopes built by the famous instrument builder **Eustachio Divini** were tested against the telescopes of the young clockmaker **Giuseppe Campani**. Although no winner was decided, Campani gained fame and publicity.

The two telescopes were bought for the Medici collection, and are on display today at the Science Museum in Florence.

CLOCKS AND TIME

At the end of the 15th century clockmakers in Nuremberg replaced pendulums with springs. The inventive development provided smaller carry-on watches, as well as marine chronometers, essential for determining latitudes and longitudes during mid-ocean sails.

1335 A mechanical clock in Milan

1581 Galileo studies pendulum cycle times, its dependence on length but independence on mass. He reports to the Dutch authorities. This information must have reached **Huygens**.

1659 Jacopo di Michelangelo Viviani 1622-1703 draws a clock by the instructions given by Galileo to his son **Vincenzo**. The Florentine clockmaker **Eustachio Porcellotti** use these drawing and built “time measurer“

1656 Christian Huygens builds a pendulum clock. All Parisian clocks are synchronized.

Can you try to imaging our world without world-wide synchronized time ?

Christiaan Huygens, 1629-1695

A Dutch scientist. Among his many inventions:

The rings of the Saturn, and its satellite Titan,
Orion constellation and few stars the belong to Orion,
Distance of stars based on their brightness (wrongly
assuming they have similar sizes and temperatures),

He invented the spring pendulum, and described
harmonic vibrations and the time of its cycle:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

He also studies string vibrations, and used Logarithms
to define well tempered harmonics. String vibration frequency, f , is inversely
proportional to length, L , and depend on the square root of the tension, T :

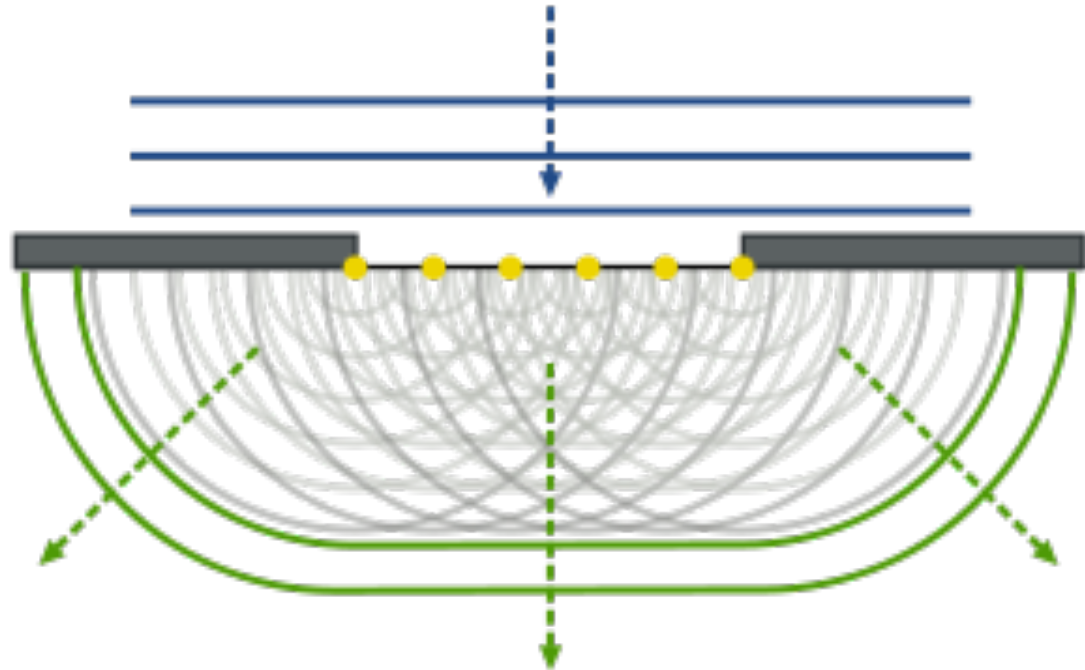
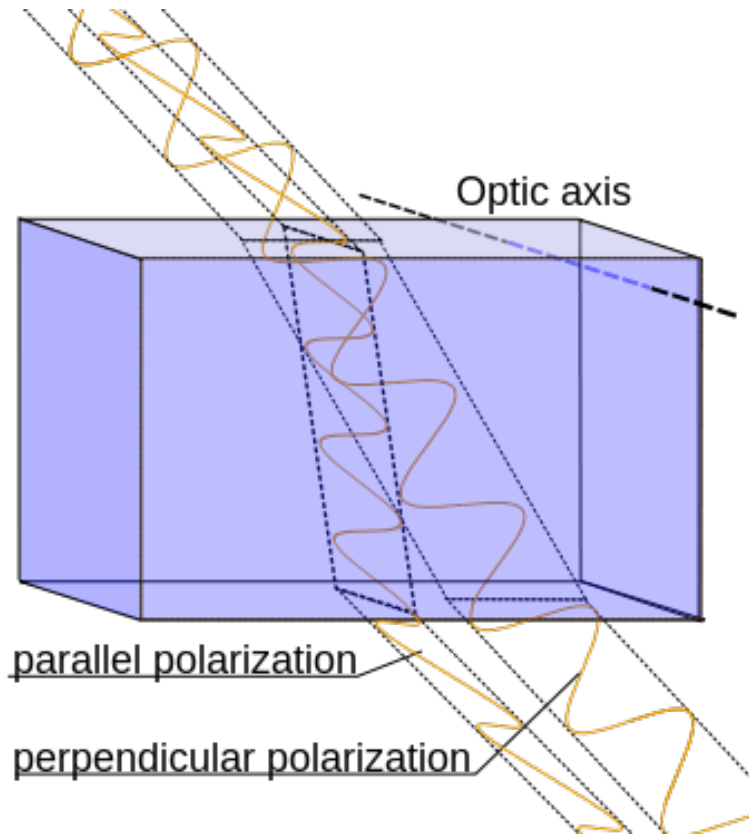
$$f = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

μ is the string mass per unit length.

Huygens calculated the path of projectiles, and wrote Newton's Second law
of motion. He expresses the centripetal force $F=mv^2/r$ and concludes from
Kepler laws that Gravity has square dependence on distance.



In optics Huygens-Fresnel principle builds light propagation wave-front based on Rømer's finding of light velocity, c . Each point on the present wave-front is a light source extending a spherical wave from it with radius $r=c*\delta t$, and the new wavefront is the envelope of these spheres.



1690 Since wavelength alone cannot explain birefringence, viewed in Calcite crystals, Huygens deduced that light is a lateral wave with two possible polarizations. Interesting that the theory used pulsed light, not continuous waves.

1651 Investigates optics of spherical lenses in telescopes, and was inspirational to **Herschel**.

Ismael Bullialdus

1640 - Proposes square distance dependence of gravitational force.

Cavalieri

1647 - Relates lens radii of curvature to its focal length:

$$D = \frac{1}{F} = (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

Pierre de Fermat

1665 - Derived the minimal time principle for light path, which explains refraction.

Francesco Maria Grimaldi

1665 - discovers that light path in a narrow slit does not follow straight lines. Diffraction patterns can only be explained by the wave nature of light.

John Wallis

1668 – Proposes conservation of momentum.

1676-1689 - Gottfried Leibniz

1676 – Developed a (limited) theory for conservation of energy.

1673 — Ignace Pardies

1673 – Proposes a wave explanation of diffraction, based on change in speed of light.

René Descartes

Confirmed the angles of the primary and secondary rainbows with sun direction.

AVIATION

John Wilkins, Bishop of Chester 1614-1672

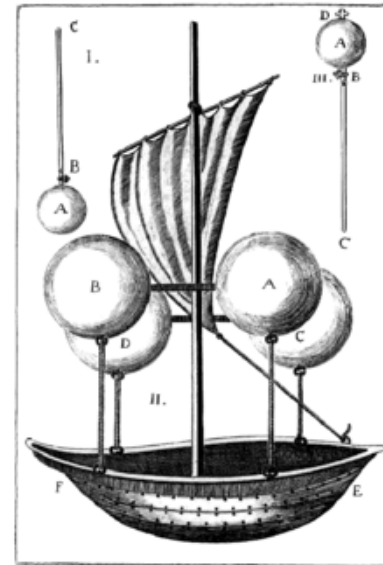
1638 - proposes what will become a missile in his book “Discovery of the world of the moon”.

Evliya Çelebi (Derviş Mehmed Zillî) 1611-1682

1638 - The Ottoman explorer, tells in his travel stories about his floatation with artificial wings of **Hezarfen Ahmet** from Galanta tower in Istanbul over the Bosphorus, landing in Dogancılar square in Üsküdar.

Jesuit Francesco Lana de Terzi 1631-1687

1670 Proposes a design for Flying boat, but God would not allow to build it, since every city could be conquered.



Jacob Besnier

1678 – A French blacksmith who reports flying with a flipping wings machine.

HEAT ENGINES

Johann Joachim Becher

1669 – Studies minerals. A theory for burning: oily soil causes fires: the idea of phlogiston – the liquid of heat.

Christiaan Huygens 1629-95

1673 – Builds an engine using the force of gunpowder explosion.

Denis Papin 1647-1712

1679 - Develops a closed container and measured increase in boiling point at increasing pressure, demonstrating the dependence of boiling point on height.

1690 – Invents a pump with a steam-driven piston

Thomas Savery 1650-1715

1698 – Steam engine based on the vacuum created by steam condensation. Registered a patent. Was implemented to pump water out of coal mines, and to supply water to villages at high altitudes.

Thomas Newcomen (1664-1729)

1712 – A steam suction piston engine, pumping water from mines.

Engine technical developments brought the industrial revolution.

Edmond Halley 1656-1742

Had an amazing career. Was a Captain on boats, drew maps (a cartographer) and professor of geometry in Oxford. Headed the royal mint, and was the royal astronomer. Invented the diving bell, wrote about magnetism, tides, planet orbits, and medical effects of Opium. Invented weather maps, life expectancy tables, methods of dating earth and its distance from the sun, and a method to preserve fresh fish.

1682 – observed a comet and realized it is the same reported in years 1456, 1531 and 1607. The comet was named at his honor after he died.

During a dinner with Robert Hooke and Christopher Wren they offered a prize for explanation of the elliptic orbits of the planets. Halley brought the problem to Newton, who claimed he already worked it out, but only after a few months he eventually sent them the solution, connecting the elliptic orbits to the square dependence of the Gravitational force.

1693 – Halley writes the lens equation for object, o, and image, i, distances for a lens of a given focal length, f:

$$1/o + 1/i = 1/f$$

John Hadley 1682-1744

Parabolic mirror, used in Newton's telescope.

ELECTRICITY AND MAGNETISM

At the beginning of the 17th century magnetism and electric charges were confused. Both showed force exertion from a distance, (maybe the driver to understand gravity too). Several methods were successful in charging bodies with electricity (including friction of certain isolating materials, and lightening). They enabled to discover the two kind of electric charges with attraction or repelling between them, depending on both kinds.

But only when continuous generation of charges was invented, conduction and discharge of electricity by metals was discovered, as opposed to magnets that can only be broken into two each having two poles.

ELECTRICITY

Otto von Guericke 1602-1686



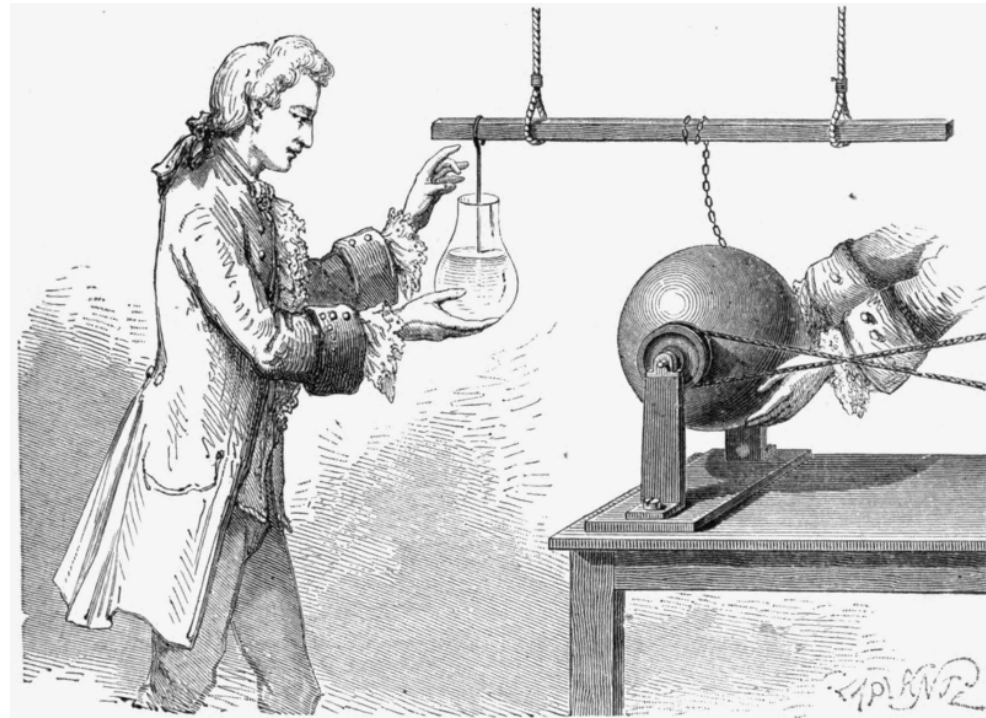
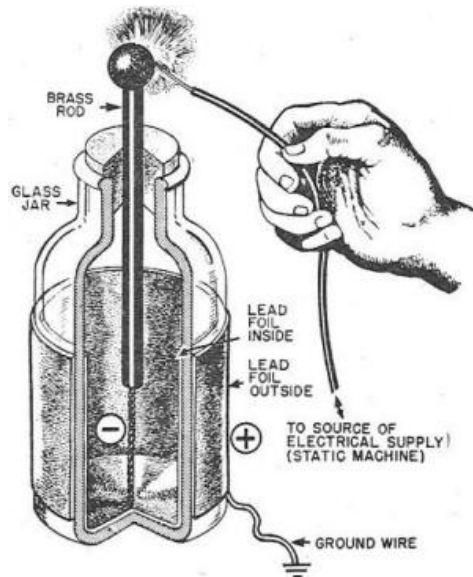
1677 Electric generator from friction. Produced unstable electric current.

Stephen Gray 1666-1736

1729 Conduction of electricity in metal wires

Pieter van Musschenbroek from Leyden

1745 Charges electricity in Leyden Jars. Covered the inside and outside of a glass jar with thin lead plates. The first capacitor, with increasing capacitance for thinner glass between the plates. Could be charged by thousands of volts, and provided a stable source of electric current for experiments.



Benjamin Franklin 1706-1790

1749 In America “catches” lightening with a kite, and almost died from the electric shock.



Henry Cavendish 1731-1810 (see Enlightenment)

Hits himself by electric shocks till loss of conscience.

Formulates in his notebooks the laws of electrical conductivity such as Ohm's law. But were only discovered by Maxwell at the end of the 19th century. He also discovered without publishing many findings later rediscovered without knowing Cavendish work: Laws of energy conservation, Dalton's law of partial pressures, Richter's law of Reciprocal Proportions, Charles' gas law, Freezing mixtures (Larmor, 1915), local cooling of the atmosphere (Kelvin & G.H.Darwin), Predicted existence of Nobel gases (Rooseboom) equilibrium in reactions (Pickering) as well as the effect of friction in tides on the slowing down of earth rotation.

The famous lab in Cambridge carries his name.



GASES

Otto von Guericke 1602-1686

A physicist and the mayor of Magdenburg. Determines the weight of air. 1654 builds a vacuum pump. Demonstrates to the German Reichstag the power of vacuum, holding two half spheres made of strong metal "Magdeburger Halbkugeln", against 16 horses pulling on it. Reminding the release of the Gordian Knot by Alexander the Great: a sharp knife and a prick of a needle will annihilate this holding power. Guericke found that a bell does not sound in vacuum, and a candle cannot burn. He built an electricity generator, and conducted electricity to a distance of several meters using a wet thread.



PHYSICAL PROPERTIES OF GASES

1662 Boyle Marriotte Law

$PV = \text{const}$

1699 Amontons' law

$P/T = \text{const}$

1780 Charles' law & Gay-Lussac

$V/T = \text{const}$

Robert Boyle 1627-1691

1660 – Uses a vacuum pump that he developed and finds experimentally „Boyle's law“ stating that volume times pressure is kept constant. The law appears in an earlier publication of **Henry Power**.

Discovers that sound is not heard in vacuum, and concluded that air acts as a spring to convey sound.

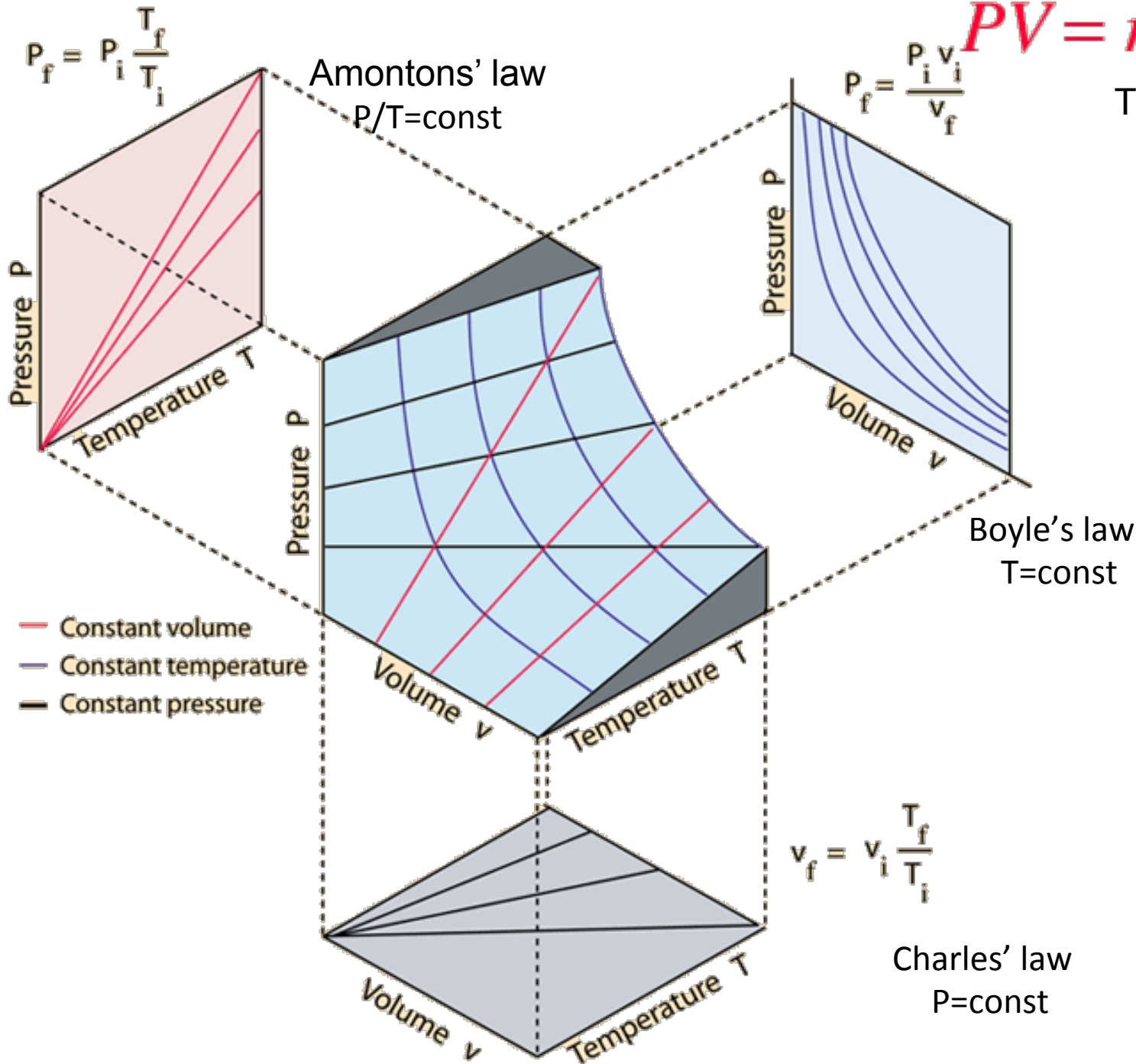
Edmé Mariotte 1620-1684

1676 Discovers independently Boyle's law, and adds that it is correct only at a constant temperature.

BAROMETER – see LIQUIDS

$$PV = nRT = NkT$$

The state equation
For ideal gases



LIQUIDS

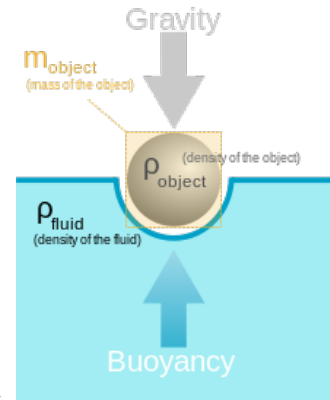
PAST LANDMARKS IN FLUID MECHANICS

Intuitive understanding of liquid properties is shown in the design of boats, arrows and irrigation channels.

287-212 BC Archimedes describes the law of floatation. Understands that liquids exert equal forces at all directions. Designs Archimedes screw – to pump water.

120 BC In Alexandria of the Ptolemy rulers – siphon and compression pump with valves developed by Hero and Ctesibius.

Wheels with buckets to pump water were common in Egypt, and valves in the buckets were introduced to facilitate their sinking into the Nile water.



Sextus Julius Frontinus 40-103 B.C.

The senator Frontinus designed the aqueduct to Rome. He understood that the amount of water flowing out of a hole depends not only on the size of the hole, but also on its depth below the water level.

Al-Khazini 1115–1130 Abu Rayhan Biruni 973–1048

Developed experiments to study liquids. Biruni finds the difference in density of pure and salt water, and cold and hot water.

9th century - Banū Mūsā three brothers in Baghdad design valves that open and close water flow to fountains.

1206 - Al-Jazari composes inventions notebook with hydraulic systems with feedback, pumps that lifts water with two cylinders and linked valves – one open and one close.

Leonardo da Vinci 1452-1519 – describes capillarity in thin pipes.

Galileo Galilei 1564-1642 - liquid dynamics, free fall in liquids, syphon, water flow in rivers and pipes.

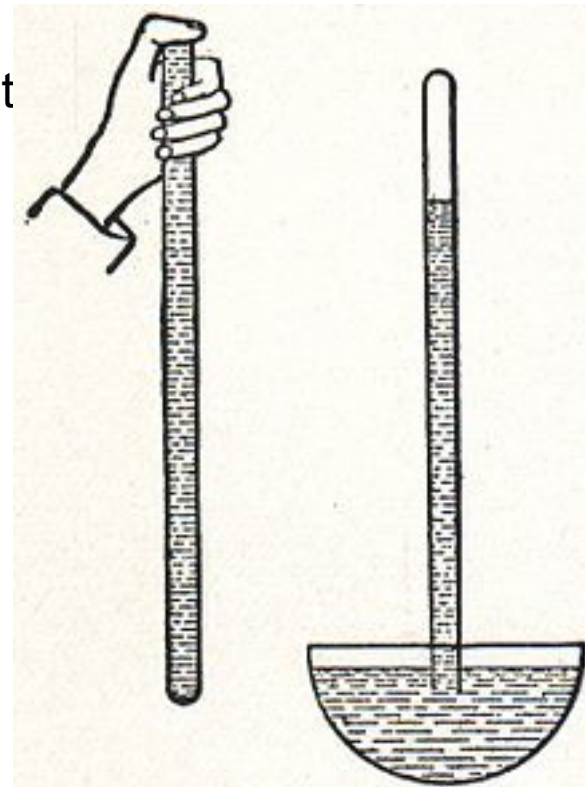
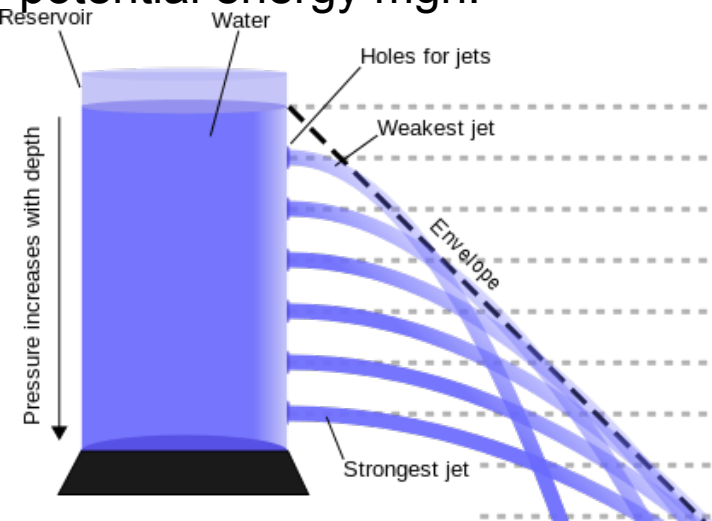
17th century

Evangelista Torricelli 1608-1647 Galileo's student.

1644 Invents the Mercury barometer. Creates Vacuum above The Mercury column, and understood that it is not held by the vacuum above it, which changes length depending on the tube, but by the atmospheric pressure, and the height of the Mercury column changed only when it changed. Mercury was heavier than water, allowing shorter column, and has low vapor pressure, diminishing temperature effects.

1643 Torricelli's law: pressure only depends on the weight of the liquid above.

1648 Raffaello Magiotti confirmed experimentally: Speed of liquid flow from holes in a barrel depends on the square root Of the liquid height above the hole: $v = \sqrt{2gh}$. This is a result of equality between liquid kinetic energy $mv^2/2$ and potential energy mgh .

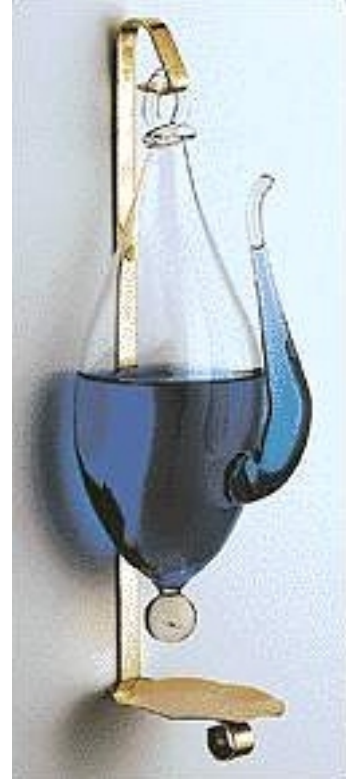


Johann Wolfgang Von Goethe 1749-1832

Goethe (yes, the same one who wrote “Faust”) was a “phenomenological scientists”, and his interpretations of nature (color theory, geology) are generally wrong. Yet, his interest in natural sciences besides being a Politician and writer, must be admired.

Goethe built a differential barometer, using water, does not require Mercury (that is forbidden in use since 2007), yet without the need for 10 meter long column...

Modern barometers use metal springy capsule with vacuum inside, That is stretched by a spring. Changes in external pressure cause it to Shrink or expand, and the motion is conveyed to a dial. In old weather stations the dial is a pen scribing on a rotating drum (1 rotation per day). Today the recording is digital, and sent to a center on line.



Benedetto Castelli 1578-1643

1629 studies fluids in motion, especially water flow in channels and in pipes.

Edme Mariotte 1620-1684

1676 Boyle-Mariotte law: $PV = \text{const}$, and note the constant changes with temperature. Discovered the blind spot in the eye, Studies recoil of cannons, freezing of water, barometers, light colors, And trumpet sounds. Was the designer of the fountains in Versailles, an impressive hydrodynamic project!

1685 write about flow of water and other liquids, published after his death



Domenico Guglielmini 1655–1710

Friction between flowing water and the channel walls or pipe.

Gassendi

1649 demonstrates changes in Torricelli's barometer Mercury column height with altitude.

Blaise Pascal 1623-1662

1646-7 After hearing about Torricelli's experiments with barometers, and reproduced his Mercury barometer, he asked what determines the height of the Mercury column, and what is above it in the closed tube end. According to Aristo moving objects are driven by the medium they move in. Thus, propagation of light through the tube region above the Mercury "prevents existence of vacuum". He assumed that region contains Mercury vapor, and therefore expected that alcohol with larger vapor pressure will have much longer volume above its column. Yet, when Pascal measured height for different liquids, he found that it is only determined by the length of the tube.

1648 Pascal rediscovers that atmospheric pressure decline with altitude (his brother-in-law made measurements on top of Puy de Dome), and concludes that above the shell of the atmosphere there is vacuum.

Pascal's law in hydrodynamics: He demonstrated that pressure on a piston depends on weight per unit area, and not total weight on the piston. He concluded that pressure in liquids act equally throughout all its volume, and create force perpendicular to the container. He demonstrated that water cannot be pumped from a pipe to height above 10.3 meters. He also showed that pressure only depend on height of liquid, since the barrel leaked from the water pressure of a thin pipe, although water weight in it was small.

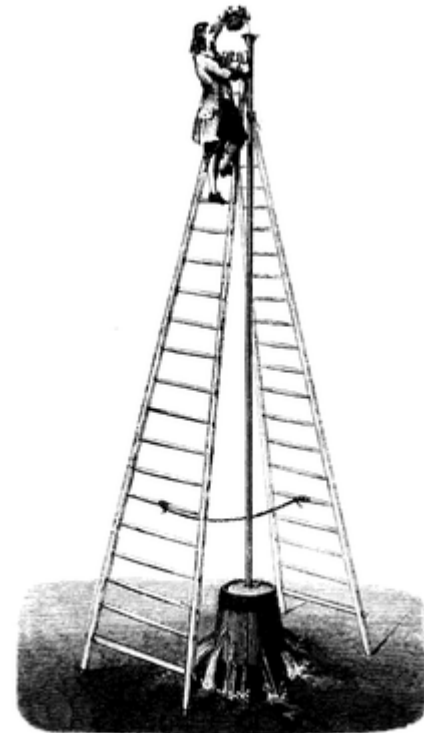
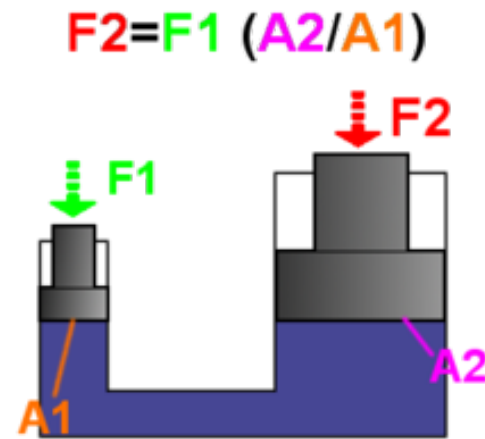


FIG. 45.—Hydrostatic paradox. Pascal's experiment.

Isaac Newton 1642-1727

Defines shear strain in solids and liquids. Studies flow from holes. Experimented with waves in liquids

Change of the velocity u of liquid with distance y from a wall: $\tau = \eta \, du/dy$

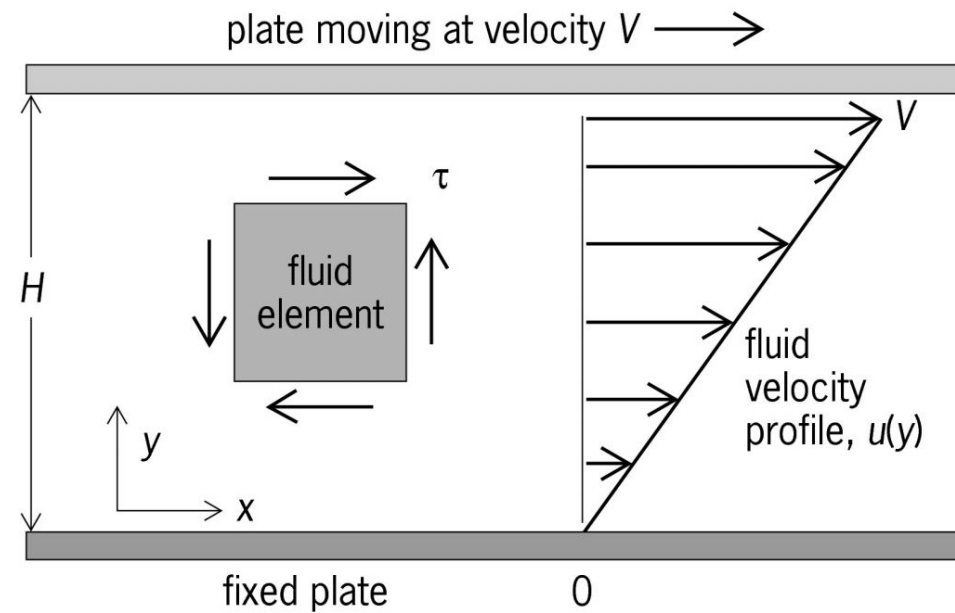
τ Drag

η Viscosity

du/dy Derivative of flow speed perpendicular to flow direction

Force, f , to drive a plate at a constant speed, with liquid filled between this plate and a stable plate below:

$$f / \text{unit area} = \eta \, V/D = \eta \, du/dy$$



Robert Hooke 1635 – 1703

1678 Spring force-elongation relation: $F=kX$

Since springs distortion is large and easy to measure, Hooke's law preceded the laws of stress strain relations in solids (next slide).



Within the elastic regime linear relations:
 (for small distortions. Breaks for large distortions):

Distortions by force acting perpendicular
 to the block surface

$$E \delta \ell / \ell = F_n / A$$

E – Young modulus

A

Shear distortion, by force acting parallel
 to the block surface

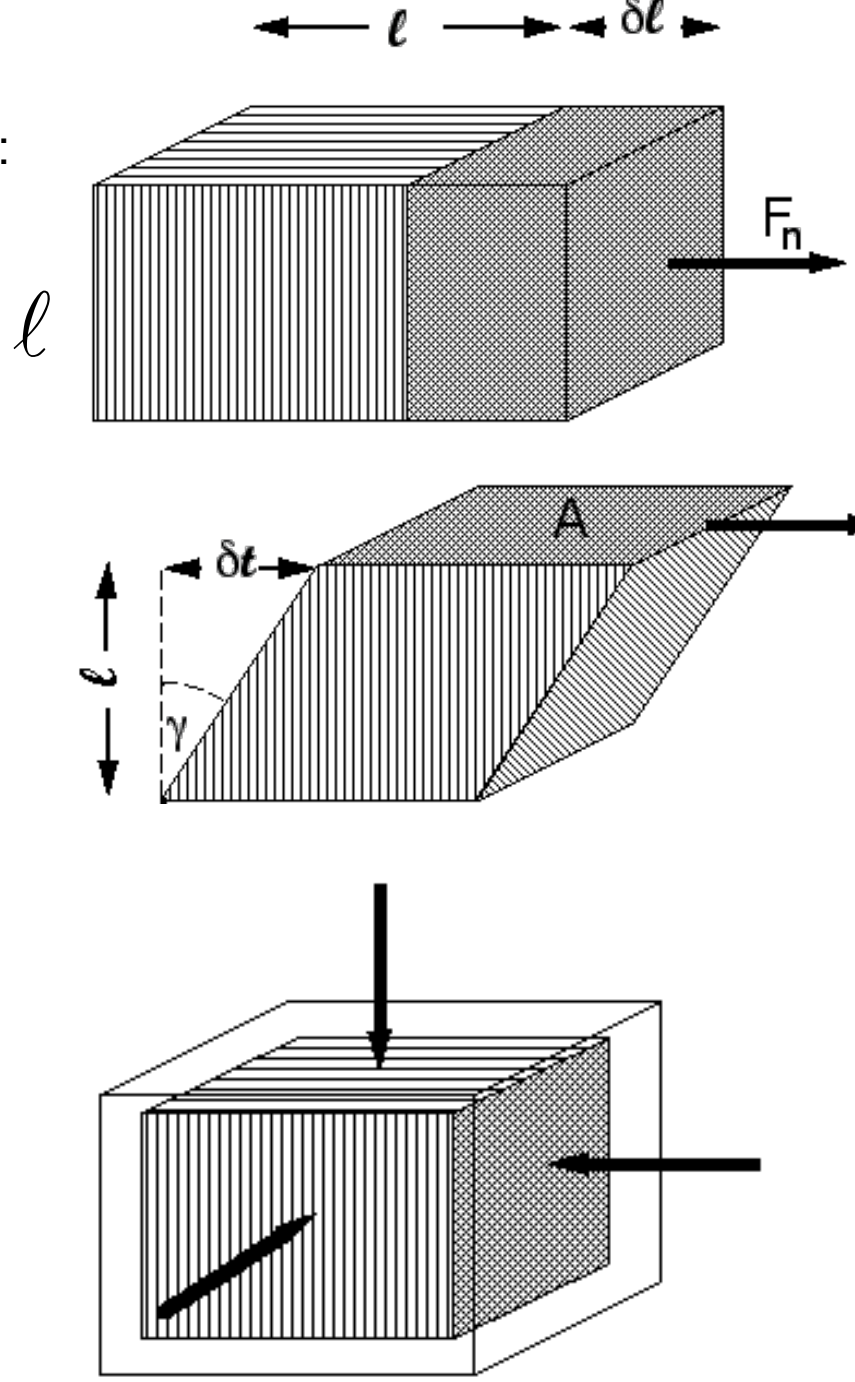
$$F_t / A = G \gamma$$

G – Shear modulus

Change in volume due to hydrostatic pressure:

$$P = K \delta V / V$$

K – Press modulus

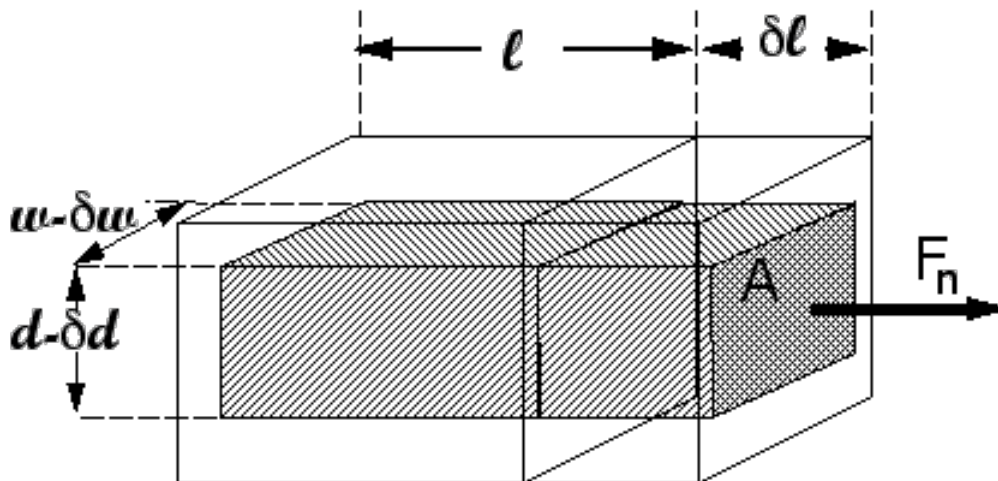
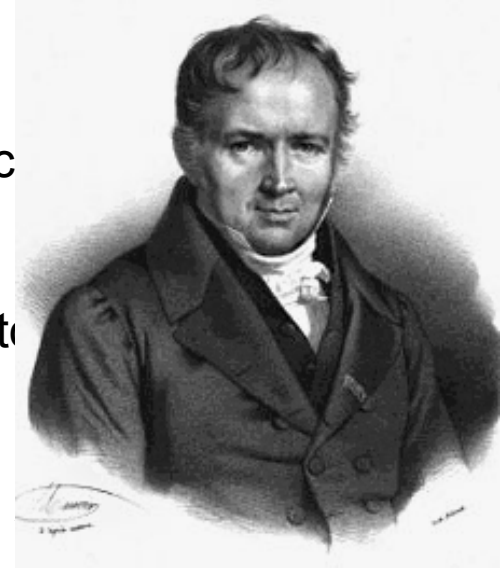


Siméon Denis Poisson 1781– 1840

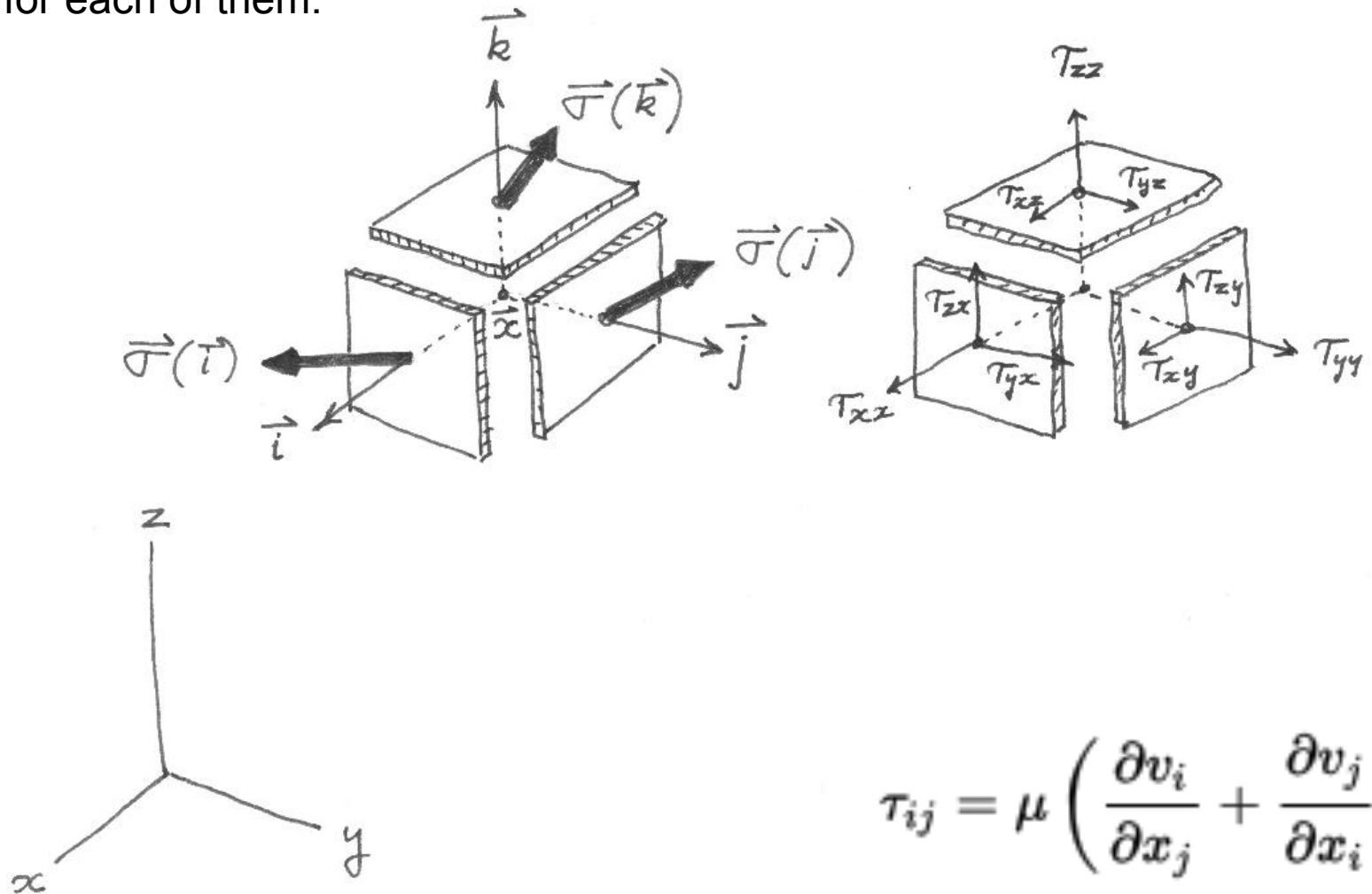
Poisson understood the relation between the different moduli, since if a solid is expanded in one axis, it will shrink in perpendicular directions.

Poisson ratio, ν , is the ratio between the relative change in length to the relative change perpendicular to it.

$$\nu = [\delta\omega/\omega]/[\delta\ell/\ell] = [\delta d/d / \delta\ell/\ell]$$

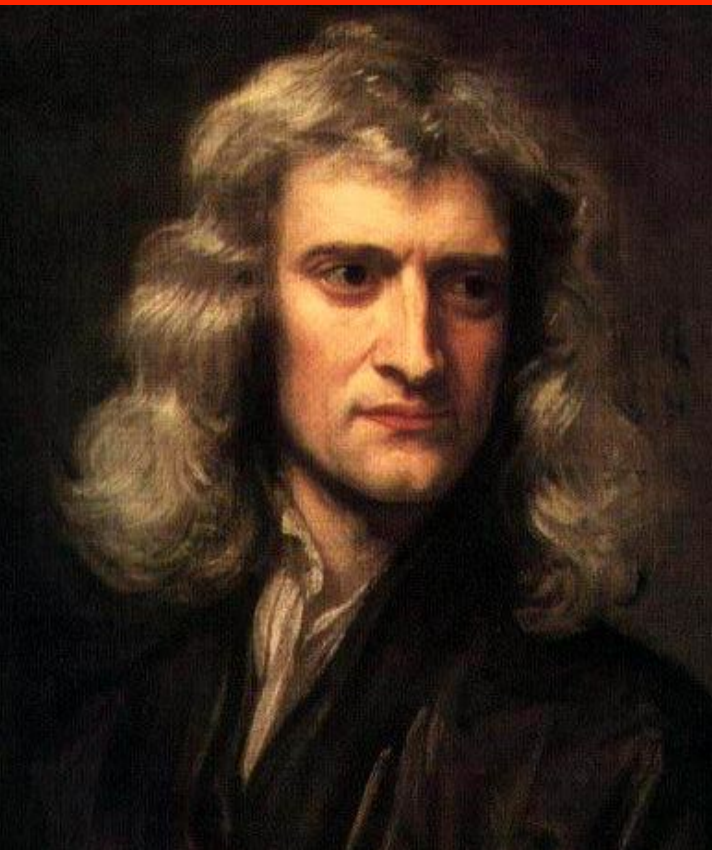


The actual strains and stresses inside solids are defined by combinations of the above moduli. In future centuries, general tensor equations were described, to relate force at any direction with the distortions in all directions. The solutions of such equations are of seminal importance in engineering, avionics, and mechanical engineering. They are done by computers using software called “**finite element analysis**”, since the body studies under forces is divided into small cubicles, with forces and distortions calculated for each of them.



$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

NEWTON



Isaac Newton 1642-1727

Newton was born in Lincolnshire district, England at the year Galileo died. At school he fancied Chemistry and Pharmacy.

1661 he studies mathematics in Trinity college, Cambridge, and paid his tuition as a butler and room cleaning services for rich students.

1664 He was awarded a scholarship. 1665-6 schools closed due to the plague, and Newton returned to his hometown, and came up with ideas about Gravity (the falling apple...) and light colors, but did not publish them until he was threatened to lose priority credits.

1667 returned to Cambridge, and worked in Chemistry. 1668 read **Nicolas Mercator's** book about infinite series, and wrote his own extended work, which **Isaac Barrow**, his tutor, rushed to send for publication in London.

1669 Newton is appointed as a Lucasian Professor of Mathematics, the prestigious chair founded in 1663 by Parliament member for Cambridge Henry Lucas, and previously held by Isaac Barrow. (Lately Stephen Hawking held this professorship).

1672 Newton introduced his telescope at the Royal Society, with parabolic mirror he polished himself. He also presents his theory on white light dispersion into colors in prisms and lenses (chromatic aberration). As a result he is appointed a Society member.

James Gregory 1638-1675 and **Guillaume Cassegrain 1625-1712** Built before Newton similar telescopes with Parabolic and Elliptic mirrors with a center hole for the eyepiece.

1670 Newton is interested in religion: "Christianity drifted away from the source". King Charles II issues a special regulation that the holder of the Lucasian chair is dismissed from prayers...

1684 Three members of the Royal Society:

Christopher Wren 1632-1723, Robert Hooke 1635-1703 & Edmond Halley 1656-1742

Discuss if the elliptical orbit of planets is connected to the square law dependence on distance to the sun. Halley traveled to Cambridge to present the problem to Newton. Newton claimed he already solved the problems years back, but failed to find his notebook. He sent the solution three months later...

1686 The Royal Society published “The history of fishes”, spending a fortune without selling the volume. They therefore missed funds to publish Newton’s “PRINCIPIA”, and the publication was funded personally by **Halley**, the society secretary (who, in addition, received volumes of “fishes” instead of the secretary salary...).

But the publication of “Principia” turned Newton into a celebrity. He moved to London, appointed the Lord of the Mint, and got knighthoods from Queen Ann.

Newton delayed publishing the third part of “Principia” due to the dispute with **Hooke** over priority of the square law dependence of Gravity on distance. Halley tried to mediate, but the struggle between Newton and Hooke continued until Hooke died 1703. Similar dispute over priority of differential calculus with **Leibnitz**.

Careful scanning of works by earlier scientists indicate that almost all of Newton’s claimed new inventions were proposed before him. Yet Newton was unique in his ability to present the scientific community with complete theories clearly and convincingly. Yet, **Newton’s** mechanical theory was not accepted readily. The translation from Latin to French by Emilie du Chatelet (who was Voltaire’s lover) helped its distribution and acceptance in France even before it did in England.

Newton must have been quite esoteric character. In his attempts to understand vision he pricked his eye with shoemakers needle and scrabbled the eyeball borders. Fortunately, he missed the retina... he also fixed his view on the sun for long time, and needed to enclose himself in a dark room for long time in order to recover. Probably the foggy weather in Cambridge attenuated the sunlight and prevented his permanent blindness.

Despite his personality, Newton became later a parliament member and the president of the Royal Society.

Newton's scientific contributions span mechanics, optics, mathematics, chemistry and theology. He built telescopes, and although he hardly used them for observations, he was responsible for the astronomical revolution replacing the classical heritage, and mainly applying terrestrial laws of mechanics with Gravity to describe the orbits of celestial bodies.

1727 Newton dies, and buried in Westminster Abbey, despite the explicit resent of the Anglican church authorities.

Newton's three laws of motion

The First law: Every body persists in its state of rest or motion in a straight line unless it is compelled to change that state by force impressed on it.

The Second law: Force is equal to change in momentum (mv) per change in time. For constant mass, force equals mass times acceleration, $F=ma$.

The third law: For every action, there is an equal and opposite reaction.

The third law means that if body A acts with force F on body B, body B reacts with force $-F$ on body A. Therefore there is no balance of forces on either body A or body B !!!

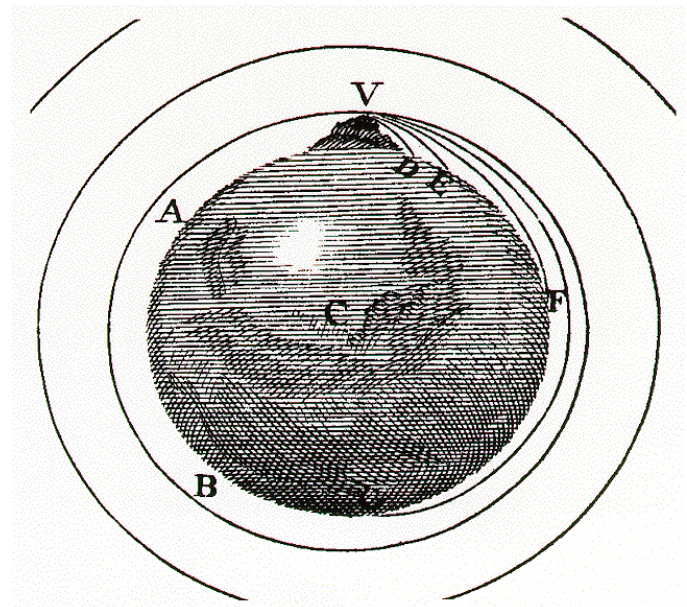
Aristo claimed that bodies fall in constant speed that is proportional to its weight. Since fall in air was too fast to measure, he deduced his law from fall of bodies in water, where, as we know, bodies reach a terminal speed due to frictional forces that balance gravity.

Galileo is known for his public demonstration of falling balls from the tower of Pisa, but he must have made many measurements on balls rolling down inclined slopes, since the motion was much slower. His conclusions:

1. Bodies accumulate speed in constant rate = constant acceleration.
2. If friction or floatation is negligible, all bodies fall with the same acceleration.
3. Balls rolling without friction at a constant speed and direction.
4. Vertical and horizontal motion can be quantified independent on each other, therefore the orbit of a freely thrown projectile is parabolic.

However, Galileo did not relate terrestrial bodies motion with celestial bodies orbits.

Newton concluded from shooting cannon shells That if he position the cannon on a sufficiently high mountain and the shell will be shot at a sufficiently high speed, it will orbit around earth like the moon. Therefore, he deduced, the same Gravitational force that makes bodies fall acts on planets to make them circle around the sun. Since gravity cause falling body to accelerate independent on their mass, the second law implies that gravity must be proportional to mass, and from the third law, proportional to the product of the two masses attracting each other:



$$F = Gm_1m_2/r^2 \quad G=6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

As we described above, Newton derived the connection between Kepler laws and the square dependence of gravitation on distance. He used the knowledge about Conic Sections described by the Greek geometers, and their second order equations.



Interestingly: Newton wrongly claimed that the sun is not attracted by other stars due to their isotropic and homogeneous distribution in the universe, overlooking the uneven “sky density” known as the milky way.

Lets make a calculation:

$R=384,000$ km the radius of the moon orbit

$r=6,350$ km the radius of earth, $r=R/60$

The gravity force to the moon is $1/60^2=1/3600$

Times the gravity force on the surface of earth.

The moon cycle is 27.3 days, at radial speed $V=1$ km/sec

Earth gravity acceleration $g=10$ m/sec

In one second a body on earth falls $gT^2/2=5$ m

The moon "fall" to earth $5/3600=1.4$ mm

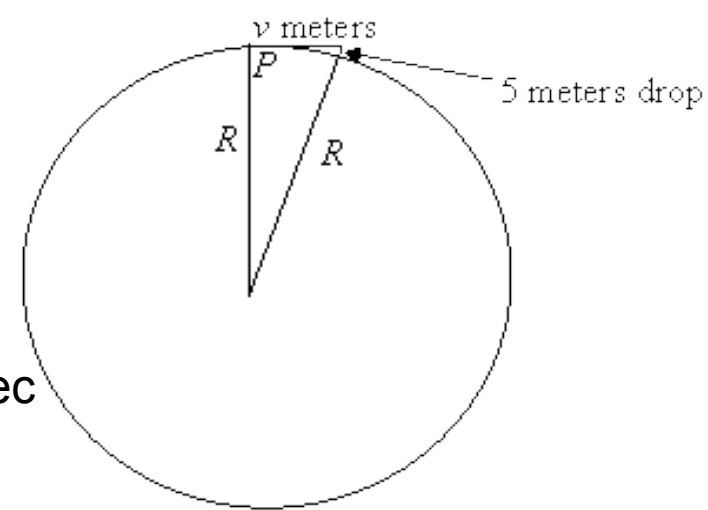
On earth surface a shell at 8 km/sec will "not fall down".

In a "stable" orbit gravity = centripetal force.

$$(R + 5)^2 = R^2 + v^2$$

What is the height of a satellite orbit where he stays over one point on earth ?

Why can a body orbit in elliptical path, and not only in a circle?



We can measure g on earth surface, and since $g = Gm_E/r_E^2$ we can know G if we knew earth radius and mass.

1798 Cavendish measured G from the minute force between two large lead spheres, using his torsion balances (see 18th century). He concluded that earth mass is 6 billion trillion tons (6×10^{24} kg), confirming Newton's estimate that was not based on experiments (today best estimate = 5.9725).

Effect of a third body on the elliptic orbit of a planets:

1781 Uranus was seen in telescopes, but deviated from its expected orbit, and another planet was suspected to be the cause.

1846 Neptune was discovered independently by **Urbain Jean Joseph Le Verrier** (1811-1877) and by **John Couch Adams** (1819-1892)

1930 Pluto was discovered.

1669 Newton publishes for the first time a paper about calculus, a method to calculate the rate of changes, and convert it to a summation problem. Two years later he proposes the terminology. The first manuscript was published **1711**, and the second at **1736**.

1671 Newton disproves **Hooke's** color theory by showing that white light is dispersed into colors by a prism, and that a single color cannot be changed to other color.

1675 Newton argues against light as vibrations in the aether (Huygens' light theory). He believes that something else propagates through the aether: not quite particles, but kind of induction of pulses that change density. With respect to Huygens' birefringence in Calcite, he finds analogy between the beams and magnetic poles, and call this property of light "polarization". Longitudinal waves, such as sound waves, cannot be polarized.

1704 Newton publishes "Optics" particle theory of light and colors.

1744 A letter to **Boyle** from **1679** is published, describing aether as air resistance that sets aside two balls while they approach each other. The aether also holds them together, last explanation to electric attraction forces.

1684 Newton proves that planets are attracted to the sun by gravity force inversely proportional to the distance square, which explain the elliptical orbits according to Kepler laws. 50 years later, **Bernoulli** proves also the inverse statement: orbits that are conic section implies square law dependence of gravitational forces.

1686 Newton use a pendulum clock with constant length and varying mass to prove at high precision (1 in 1000) the “weak principle of equivalence” first stated by Galileo: all bodies free fall have the same acceleration, independent on their mass. This principle means that the gravitational mass is the same as the accelerated mass.

The principle was extended by **Einstein** in his relativity theory, claiming that force and acceleration are equivalent (or a force in one system can diminish an accelerated system).

1687 Newton publishes the physics of classical mechanics, for terrestrial and celestial bodies, and employs infinitesimal calculus to formulate its laws.

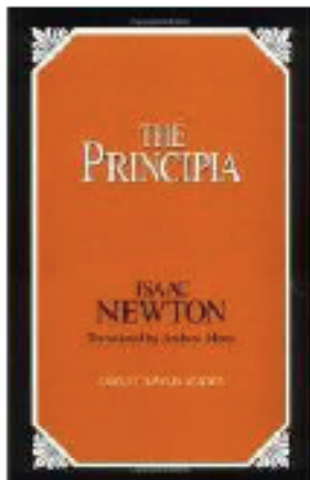
In a second book he presents drag forces in liquids. He considers solids as composed of closely packed atoms, therefore held by strong forces.

1713 A new edition of “principia” referring to aether as the mediator of forces between neighboring particles.

SUMMARY OF NEWTON'S CONTRIBUTIONS

1. Unification of terrestrial laws of mechanics (3 laws) and celestial laws (+Gravitation).
2. Use of calculus to express dynamic laws.
3. Belief in absolute coordinate system.
4. Belief in the atomic model. Material properties reflect attraction between atoms.
5. Light is also consisted of particles (not waves).

In addition to the use of mathematics, following **Galileo** and **Descartes**, to formulate physical laws, he presents causal physical theory based on experimental quantification of nature behavior, following **Gassendi, Roberval, Boyle, and Hooke**. He also used mathematics to extend physical laws from simple measurable cases to all cases, for example from the moon to all planet motion.



“Principia”
Newton’s book
that is still
a best seller
Today.

CHEMISTRY

Michal Sedziwój 1566-1636

1605 Publish a book about alchemistry, proposing that air contains the nutrition for life.

Jean Beguin 1550-1620

1615 A first alchemistry book with chemical reaction equations.

Jean Rey 1582-1645

1620 measures a small increase in weight of Lead and Tin when heated to high temperature. Concludes that something from the air combine with the smallest particles in the metals.

Heating was commonly used to convert lime stone (Calcium Carbonate, CaCO_3) into CaO and CaOH_2 as building plaster.

René Descartes 1696-1750

1633 Discusses primitive elements (Air, Fire, Earth, Water) and their compositions.

Johann Baptista van Helmont 1580-1644

1648 His son publishes after his death a manuscript considered the transition from Alchemistry to Chemistry, and strongly influences **Boyle**. Describes many experiments, implies conservation of mass. Coined the term "GAS" and links the production of the "sylvestre gas" (CO_2) in burning charcoal to the same gas emitted during fermentation, and the gas makes the air in caves un-breathable. Metals digested by acids can be recovered.

Robert Boyle 1627-91

1661 Publishes “The skeptic chemist” defining atoms and molecules, primitive element, chemical reaction, chemical analysis, bases and acids.

1670 Discovers and isolates the inflatable gas (Hydrogen) emitted in reactions between acids and metals.

John Mayow 1641-1679

1674 Concludes from experiments with rats and candles that air contain two kinds of gases.

Hennig Brand 1630-1692 or 1710

1669 Tries to extract gold from urine, and discovers Phosphorus at **1674**.

Nicolas Lémery 1645-1715

1675 Publishes a chemistry book presenting the theory of corpuscles – atoms.

Johann Joachim Becher 1635-1682 or 85

A theory that a burning material release “fire-earth”. The elements are Earth, Fire, Air and Water.

Georg Ernst Stahl 1694-1734

1697 calls “fire-earth” phlogiston – the agent of burning, emitted in burning and in rust.

Abraham Darby I 1678-1717

1670 Develops an iron fusion furnace heated by Coke instead of coal. Coke is produced by heating charcoal in deoxygenated atmosphere, it is a porous grey stone, burning at higher temperatures than charcoal.

Henry Cavendish 1731-1810

The first to isolate pure Hydrogen, and demonstrate that its burning produces water.

Pilatre de Rozier 1754-1785

Blew hydrogen from his mouth into a flame, and his eyebrows were badly burnt.

Isaac Neuton 1642-1726

1692 Hierarchy of particles that cannot be broken, and complex particles. If we could decompose gold, it will become liquid. If we ferment gold it may turn into something else. Attraction forces between particles turn them into a variety of materials we find in nature.

SUMMARY

The 17th century marks a turn point in chemistry. Greek theories about primitive elements is confronted with controlled chemical reactions (mainly by heating) and quantitative measurements of mass changes that breed new models for particles and their composition into compounds. Air is understood to contain more than one recognizable gas component. Earth can create water in reactions, therefore is not a primitive element. Isolation of Hydrogen, Carbon dioxide, Phosphorus etc. pave the road to assemble a more realistic list of primitive elements in the next century, and develop analytical methods to identify chemical compounds.

Nevertheless, modern chemistry is still far ahead.

BIOLOGY and MEDICINE

Gaspar Bauhin 1560-1625

1596 publishes "Phytopinax" describes 6000 plants and classifies in a method predicting Linnaeus

Joachim Jung 1587-1657

1620 Precise definitions of plant organs and parts.

Gasparo Aselli 1581-1625

1620 discovers the lacteal vessel of the lymphatic system supplying fatty acids to the blood, and jugular vein in the neck.

Matteo Realdo Colombo 1516-1559

Describes the blood cycle

William Harvey 1578-1657

1627 - Disproves Descartes' description of heart functions, describes blood cycle, heart valves: the heart as a pump.

1628 - Studies blood dynamics from surgeries on animals.

1651 - All life emerge from the ovule, including mammals. No spontaneous creation of life.

Marc Aurelio Severino 1580-1656

1645 – Gills are for fish breathing. Integrates vertebrates, including mammals.

Santorio Santorio 1561-1636

1614 Takes his weight before and after eating, sleep, work, sex, fast, drink and defecation. Finds that most eaten weight is lost by perspiration.

Jean Baptiste van Helmont 1580-1644

Experiments with plants, showing they absorb food from water and air, and need light. Published by his son **1648**.

Physiological changes are mediated by chemical reactions. Bile liquids neutralize acid digestion.

Thomas Bartholin 1616-1680

1652 discovered the lymphatic system and its connection to the blood cycle.

Thomas Sydenham 1624-1689

mid 1660th – diseases are organisms inside the host. Preaches direct connection with patients, and classification of diseases and their causes. Uses Quinine and Opium as drugs.

Jan Swammerdam 1637-1680

1658 – discovered red blood cells using Leuwenhoek's microscope, probably built for him by **Johan Joosten van Musschenbroek** in Lyden.

Marcello Malpighi 1628-1694

1661 – Flow in blood capillaries. Studies lymph.

Thomas Willis 1621-1675

1664- Discovers that base ganglions in the brain control motion and sensory receptors. Discriminate between the grey cells in the cortex, and the internal brain white cells. Abolishes Galen's brain rooms model.

Robert Hooke 1635-1703

1663 – Observes through his compound microscope cell walls in thin cork sections.

Francisco Redi 1626-1697

1668 - In a series of well controlled experiments Redi adjudicated spontaneous creation of life, demonstrating that meat maggots (fly embryos) do not appear in containers well covered by cheese cloth.

Marcello Malpighi 1628-1694

1672 – The first description of chick development, including somites, muscle development, blood and nervous systems.

Nicolas Malebranche 1638-1715

1674 – A theory that every embryo is wrapped by his parents embryo...

Guillaume Lamy 1644-1683

1677 – An physician and Epicurean philosopher, uses soul and spirit interchangeably.

Anton van Leeuwenhoek 1632-1723

1674 – painfully perfects a single lens microscope, by carefully selecting glass beads he created by dropping melted glass into oil.

The microscope has fine screw mechanism to translate and bring to focus the object in front of the lens.

Leeuwenhoek saw protozoa, that he called “Animalcules”

1677 – Discovers sperm cells

1683 – Observes blood cells and bacteria. Denies spontaneous creation of micro-organisms, and believes they are carried in air.

First to describe nervous systems, and ants life cycle from eggs, to warms, to cocoon, and the adult ant.



Nehemiah Grew 1641-1712

1676 – Propose the true nature of ovules and pollen.

Daniel Sennert 1572-1637

1649 – Studied the corpuscular theory of matter. Fertilization is reunification of the separated smallest parts of the body.

Giovanni Alfonso Borelli 1608-1679

1679 – Describes the motile systems of vertebrates as mechanical devices: the father of biomechanics. Concluded that human muscles could not fly its mass like a bird. Believed that something penetrated the sick body, and can be cured by chemicals.

John Ray 1627-1705

1682 – Describes 18,000 different plants

1693 - Denies Descartes claim that animals have no emotions.

John Locke 1632-1704

1690 – describes brain as an empty tablet acquiring knowledge by the five senses. Believes man is created good, and has full rights to fulfill his desires.

Rudolph Jakob Camerarius 1665 -1721

1694 – Reported sexual fertilization of plants and flowers.

ONSET OF MODERN MEDICINE

Contagious diseases – epidemiology replaced by bacteriology
Surgeries with tranquilizers and anesthesia

Ambroise Paré 1510-1590

The father of modern surgeons – treatment of injuries in battle field.

Matteo Realdo Colombo 1516-1559 William Harvey 1578-1657

Understanding of the blood cycle.

Herman Boerhaave 1668-1738

The father of modern physiology.

Pierre Fauchard 1678-1761

The father of dentists.

Guy de Chauliac 1300-1368

A surgeon. Wrote influential text on surgery.

Realdo Colombo 1515-1559

Anatomist and a surgeon, described the small blood cycle from heart to lungs.

Michael Servetus ~1511-1553

Describe the small blood cycle.

Ambroise Paré ~1510-1590

“Barber-Surgeon”. Replaced cauterization - burning after limb amputation by ligature of arteries (first used by Galen). Inspected the action of bezoar stones, found in the guts of goats, and immersed by royalties in their drinks to prevent poisoning.

John Hunter 1728-1793

A surgeon.

Amato Lusitano 1511-1568

Describes Venus valves and their function.

Garcia de Orta ~1501-1568

First to describe cholera and other tropical diseases, and treatment by plant extracts.

Sir Thomas Browne 1605-1682

Physician and neurologist.

Thomas Sydenham 1624-1689

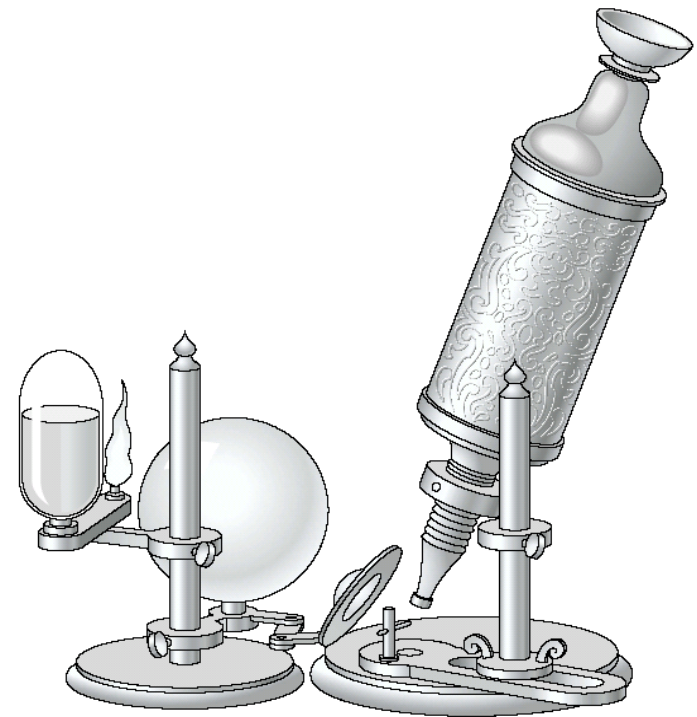
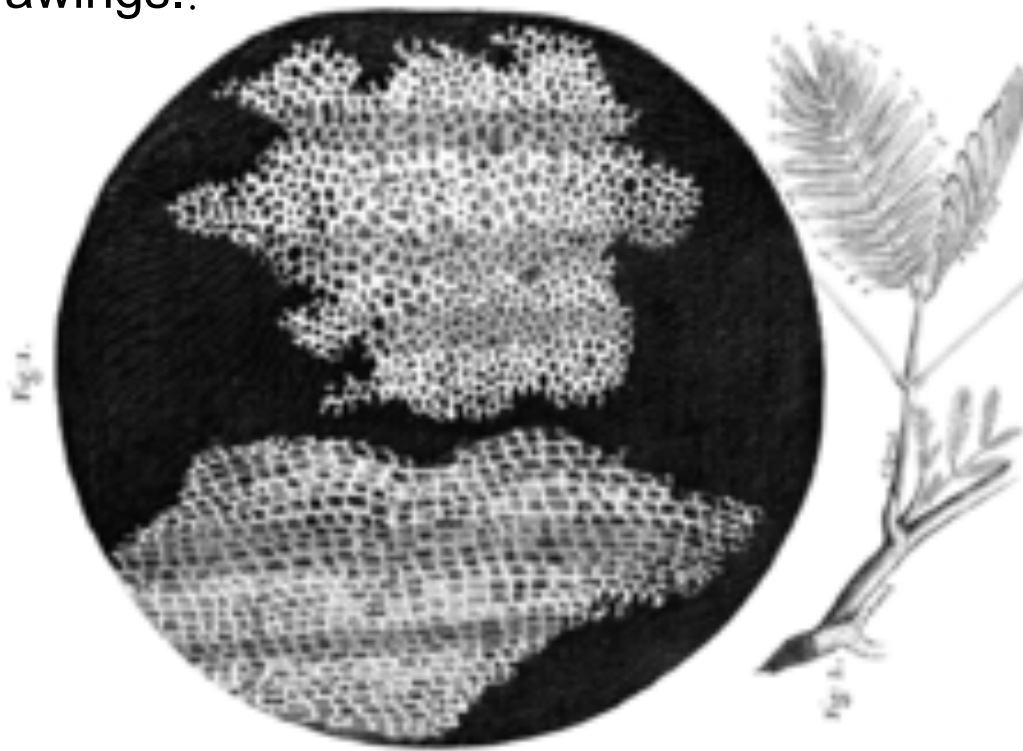
A physician called “The English Hippocrates”

Robert Hooke 1635-1702/3

1660 An English naturalist, designed and built a compound microscope – objective + eyepiece lenses. Light source was the sun, or alcohol lamp with concave mirror to concentrate the light on the sample.

1665 Observed boxes similar to honeycomb in thin slices of cork, and name it “cells”, small chambers. Since the Greek model of atoms was long abandoned, and materials were believed to be continuous, this observation that plants are segmented was sensational.

1667 compare plants to fossils. “Micrographia” is a collection of biological drawings..



Landmarks in the History of Microscopy

Interdisciplinary step-by-step progress in science

- 1900BC Egyptian queens use for cosmetics flat and spherical mirrors.
- Phoenicians Glass spheres filled with water divert light and magnify.
- Thales, 600BC Saw plant leave “cells” through morning dew droplets.
- Greeks Eye sends rays that “sense” the world/
- Alexandria school: +/-200, Euclid, Hero and Ptolemy optics book.
- Middle age Arab scholars: Ibn al-Haytham (Alhazen) physical nature of light.
- ~9C vision glasses in China & Arabia (Ibn Firnas) -> 15C Europe (Cusa).
- 14th century Developing art of lens polishing for eyeglasses.
- 1590 Zacharias Janssen, Holland (polish lenses), builds two-lens microscope/
- 1611 Kepler builds telescopes, suggests microscopes/
- 1655* Robert Hooke microscope - cork “cells”.
- 1674* Anton van Leeuwenhoek use 1.5mm glass sphere magnifiers – protozoa.
- 1683* Leeuwenhoek sees bacteria and blood cells.
- 1733 Chester Hall use doublets of two glass types to correct chromatic aberration.
- 1830 Joseph Jackson Lister combines lenses to correct spherical aberration.
- 1830 Airy, diffraction rings in star images.
- 1833* Brown, nucleus in orchids/
- 1838* Schleiden & Schwann cell theory/
- 1876 Abbe’s theory of diffraction in light microscopy.
- 1879* Flemming, mitotic chromosomes.

- 1881* Cajal use stains to see tissue anatomy.
- 1882* Koch, microbiology (Cholera, Tuberculosis)/
- 1886 Zeiss and Abbe design and build a diffraction limited microscope.
- 1898 Golgi use silver nitrate staining to see “his” apparatus.
- 1903 Richard Zsigmondi ultra-microscope. Scattering from suspension of small particles. Nobel prize 1925 for his studies of suspensions.
- 1924 Lacassagne use Marie Curie’s radium in Autoradiography.
- 1924 de Brogli, electron’s wave character.
- 1930 Lebedeff, interference microscope.
- 1931* Ruska, transmission EM. Commercialized: 1939 (Siemens).
- 1932* Zernike, phase contrast microscope-> Cells in culture (Nobel prize 1953).
- 1938 Ernst Ruska builds the electron microscope. (Nobel 1986).
- 1941* Coone, fluorescence microscopy.
- 1940-60 Petráň spinning disk microscope: confocality principle.
- 1945* Porter, cells fixed in Osmium. Palade: organelles. Huxley: muscles.
- 1952 Nomarski, Differential Interference Contrast (DIC).
- 1940-60 Koana, Naora, Minsky (confocals) Petráň (Tandem scanning->spinning disk).
- 1968 Gabor, lasers .
- 1975 Ploem “pack”: excitation emission and dichroic filters.
- 1977-80 Sheppard, Brakenhoff, Koester, & Amos, White - laser scanning confocals.
- 50’ s TV technology develops.
- 70’ s Digital image processing.
- 1981 Gerd Binnig and Heinrich Rohrer scanning tunneling microscope, atomic resolution. Nobel 1986.

- 1981 Allen & Inoue, Video-enhanced microscopy.
- 1983 Sedat & Agard 3D digital microscopy using “wide field” + deconvolution.
- 1985 Boyde, Kino Nipkow-disk tandem confocal (spinning disk).
- 90' s Near field, Tunneling and Atomic force microscopy.
- 80' s pSec pulsed lasers.
- 1997* Watt Webb, two photon microscope.
- 2000- Breaking the Abbe resolution limit: PALM, STED, SI (Nobel 2014).



Z. Janssen



A. van Leeuwenhoek



E. Abbe



R. Zsigmondy



F. Zernike



E. Ruska



G. Binnig and H. Rohrer

ASTRONOMY

and

EARTH SCIENCES

Francis Bacon 1561-1626

1620 - Notes that the matching coastline of Africa and America is not accidental.

Pierre Gassendi 1592-1655

1631 Follows the passage of Venus in front of the sun, and establishes that its distance to the sun is shorter than earth distance to the sun.

John Wilkins 1614–1672

1638 – Published “A world in the moon” proposing life there.

Christiaan Huygens 1629-1695

1655 – Discovers Titan, the large moon of Saturn, and that “ears” of Saturn that Galileo considered to be a moon are continuous rings. Was first to observe patterns on Mars. Estimated centripetal forces of moon from mv^2/r .

Giovanni Domenico Cassini 1625-1712

1665 – Observed a large red spot migrating across Jupiter.

1668 – Publishes precise tables of the orbits of Jupiter, Venus and Mars. Which credit him to get the position of the head of the observatory in Paris. Huygens and Rømer worked under him.

Jeremiah Horrocks 1618-1641

1640 – Moon orbit is elliptic, and it is as if it falls towards earth.

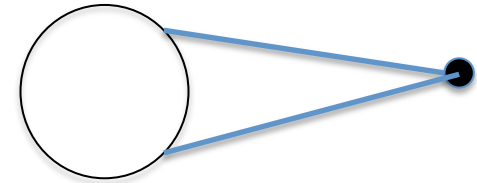


Giovanni Domenico Cassini 1625-1712

1671 -1684 Discovers the four moons of Saturn, and the split-ring pattern.

1671 - First successful measure of Mars distance by parallax.

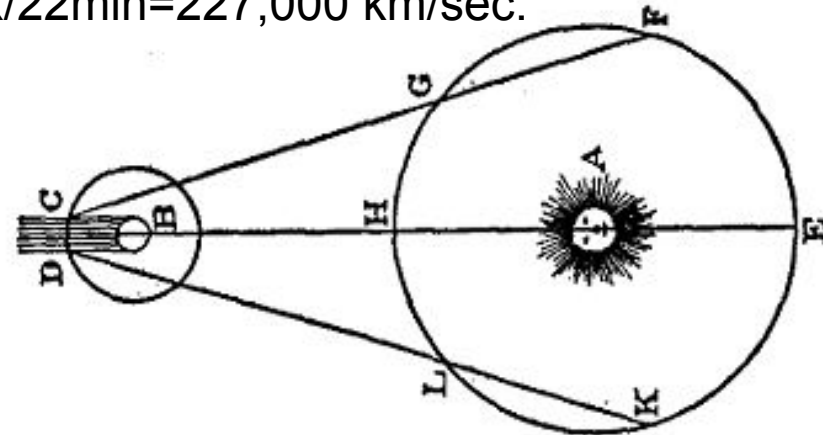
Jean Richter led a delegation to French Guiana to establish the angle of Mars (from the average of rising and setting angles) simultaneously at two positions on earth at a known distance, the basis for triangulation.



Ole Rømer (Copenhagen) 1644-1710

1675 – Repeats Galileo's gravity experiments, and applies to Jupiter moons.

1676 – Measured the speed of light from the change in the measured orbit cycle of Io, Jupiter's moon, when earth approaches F->G or moves away L->K from Jupiter. He measured 22 minutes difference out of 4.5 hours per orbit cycle. He did not calculate the speed of light since he may not have relied on the value of earth speed. Huygens mistakenly took the 22 minutes to be the time light travels through the earth diameter, while the 22 minutes are time for light to travel across earth orbit diameter around the sun: $2R=300 \times 10^6$ km, thus the speed of light is $2R/22\text{min}=227,000$ km/sec.



Nils Steensen, (Steno) 1638-1686

1669 – Fossils are life buried at the bottom of the sea, and by underground forces elevated and revealed by rain erosion.

Henry More 1614-1687

1672 – Cambridge theologian who influenced Newton. “Most of the universe is vacuum.

Richard Norwood 1590-1675

1637 – Measures the length of one longitudinal degree from triangulation as 11072 km.

William Crabtree 1610-1644 & Jeremiah Horrocks 1619-1641

Instrument builder and a teacher, who predicted the passage of Venus over the sun and confirmed their prediction.

**1675 King Charles II funds the Greenwich observatory
to assist navigation of ships.**

Edmond Halley 1656-1742

1678 – Observes 341 new stars from St. Helena at the southern hemisphere. After viewing Mercury passage over the sun, he suggests that the coming Venus passage at 1761 can determine our distance from the sun.

HALLEY'S COMET

Observations documented at 260, 1059, 1066.

1611 Shakespeare writes that a comet prophecy a war. Halley followed comets at 1531 and 1607, and determined both are the same comet with 76 years orbit cycle, to appear again 1758, 1835, 1910, 1984, 1985, 2062. It has elliptical shape 16x8x8 km. Was named after Halley after his death.



Precession of the equinoxes

Earth is a oblate spheroid, flatter at the poles, and more convex at the equator. The attraction by the sun and the moon cause earth rotation axis to make a precession with 26,000 years cycle, moving the date of the equinox (equal time length of day and night). This drift was known at antiquity, but was first explained by Halley.

Jean Picard 1620-1682

1669 – Assuming earth is a sphere, made complex calculation to find the length of a longitudinal degree as 110.46 km.

Giovanni Cassini 1625-1712 & Jaques Cassini 1677-1756

Wrongly claim that earth is a prolate spheroid. Also Newton made this mistake.

GEOGRAPHY, GEOPHYSICS, EARTH SCIENCES

William Gilbert's 1544-1603

1600 “De Magnete” Compass turns north since earth is a huge magnet

Newton 1642-1727

Gravitational forces, tides, earth precession, earth shape, density and gravity.

Pierre Bouguer 1698-1758; Alexis Clairaut 1713-1765; Henry Cavendish 1731-1810
Earth magnetic field

**Alexander von Humboldt 1769-1859; Edmund Halley 1656-1742;
Carl Friedrich Gauss 1777-1855**

Seismology.

John Milne 1850-1913; Robert Mallet 1810-1881

The age of earth estimated from temperature change with depth.

Arthur Holmes 1890-1965; William Thomson, 1st Baron Kelvin 1824-1907

Water cycle.

**Marcus Vitruvius 70-15 BC; Leonardo da Vinci 1452-1519;
Bernard Palissy 1510-1589;**

Pioneers in hydrology.

Pierre Perrault 1608-1680; Edme Mariotte 1620-1684; Edmund Halley 1656-1742

Rains, drainage basins, river beds and their draining capacity.

Henry Darcy 1803-1858; Dupuit-Thiem 18054-1866

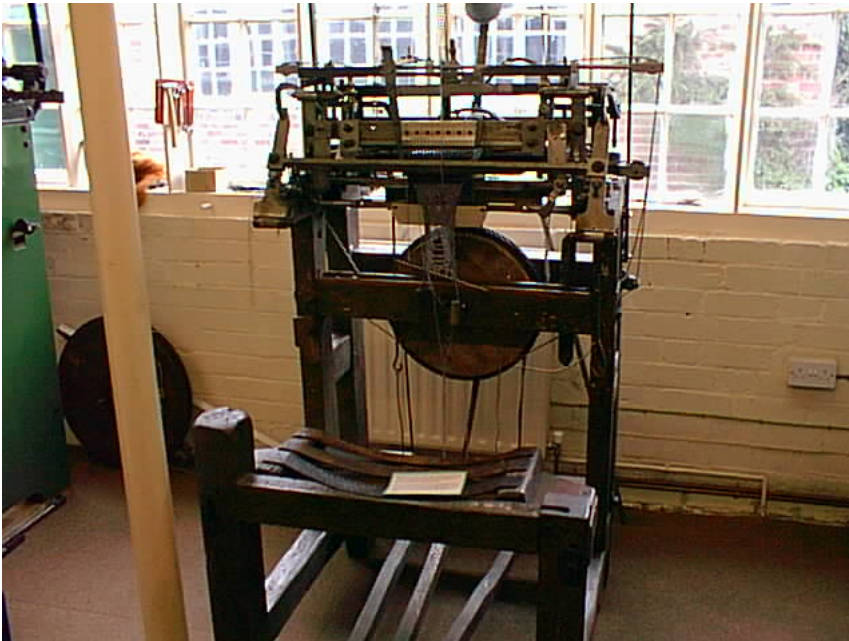
Hydrology of underground water: Darcy law - water flow in porous media. Dupuit well formula – steady state flow from aquifer to a well.

ECONOMICS

1530 Florence issues lottery to fund state expenses.

1589 William Lee 1563-1614, the Englishman, invents stockings frame knitting machine. Textile industry thrives at the 16th century.

1846 Elias Howe 1819-1867 builds a general purpose sawing machine



Stocking frame knitting machine



Howe's sawing machine

APPENDIX: List of scientists at the age of reason

Guldin Paul 1577-1643 Swiss math [in 17-cent]

Bramer Leonaert 1596-1674 Dutch painter

Etienne Pascal

Mersenne Marin 1588-1648 French mathematician, music theory & acoustics

Desargues Girard 1591–1661 French mathematician and engineer.

[projective geometry](#)

Gassendi Pierre 1592-1655 [transit of Mercury & Venus](#) as [Kepler predicted](#)

Beaune René ???

Albert Girard 1595-1632 mathematician First to use sin, cos etc. Area of a spherical triangle: [Girard's theorem](#)

Descartes René 1596-1650

La Faille

Gellibrand Henry 1597–1637 English mathematician. Earth's magnetic field.

Claude Hardy 1598-1678

Carcavi Pierre de 1600-1684

Vlacq Adriaan 1600–1667 Dutch book publisher and author of mathematical tables

Delamain Richard 1600–1644 English mathematician. circular slide rule & sundials.

Fermat Pierre de 1601-1665

de Beaune Florimond 1601-1652 mathematician

Billy ???

Roberval Gilles Personne de 1602-1675 mathematician

Boulliau Ismaël 1605–1694 astronomer and mathematician

Nicolas Steno [1638 –1686](#) Danish anatomy and geology

Athanasius Kircher (Kirchner) 1602–1680 geology, egyptology

Grimaldi Francesco Maria 1618 –1663

Frenicle de Bessy

Caramuel

Fabri

Torricelli Evangelista 1608–1647 Italian physicist and mathematician, barometer

Le Tenneur

Stampioen

Pell

Arnauld

Tacquet

Wilkins

Schooten

Wallis

Mouton

Ricci

Jean Picard

Brouncker

Nicolaus Mercator

Dechales

Rahn

Viviani

Sluze

Angeli
Collins
Cassini Giovanni Domenico 1625-1712 Italian mathematician & astronomer

Bartholin Caspar 1655–1738 Danish anatomist
de Witt Johan 1625-1672 Dutch statesman an accomplished mathematician

Mengoli
Boyle Robert 1627-1691 chemist & physicist

Jonas Moore

Hudde
Huygens Christiaan 1629-1695

Barrow

Richer

Cocker

Wren Sir Christopher Michael 1632-1723
English architects

Heuraet

Hooke Robert 1635-1703

Neile

Malebranche

Gregory

La Hire

Mohr

Lamy

Ozanam

Seki

Newton Isaac 1642-1727

Leibniz Gottfried 1646-1716

Flamsteed

Papin

Giovanni Ceva

Tommaso Ceva

Raphson

Tschirnhous

Le Févre

Rolle

Jacob Bernoulli

Varignon

Reyneau

Halley Edmond 1656-1742

Fontenelle

David Gregory

Saurin

Lagny

de l'Hôpital

Craig

de Moivre

Johann Bernoulli

Whiston

Arbuthnot

Saccheri

Magnitsky

Grandi

Keill

Doppelmayr

Clarke

Jones

Riccati

Privat de Molères

Hermann

Montmort

Hadley

Cotes

Giulio Fagnano

Poleni

Berkeley

Taylor

Simson

Nicolaus I Bernoulli

'sGravesande

Castel

Godbach

Stirling

Nicolaus II Bernoulli

Bourguer

Maupertuis

Maclaurin

Camus ~1700-

