ROMANTIC AGE

Mathematics

Collected and edited by Prof. Zvi Kam,
Weizmann Institute, Israel

The 19th century is called “Romantic” because of the romantic trend in literature, music and arts opposing the rationalism of the 18th century. The romanticism adored individualism, folklore and nationalism and distanced itself from the universality of humanism and human spirit. Coming back to nature replaced the superiority of logics and reasoning human brain.

In Literature:

England-Lord byron, Percy Bysshe Shelly

Germany –Johann Wolfgang von Goethe, Johann Christoph Friedrich von Schiller, Immanuel Kant.

France – Jean-Jacques Rousseau, Alexandre Dumas (The Hunchback from Notre Dam), Victor Hugo (Les Miserable).

Russia – Alexander Pushkin.

Poland – Adam Mickiewicz (Pan Thaddeus)

America – Fennimore Cooper (The last Mohican), Herman Melville (Moby Dick)

In Music: Germany – Schumann, Mendelsohn, Brahms, Wagner.

France – Berlioz, Offenbach, Meyerbeer, Massenet, Lalo, Ravel.

Italy – Bellini, Donizetti, Rossini, Puccini, Verdi, Paganini.

Hungary – List.

Czech – Dvorak, Smetana.

Poland – Chopin, Wieniawski.

Russia – Mussorgsky.

Finland – Sibelius.

America – Gershwin.

Painters:

England – Turner, Constable.

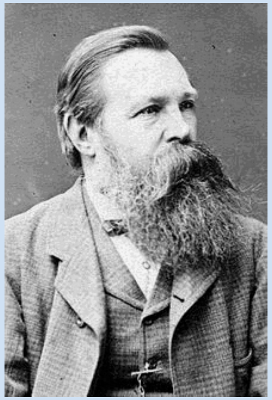
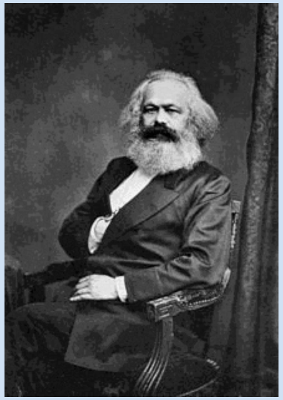
France – Delacroix.

Spain – Goya.

Economics:

1846 - The American **Elias Howe Jr.** builds the general purpose sawing machine, launching the clothing industry.

1848 – The communist manifest by **Karl Marks & Friedrich Engels** is published. Describes struggle between classes and replacement of Capitalism by Communism.

But in the sciences, the Romantic era was very “practical”, and established in all fields the infrastructure for the modern sciences.

In Mathematics – Differential and Integral Calculus, Logarithms.
Theory of functions, defined over Euclidian spaces, developed the field of differential equations, the quantitative basis of physics.
Matrix Algebra developed formalism for transformations in space and time, both orthonormal and distortive, preparing the way to Einstein's relativity.
In Astronomy – studying stars beyond the solar system, and expanding geology and earth sciences.
In Physics – Electromagnetism, atoms and interaction between radiation and matter, mechanics of solids and liquids, hydrodynamics, gases and vacuum. Classical mechanics is shaped by mathematical formulations (Hamiltonian mechanics) and implemented for string oscillations and wave mechanics, all prepared for the quantum mechanical theories.
Thermodynanmics – Heat, and mechanical Energy, Entropy, Conservation laws, Absolute Zero, Physics of liquids and gases. Theoretical foundations of thermodynamics (such as Carnot machine) gave birth to the steam engine and its improved efficiency, and brought the industrial revolution.
In Chemistry – More elements are isolated and characterized, leading to Mendel's periodic table. Organic chemistry buried vitalism.
And in Biology & Medicine – Cell biology, developmental biology, descriptive genetics and evolution. For the first time established link between diseases and microbiology. The biology of heredity prepare the grounds for molecular biology and genetics of the 20th century.

MATHEMATICS

http://www.storyofmathematics.com/19th.html

<u>19th century in Mathematics</u>

In France, especially under Napoleon, mathematics serves engineering. The three L-s stick out: **Lagrange, Laplace and Legendre** as well as **Galois and Fourier.**

Formal definitions of functions lead to the ultimate contribution of **Dirichlet** to the theory of functions at the 20th century.

Complex numbers, defined by Euler, got their extensive application in theory of functions. **Évariste Galoi** proved at 1820, using permutation groups, that 5th order equations do not have a general solution. **Augustin-Louis Cauchy** and **Carl Jacobi** contributed to functions and matrices. **Jacques Hadamard**, **Charles de la Valle Poussin** continued Riemann's study in number theory on the distribution of primes. **Henri Poincare** produced a partial solution to the three-body problem.

In Germany, **Wilhelm von Humboldt** emphasized pure mathematics, peaked with **Carl Friedrich Gauss** and the Göttingen school. Gauss studied non-Euclidean spaces, but his unpublished work was eventually explored by **Janos Bolyai, Nikolai <u>Lobachevski</u> & Riemann** (Elliptic geometry and multidimensional spaces) and **Minkovski** (space-time). **Bolzano & Wierstrass** formalized algebra and function theories, **Möbius & Klein** advanced topology **Cantor & Dedekind** infinite groups. **Gottlob Frege** made a breakthrough in logics at 1897, paving the way to **Hilbert, Bolzano and Bertrand Russell.**

In England, **George Peacock** started abstract algebra, **George Boole** developed Boolean algebra, and **John Venn** studied this algebra, **Hamilton** studied quaternions (4-vectors), and the non-commutative transformation group of three-dimensional vectors. **Arthur Cayley**, considered the most productive mathematician, studied group theory, matrix algebra, singular functions and invariants.

In Norway, **Niels Henrich Abel & Marius Sophus Lie** continuous groups.

The logical foundation to the theory of functions was linked to physics: the initial efforts to characterize smooth and continuous function properties resulted with theorems that defined them as uninteresting: if we know them in a limited region in space, we know them everywhere, and from their values along a closed contour (in two-dimensions) or closed surface (in three-dimensions) we can calculate their values everywhere inside.
Yet, electromagnetic theory needed to deal with functions that have singularities due to electric charges (e.g. the potential field depend of inverse distance to the charge, and therefore diverge at its position). The theory of functions was thus extended to singularities, and served electromagnetism as well as Hamiltonian operator descriptions of classical mechanics, creating the mathematical foundation to quantum mechanics.
The development of vector fields and matrix theory supported transformations in space-time as well as definition of operators in any dimensionality, including continuous and infinite. Classical mechanics was reformulated in terms for operators in continuous spaces (Hamiltonian, Hilbert space), and this formulation was then easily applied in quantum mechanics.
Interestingly, also group theory was intensively applied in physics − in the relation between symmetry and conservation laws, and in high-energy particles for prediction of new elementary particles from symmetry in quantum number spaces.
Last, the logical basis for computing machines and computers was established.

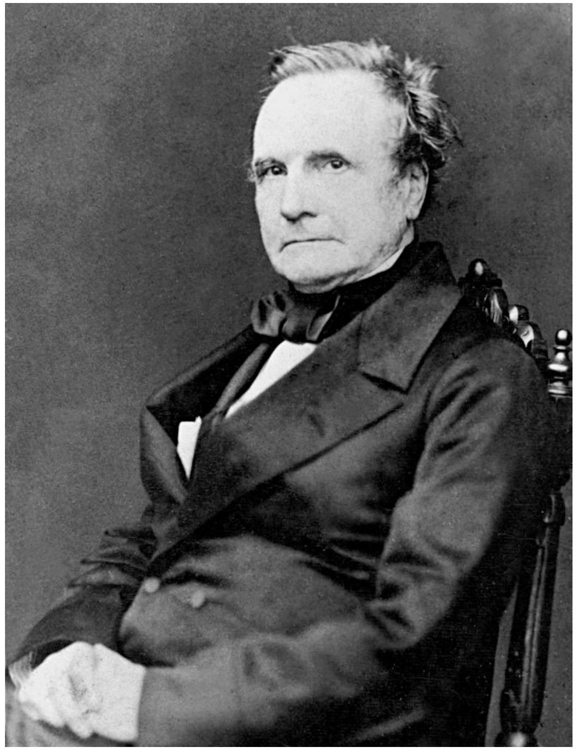
It must be emphasized that the following slides are intended to present just a brief glimpse of flavors in mathematics of the 19th century. This cannot replace organized courses with rigorous building up of each of the fields that were developed to perfection in this era, and established the foundations of present basic and applied mathematics.

Charles Babbage, FRS 1791-1871 England

The inventor and builder of a mechanical calculating machine.

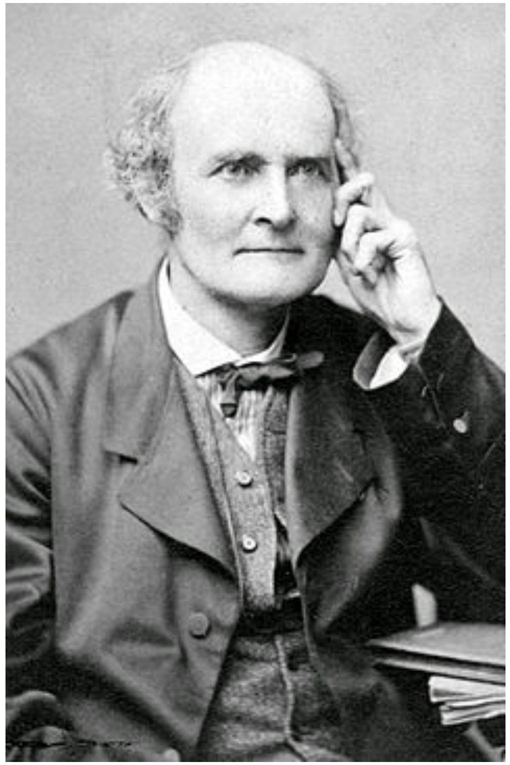
Pascal and Leibniz described the logical basis of computers: including central processing unit (CPU) memory and programs software read from punched cards and determined the order of operations.

Augusta Ada Byron (the daughter of Lord Byron and Babbage student) translated at 1843 a book of an Italian engineer, **Luigi Menabrea,** about an “Analytical Machine” and added remarks that described computer software programs capabilities beyond number crunching. The design was never realized at the time, but prepared the infrastructure for the building of computers in the future.



Arthur Cayley, **1821-1895** England

1858 Defined matrices, and extends linear transformations.
Writes a formula for inverting a matrix using determinants.



Cayley's theorem: "Every group G is isomorphic to the Subgroup of the symmetric group acting on G"

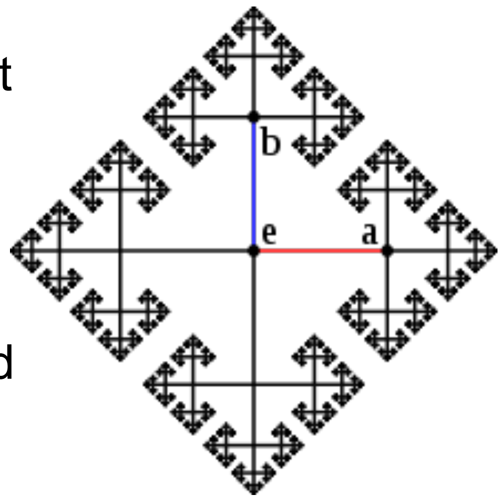
Cayley–Hamilton theorem: If A is a matrix of dimensions nxn And I_n is the unit matrix, and the characteristic polynomial of order-n in λ defined by: $p(\lambda) = \det(\lambda I_n - A)$,
If we substitute A for λ in the polynomial we get the zero matrix:

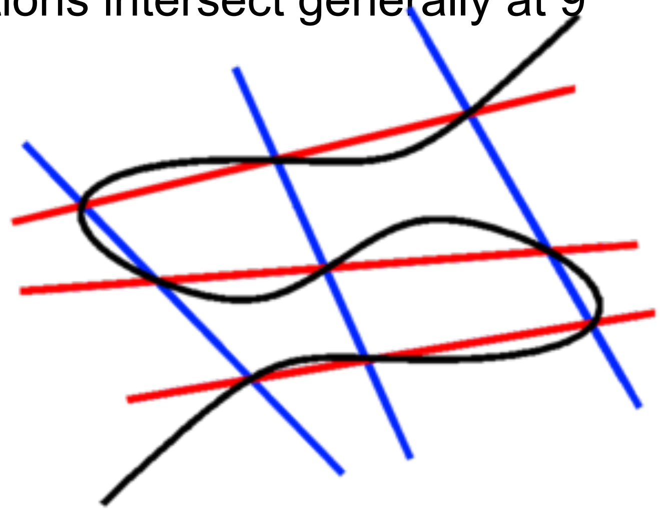
$$p(A) = 0.$$

and the solutions of $p(\lambda)=0$ are the EigenValues of the matrix.

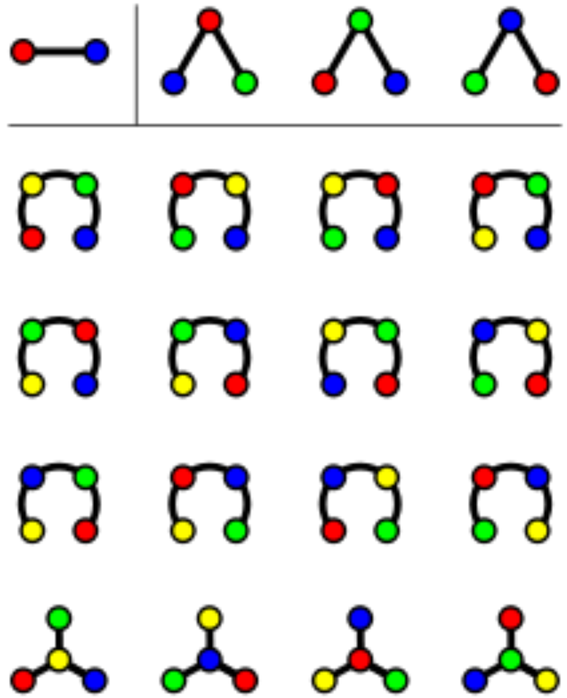
Cayley–Bacharach theorem:
Wo-dimensional projections of two cubic (third order) functions intersect generally at 9 points. Every other cubic function that crosses 8 of these points must cross the 9[th] too.

Cayley graph:
Graph that describes the structure of a group, here a group generated by two members.





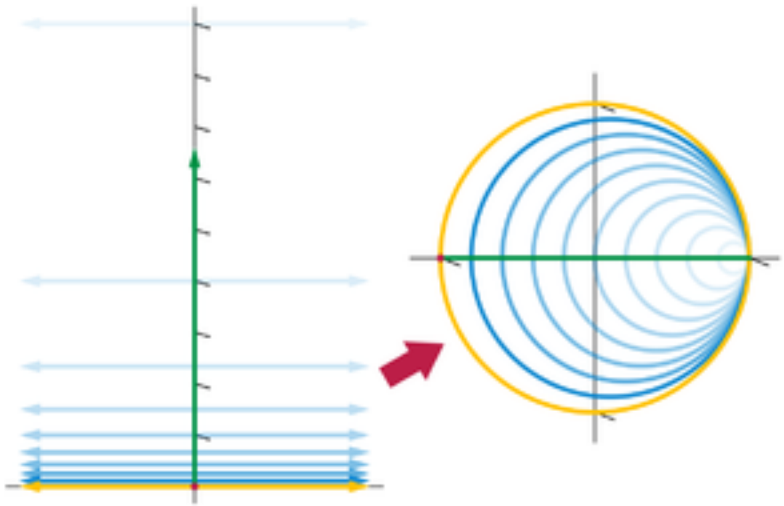
Cayley's formula: for every integer n the number of graphs with n assigned vertices is n^{n-2}. For example, all graphs with 2,3,4 color-assigned vertices:
n=2 one graph, n=3 three graphs, n=4 16 graphs.



Cayley's Ω process:

$$\Omega = \begin{vmatrix} \frac{\partial}{\partial x_{11}} & \cdots & \frac{\partial}{\partial x_{1n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial}{\partial x_{n1}} & \cdots & \frac{\partial}{\partial x_{nn}} \end{vmatrix}.$$

Cayley transform: maps diagonally-symmetric matrices
And orthogonal matrices – transforms upper half plane to
The unit circle.



EigenValues and EigenVectors of matrices

Matrix theory started by three-dimensional transformations in space, in order to solve motion of bodies, vibrational modes, inertia etc. It was generalized for linear operations in multi-dimensional spaces, including Hilbert infinite dimensionality spaces.

This field had a central role in quantum mechanics – the fact that possible solutions of the equation of motion (e.g. Schrödinger's equation for electrons) are discrete, have discrete energy levels, and can be written as Hamiltonian linear operator $\hat{H}y=Ey$.

Leonhard Euler ***1707 –1783*** *Swiss*

Euler discovered the significance of principal axes to solve modes of rigid body motion.

Joseph-Louis Lagrange 1736 -1813 France

Lagrange found that the principal axes are eigenvectors of the inertia matrix.

Augustin-Louis Cauchy 1789 –1857 France

Cauchy generalized for n-dimensions.

Jean-Baptiste Joseph Fourier 1768 –1830 France

Fourier applied eigenvectors to separate variables in the differential equation for heat propagation. Cauchy followed Fourier's work proving that symmetrical matrices have real Eigenvalues..

Charles Hermite 1822 –1901

Hermit extended the real EigenValues to complex matrices that are "Hermitian": ($A_{ij}=A^*_{ji}$)

Francesco Brioschi 1824 –1897

Showed that complex EigenValues of orthonormal matrices (row and column vectors are unit vectors and orthogonal) are on the unit circle.

Rudolf Friedrich Alfred Clebsch 1833 –1872

Clebsch extended the real EigenValues to skew-symmetric matrices $A_{ij}=-A_j$ or $A^T=-A$
Clebsch-Gordon coefficients: comes in Expanding of Angular Momentum states.

Karl Theodor Wilhelm Weierstrass 1815 –1897

Found instability (small change cause "far" solution) in equation systems with "defective" matrices, meaning that their EigenVectors span m-dimensional space with m<n, n is the matrix dimension.

François Sturm 1803–1855 & Joseph Liouville 1809–1882

The EigenValue problem applied to infinite space for the differential equations below with boundary conditions at x=a,b: (Strum-Liouville theory).

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[p(x)\frac{\mathrm{d}y}{\mathrm{d}x}\right]+q(x)y=-\lambda w(x)y,$$

Only a discrete set of λ have solutions y(x).

Karl Hermann Amandus Schwarz 1843–1921

Solved the first EigenValue for Laplace equation: $\nabla^2 \varphi = 0$

Jules Henri Poincaré 1854 –1912

Studied the EigenValues of Poisson's equation: $\nabla^2 \varphi = f.$

David Hilbert 1862–1943

Studied the EigenValues of integral operators as matrices in infinite dimensionalities: Hilbert space.

Was first to use "Eigen…" maybe following Helmholz.

Numerical solutions to EigenValues/Vectors of matrices:

The applied mathematician **Richard Edler von Mises** 1883–1953 gave an iterative solution to the largest EigenValue. The recursive equation:

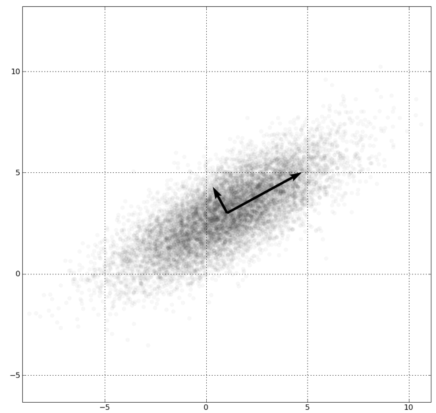
$$b_{k+1} = \frac{Ab_k}{\|Ab_k\|}$$

implies that multiplying a matrix A by an arbitrary vector b multiple times yields the EigenVector corresponding to the largest EigenValue, since $Ab_k=\lambda_k b_k$ the contribution of all smaller λ_j will diminish.

von Mises also presented the tensor equation relating stress and strain, a bedrock in construction engineering

John G.F. Francis 1934- & Vera N. Kublanovskaya 1920 –2012

Since the von Mises iterative solution converges very slowly if EigenValues are close to each other, Francis and Kublanovskaya developed independently the QR algorithm to factorize a matrix A to a product of two matrices QR where Q is orthogonal matrix ($Q^{-1}=Q^T$) and the matrix R upper-triangular (R_{ij}=0 for i<j). Then the iterations $A_{k+1}=R_kQ_k$ brin A closer to a triangular matrix with the EigenValues along its Diagonal. Today, a fast QR version is applied.



Golub-Kahan-Reinsch method for SVD-singular value decomposition also resolves the EigenVectors of matrices. It is commonly applied in multiparametric statistical analysis, in order to display projected distributions of data points in the plane of largest variation.

Mikhail Vasilyevich Ostrogradsky (1801 – 1862) Russia

1831 rediscovered the divergence theorem of **Lagrange, Gauss and Green**: the relation between volume integral on the divergence of a vector-function and integral on the surface enclosing the volume on the component of the vector function perpendicular to the surface:

$$\iiint_V (\nabla \cdot \mathbf{F})\, dV = \oiint_S (\mathbf{F} \cdot \mathbf{n})\, dS.$$

Applications: Gauss law: Integration on the electric field on a closed surface equals the electric charge inside the surface:

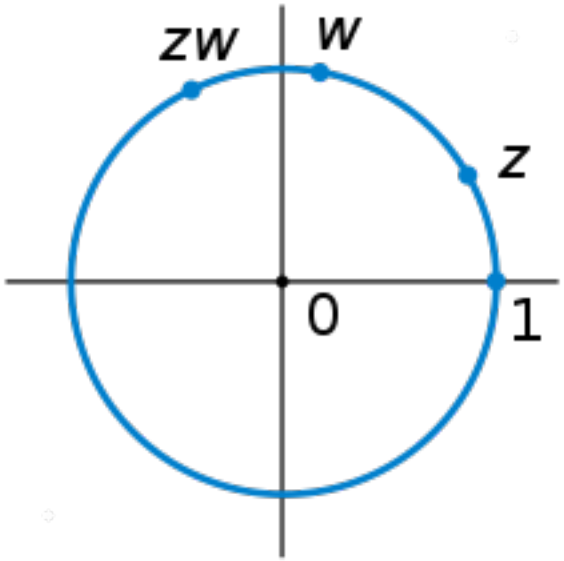
$$\oiint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q}{\varepsilon_0}$$

This relation bypasses the problem of infinite field at charge positions.

Marius Sophus Lie 1888 -1893 Norway & **Friedrich Engel (1861-1941)** Germany

Published six volumes on transformations, and show that symmetries create groups. Lie described the theory of continuous groups, and applied it to geometry and to differential equations.

100 years later, Lie groups and Lie algebra was applied to describe symmetries of elementary particles.

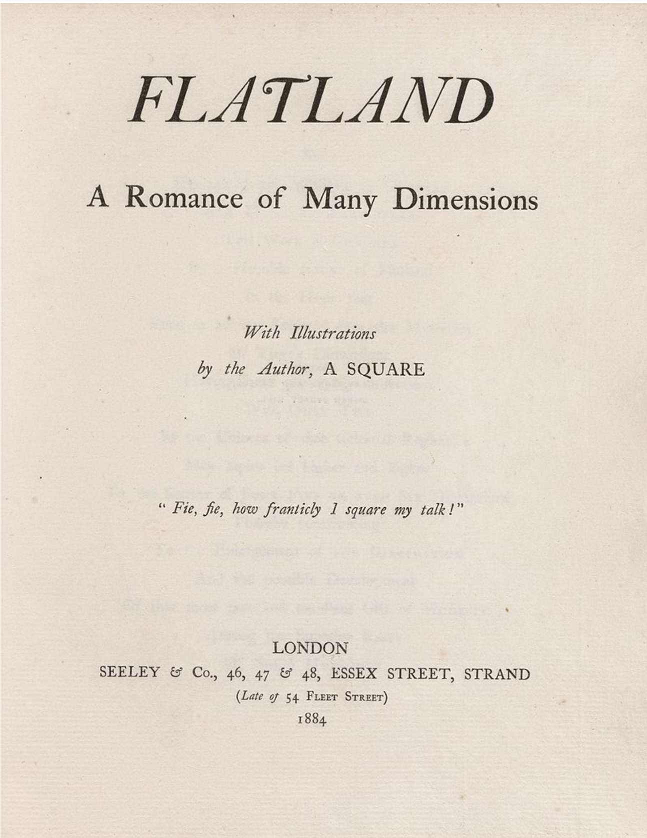


A circle of radius 1 in the complex plane generates a Lie group for multiplication.

1884, Edwin A. Abbott

1884 Published an imaginary novel about a square living with triangles and other shapes in a two-dimensional world. A sphere carries him to the third dimension to see his world from the outside, then he imagines how would a four-dimensional creature will see his three-dimensional world, e.g. the number of vertices for a line, square, cube and 4D hypercube are 2,4,8,16

This was an interesting exercise anticipating 4D properties in relativity.

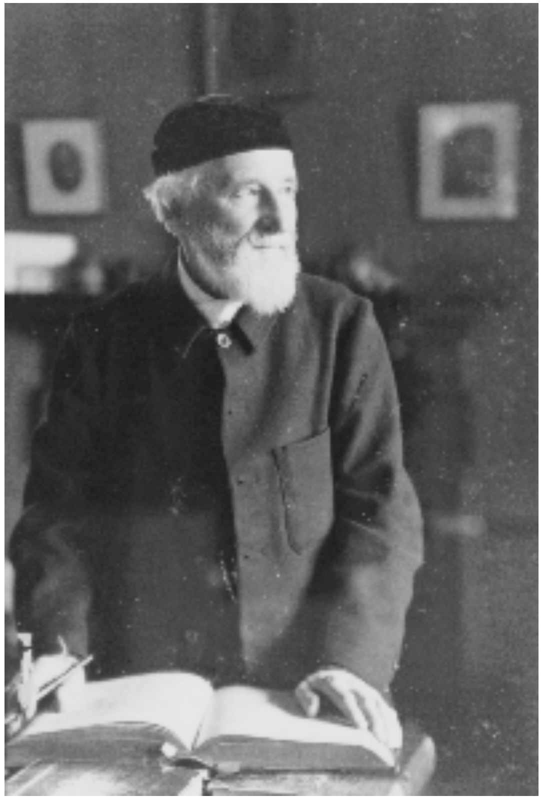

FLATLAND

A Romance of Many Dimensions

With Illustrations
by the Author, A SQUARE

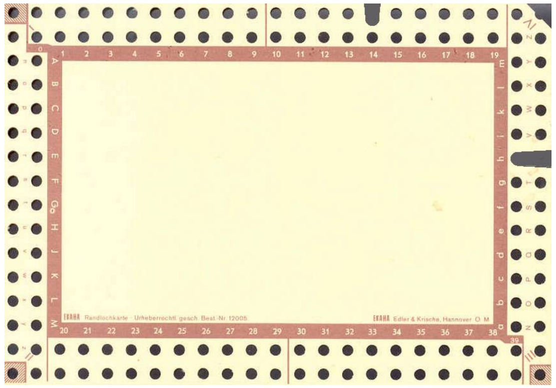
" Fie, fie, how franticly I square my talk!"

LONDON
SEELEY & Co., 46, 47 & 48, ESSEX STREET, STRAND
(*Late of* 54 FLEET STREET)
1884



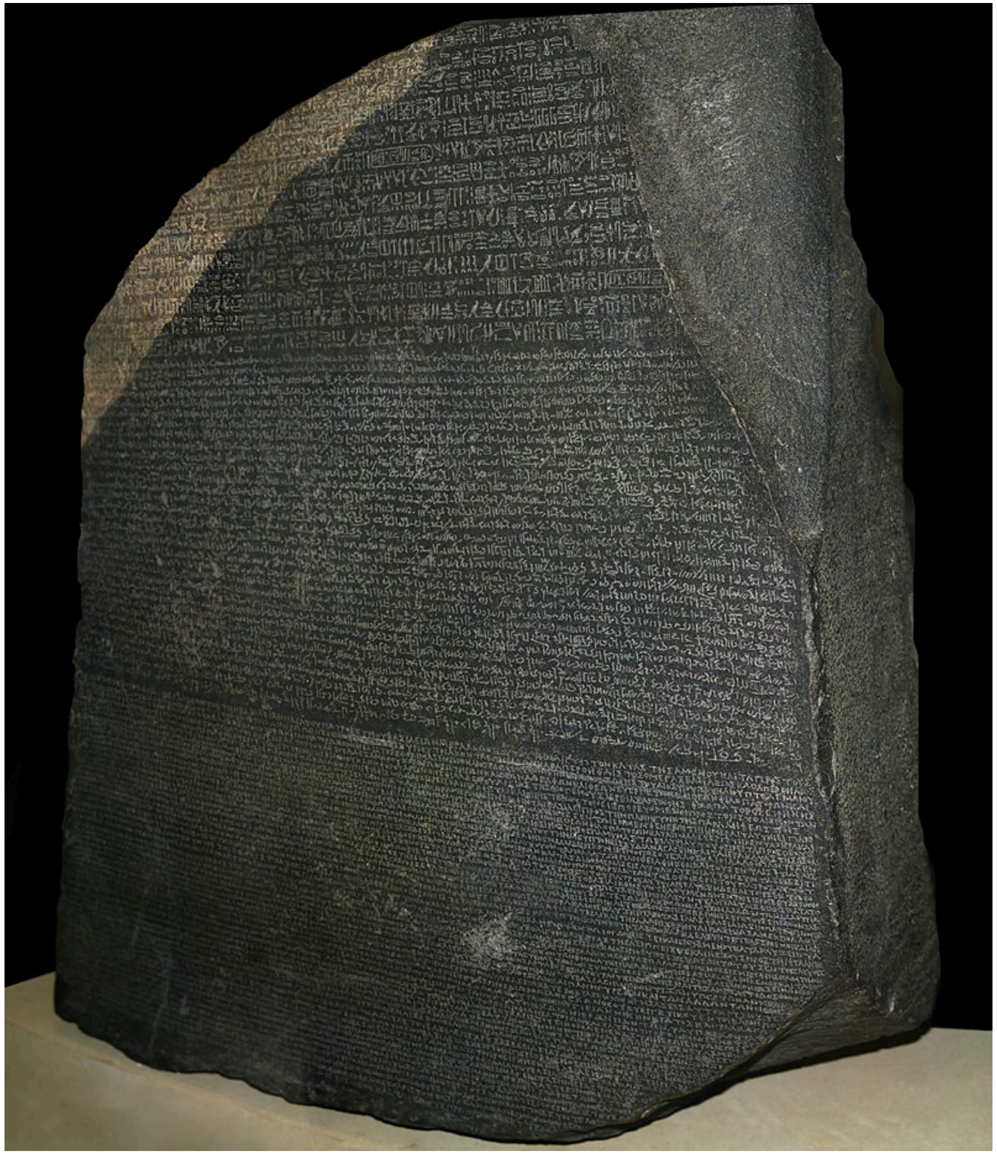
William Stanley Jevons (1835-1882) England

1862 Started mathematical economy
1874 Studied permutations of ABC for Boole-Venn logical diagrams. Designed a machine for logical computations, a kind of needle cards that can be sorted by lifting with a knitting needle.



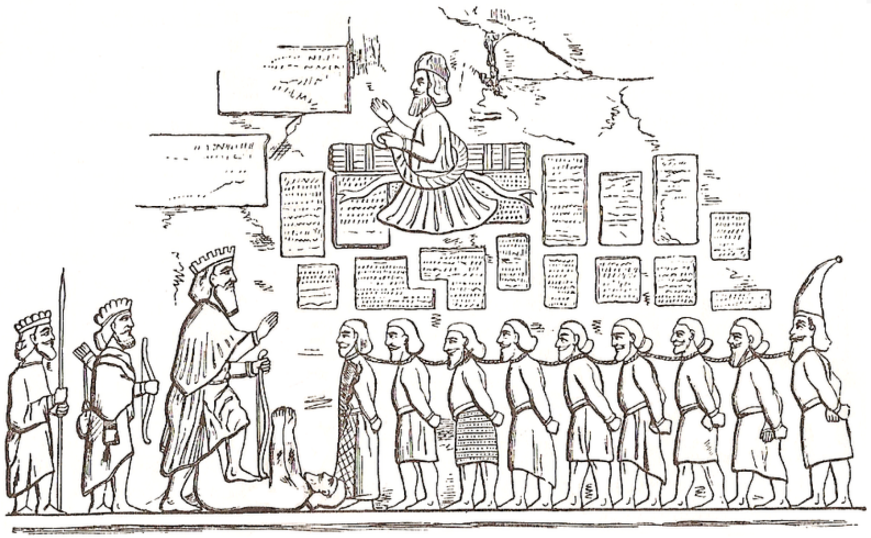
Jean François Champollion 1790 – 1832 France

1821 Used the Rosetta stone to decipher the hieroglyph script.
T. Young used it to decipher the Demotic script, one of the three ancient Egyptian scripts.



Henry Creswicke Rawlinson 1810 –1895 England

1846 Deciphered the cuneiform using multi-script table (Persian, Babylonian and Elamite cuneiform) of Darius I, found at the side of mountain Behistun.



William Kingdon Clifford 1845–**1879** England

1870 Studied non-Euclid geometry, presenting questions about the changes in universe curvature, like mountains in landscape.

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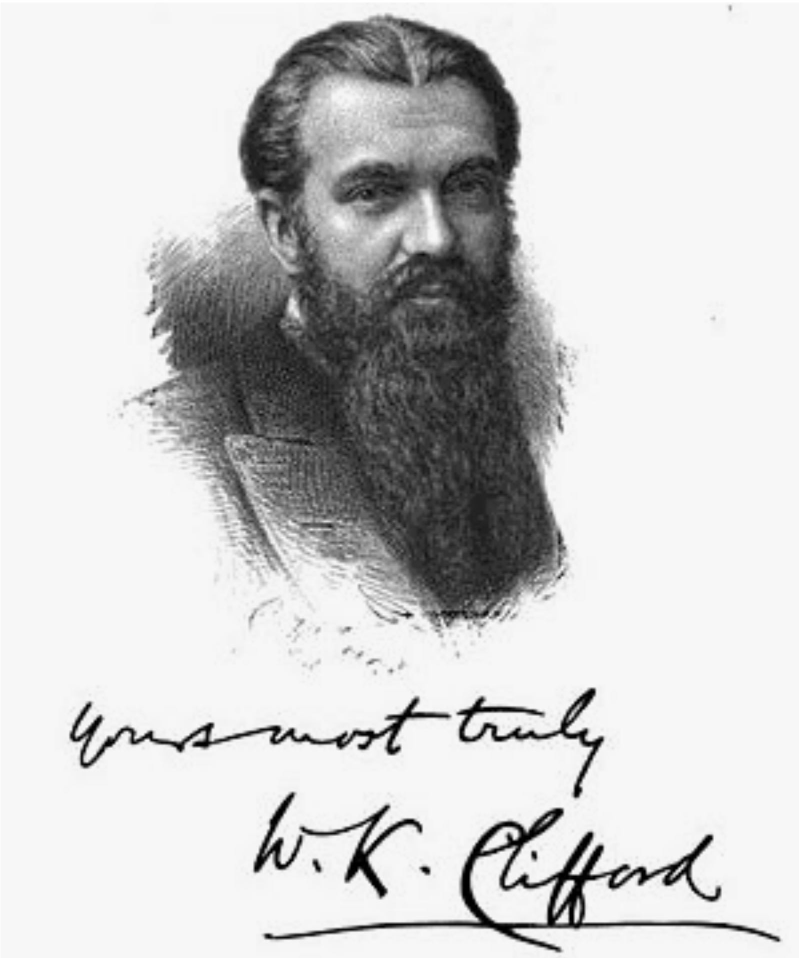
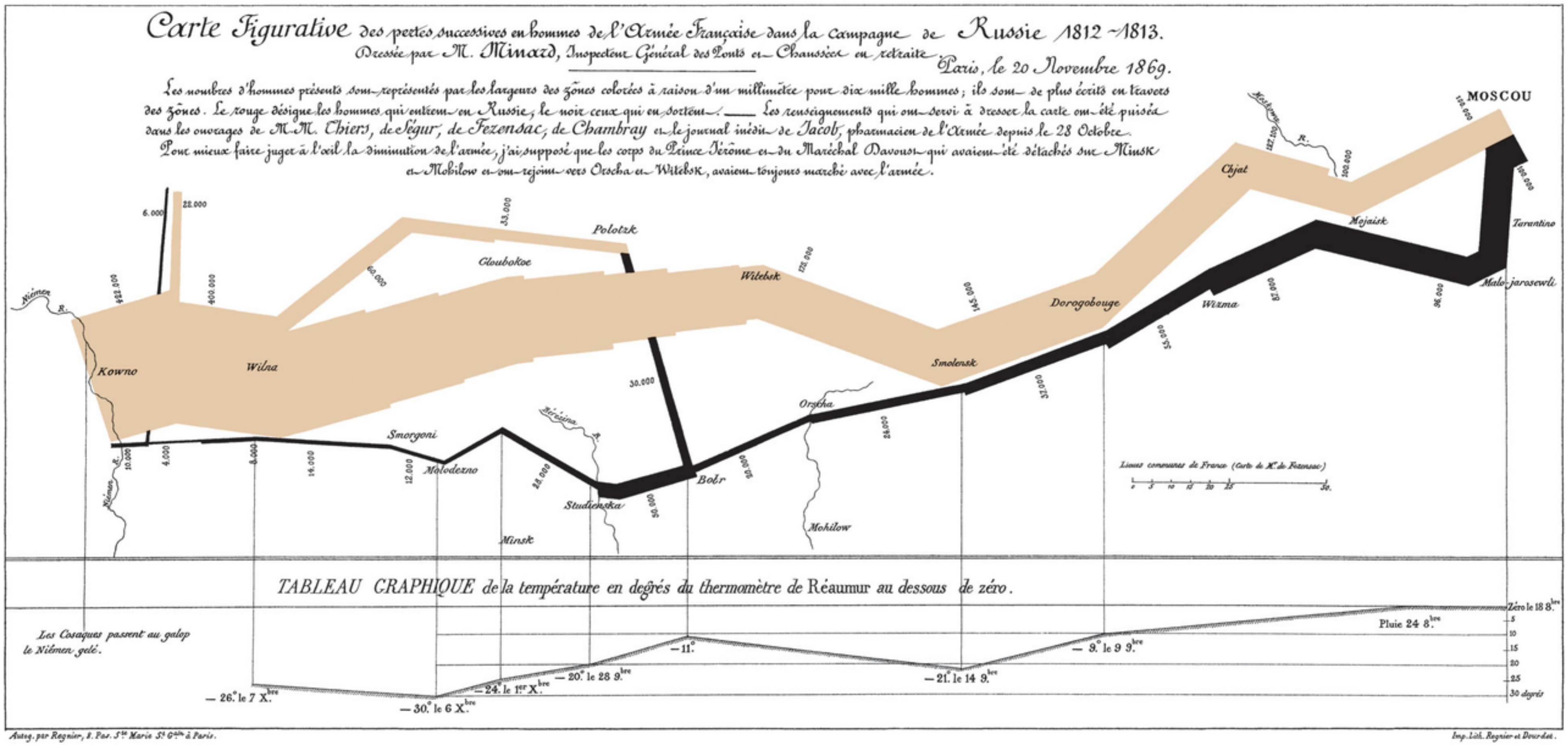


Charles Joseph Minard 1781-1870 France

1869 Draws a composite multiparametric graph of Napoleon's march to Russia and back: Size of his army is graph line thickness, x-axis position, color: brown to Russia black: back, and temperature.



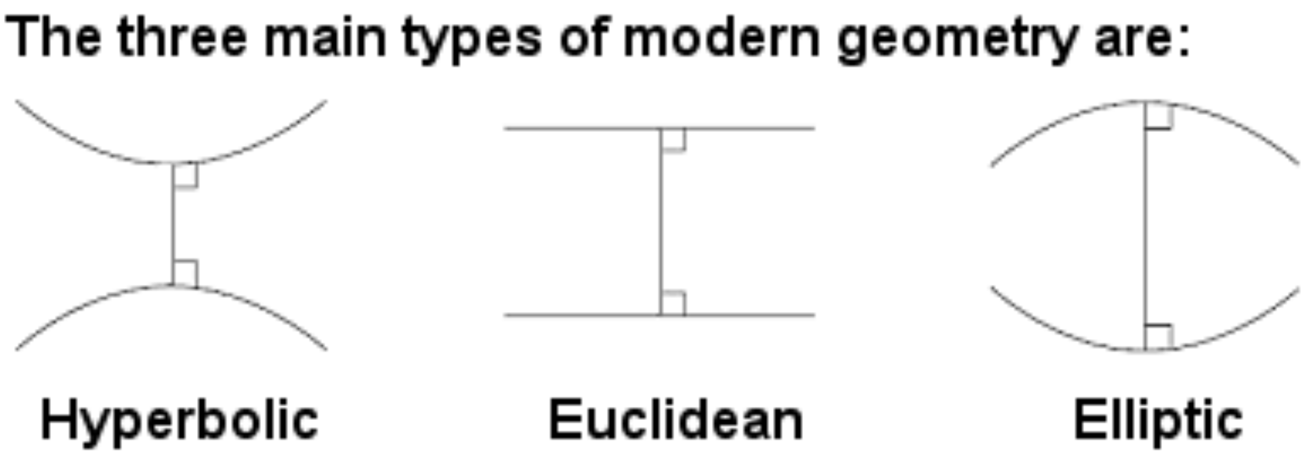
Elwin Bruno Christoffel 1829-1900 Germany

1869 Transformation between second-order forms in differential equations. Relate to Riemann's geometry and covariant differentiation.

Christoffel symbol express changes from one position to another in curvature of space. Relating to metrics, including non-Euclidean, and from it calculation of Riemann's curvature tensor or the link between force and potential in general relativity.

János Bolyai 1802-1860 Hungary

1823 Invented non-Euclidian geometry without the parallel-lines axiom.



GROUP THEORY

Niels Henrich Abel 1802-1829 Norway

Group theory: Abel function Abel integral Abelian group

1824 Abel developed a proof that 5th order equation does not have solution in terms of arithmetic operations on its coefficients.
The proof was presented to Gauss, Cauchy & Legendre, but they were not impressed…

Évariste Galois (1811-1832) France

Group theory: Galois' field, Galois' group, Galois' theory, Galois was a genius mathematician, who, due to his weirdness failed to be accepted to École Polytechnique. He was born in Bourg-la-Teine, south of Paris and studies in Lycée Louis-le-Grand. Took great interest in Geometry books of **Lagrange & Legendre**, and in algebra. He believed he found a solution to 5th order equations, but was mistaken. **Cauchy** got his manuscripts but required corrections. **Fourier**, who got his papers for a competition died before he submitted them. Also **Chevalier**, who was the editor of a mathematical journal, rejected his papers. His development of the symmetry group of polynomial equations and the relation to their solutions was published with the help of **Liouville**, who recognized Galois' ingenuity only after his death at **1832**. Galois formulated field theory and its relation to group theory 60 years before this area developed by **Liouville and Jordan**, who considered Galois as the father of group theory.

Galois was a republican, and was in prison for that. At the age of 20 he became involved in a duel fight (not known if on political or romantic grounds), and was deadly wounded. **Alexander Duma** reported that before dying he obsessively developed group theory. A letter he wrote at that time to Liouville includes the correct proof that 5th order equations have no algebraic solution, based on the permutation group of the coefficients. This was done independently by Abel in Norway.

Groups have great importance in Physics: symmetry groups of crystals, and of high energy elementary particle quantum numbers.

Galios proposed that in order that a polynomial equation could be solved in radicals of its coefficients, the structure of a group of permutations of the solutions, the Galois group of the polynomial, should have a series of maximal normal subgroups, until we reach the identity subgroup, where the quotients (number of elements / number of normal subgroup elements) must be primes. If so the n-th order polynomial can be reduced to (n-1)-th order at each step, until reaching second order.

$x^2+ax+b=0$ ->> $(x-p)(x-q)=0$ ->> $b=pq$ $a=-(p+q)$ 2nd order

The two permutations: pq & qp are a Galios group

$x^3+ax^2+bx+c=0$ ->> $(x-p)(x-q)(x-r)=0$ ->> $x^3-(p+q+r)x^2+(pr+qr+pq)x-pqr=0$ 3rd order

Six permutations {pqr,prq,qrp,qpr,rpq,rqp} form a group, with subgroup {pqr,qrp,rpq}

The quotients is 2 – a prime.

$x^4+ax^3+bx^2+cx+d=0$ ->> $(x-p)(x-q)(x-r)(x-s)=0$ ->> 4th order

$x^4-(p+q+r+s)x^3+(pq+rs+pr+ps+qr+qs)x^2-(prs+qrs+pqr+pqs)x+pqrs=0$

24 permutations of pqrs, subgroup of order 12 (quotients = 2, a prime), sub-subgroup of order 4 (quotients = 3, a prime), sub-sub-subgroup of order 2 (quotients =2, a prime). All quotients are primes, 4th order equation can be solved in radicals.

$x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$ ->> $(x-p)(x-q)(x-r)(x-s)(x-t) = 0$ 5th order

$x^5-(p+q+r+s+t)x^4+(pq+rs+pr+ps+qr+qs+pt+qt+rt+st)x^3-(prs+qrs+pqr+pqs+pqt+rst+prt+pst+qrt+qst)x^2+(pqrs+prst+qrst+pqrt+pqst)x-pqrst = 0$

120 permutations of pqrst, but the quotients are not primes.

This is an example how one field of mathematics resolves a problem in other field. A recent example: proof of the last theorem of Fermat by **Andrew Wiles.**

LOGICS

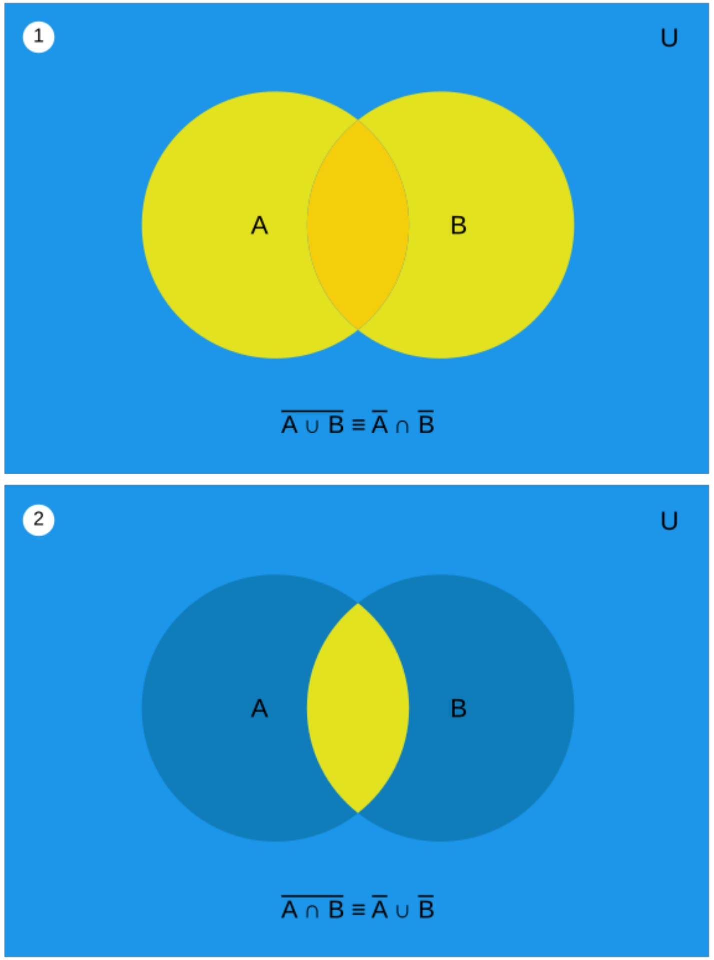
Augustus de Morgan (1736-1813) England

Logics: de Morgan laws

"***not (A and B)***" is identical to "***(not A) or (not B)***"

"***not (A or B)*** " is identical to "***(not A) and (not B)***".

1860 Developed logical relations between objects
That are associated with groups.



George Boole (1815-1864) England
Logics, probability,, Boole's rules

1839 Developed Boolean algebra, and probability laws.

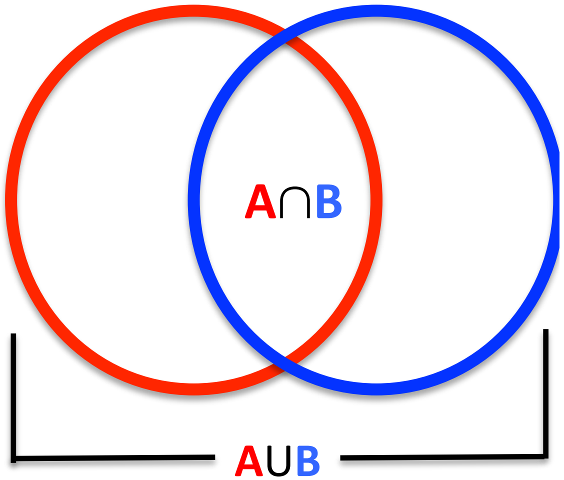
<u>Boole's inequality</u>: For every finite set of events A_i (also for countable infinite sets) the probability that at least one event would happen is not larger than the sum of probabilities of the individual events.

$$\mathbb{P}\left(\bigcup_i A_i\right) \le \sum_i \mathbb{P}(A_i).$$

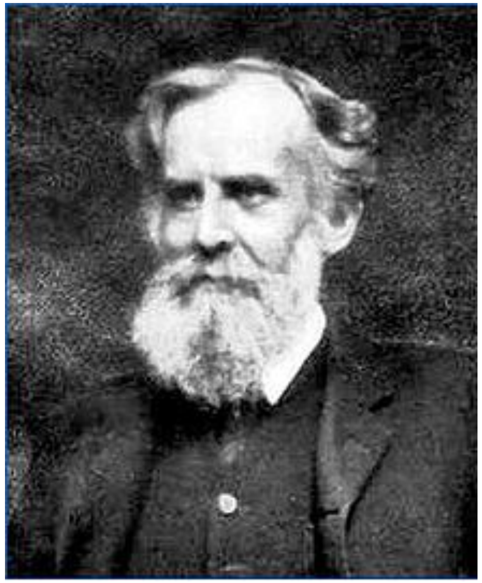
Proof by induction: for two sets of events {A,B} the probability that A or B or both happen (Union) is the probability of A + probability of B - probability of both A and B (Intersection):

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B),$$

Example: when throwing two dice, the probability that one will have 6 is 1/6 + 1/6 – 1/36



John Venn (1834-1923) England

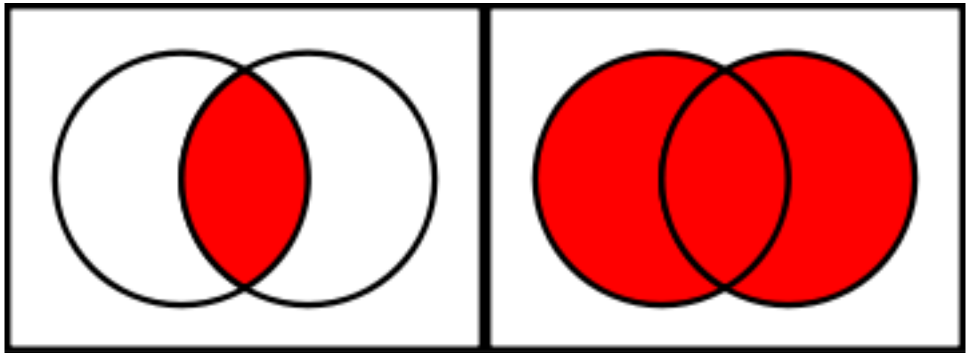


group theory, Logics, probability Venn diagram:

1881 Logical and graphical expressions to relations between sets.

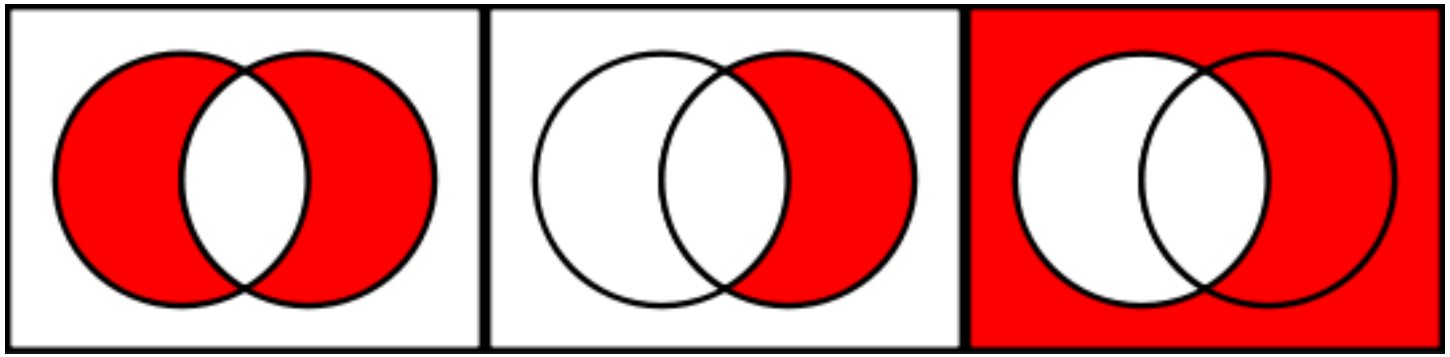
intersection of A and B Union of A and B

$A \cap B$ $A \cup B$

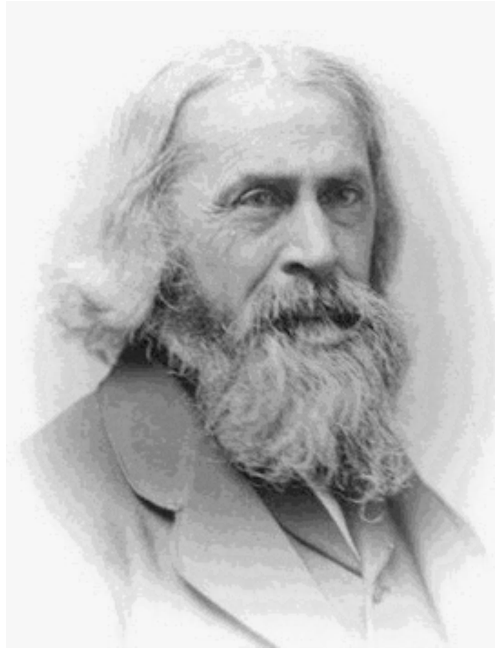


Symmetric difference the relative complement of A in B the absolute complement of A in U

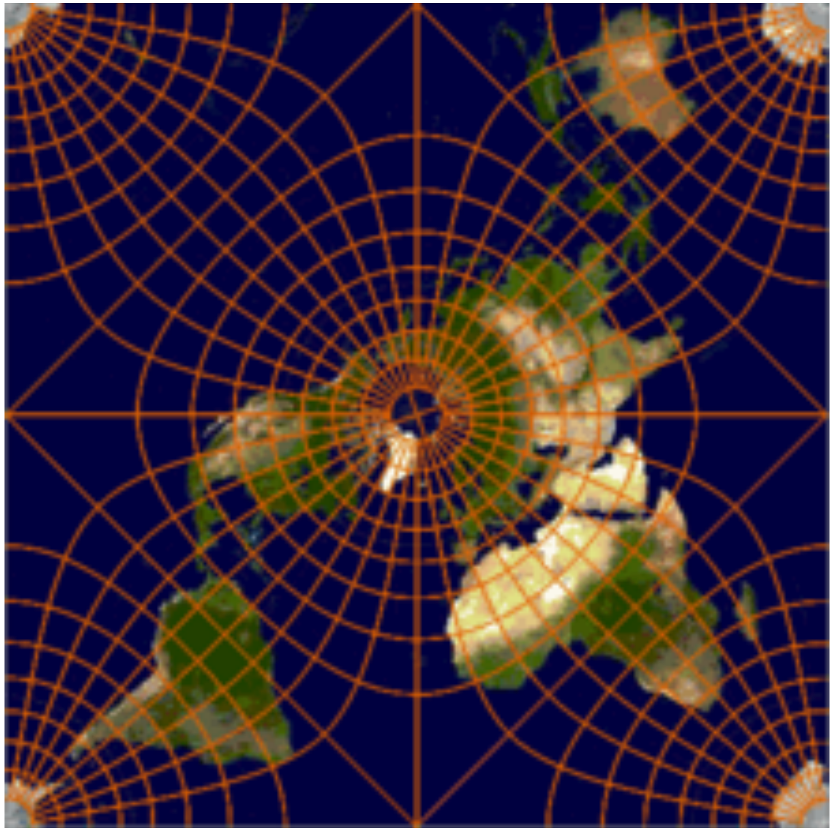
$A \,\Delta\, B$ $A^c \cap B = B \setminus A$ $A^c = U \setminus A$

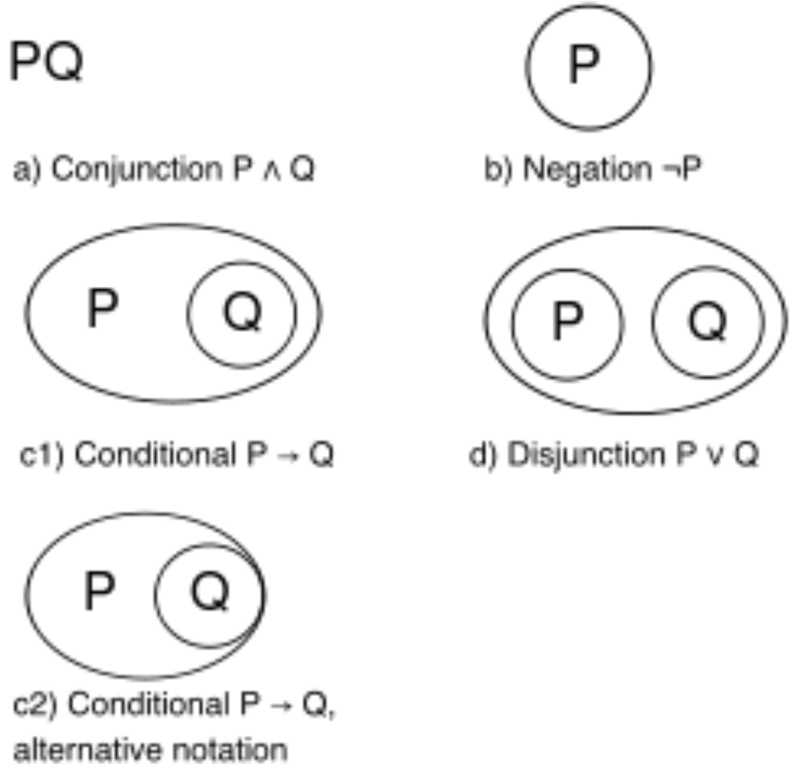


Charles Sanders Peirce (1839 – 1914) America
1897Tried to publish a topological system of symbolic logics that provide a geometrical description.

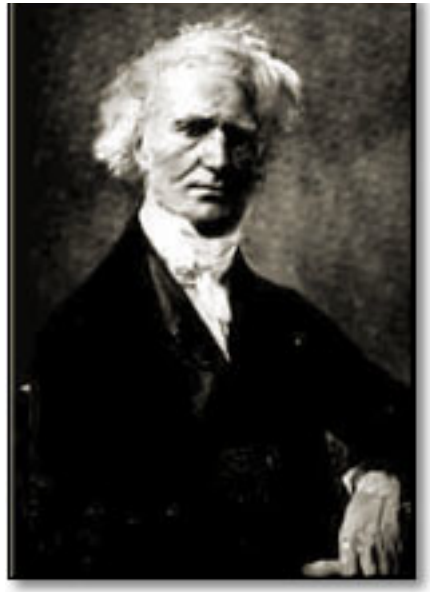


Planar projection of a sphere that apart from a few points preserve angles and have smaller area distortions.





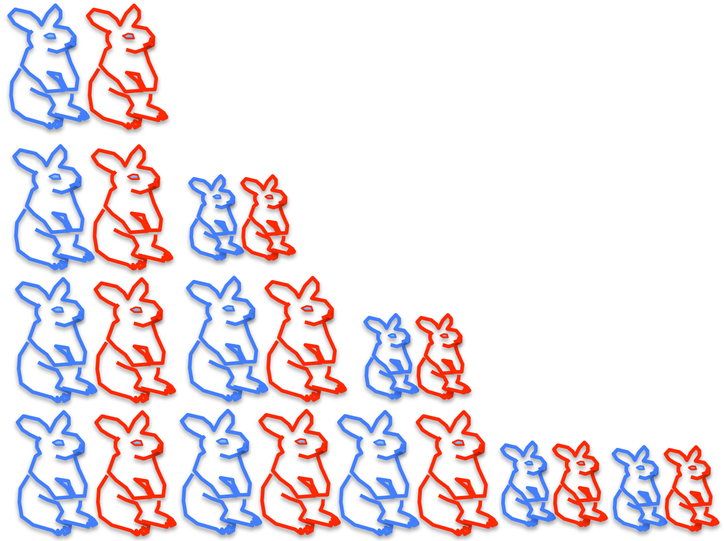
Jaque Phillip Marry Binet (1786-1866) France
number theory, Binet's formula



A close formula to Fibonacci's series recursive formula for number of rabbits: $F_{n+1} = F_n + F_{n-1}$

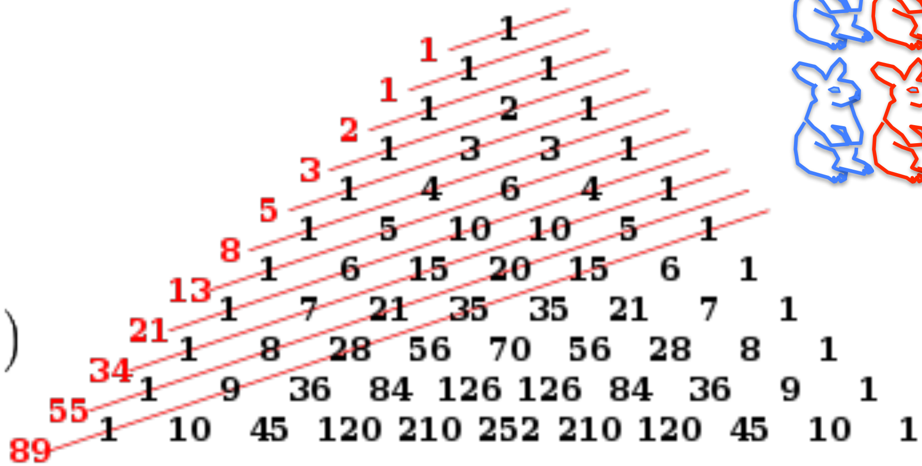
$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

This also extends the series to negative numbers:
Proof: $G_n = ar^n$ upholds $G_{n+1}=G_n+G_{n-1}$
for $r1,2 = (1\pm\sqrt{5})/2$ (solution of a quadratic equation).
The general solution is therefore: $F_n=ar_1^n+br_2^n$
Substituting initial conditions: $F_0=0$ $F_1=1$
Solves a and b as above.



The relation to the binary coefficients Pascal's triangle:

$$F_n = \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \binom{n-k-1}{k}$$



Friedrich Ludwig Gottlob Frege 1848 –1925 Germany

1879 The first system of computational logics as a foundation for mathematics.
Dealt with Fuzzy Logics: most boys love girls and most girls love boys who love girls…
Propositional calculus, Propositional logic or או Sentential calculus:
A formalistic structured system that presents logical relations between truth values of logical sentences, and conclude the logical validity of statements.
Example: **If** the insect is an ant **Then** it lives in ant house.
The inspected insect is an ant. **Unique conclusion**: The inspected insect lives in ant house.

NUMBER THEORY

Pierre Henri Brocard (1845-1922) France

number theory

Brocard's problem $n! + 1 = m^2,$

Are there other numbers than (n, m)=(4,5), (5,11), and (7,71)

Erdös assumed there are such finite numbers. Computer search reached 10^9 with no result.

FUNCTION THEORY

Fredrich Wilhelm Bessel (1784-1846) Germany

calculus Bessel function, Bessel inequality

1838 Measured distance to Cygni 61, a nearby star with lateral orbit, by triangulating from half a year position differences.

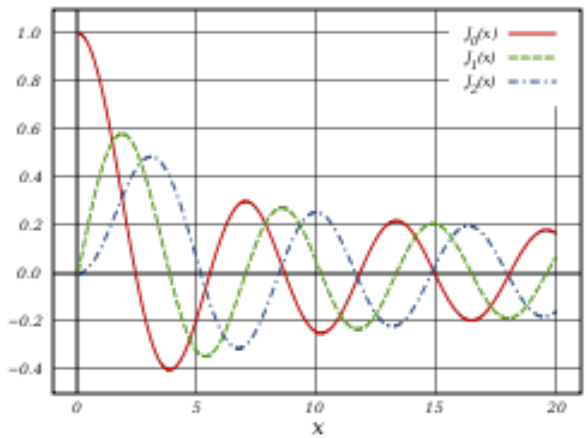
1839 Thomas Henderson measured distance to Alpha Centuri

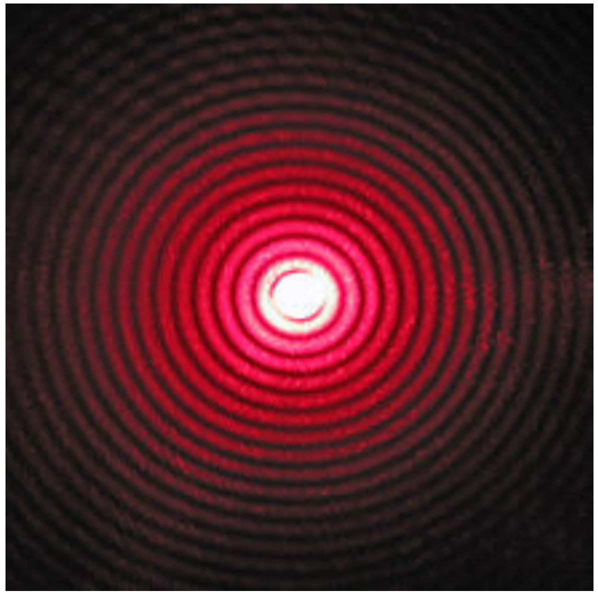
1840 Frederick von Struve measured distance to Vega.

All these established the position of the sun system in the milky way.

Bessel Function (first defined by Bernoulli) appears as solutions for physical problems with cylindrical or spherical symmetry. For example: Airy diffraction pattern of light traversing a pinhole, or star light imaged by the finite aperture of a telescope, where Bessel comes from integration on light phase exp(ikx).

Another example: Solution of Schrödinger equation for an electron in a spherical potential well.





Carl Friedrich Gauss (1777-1855) Germany

"The prince of mathematicians"

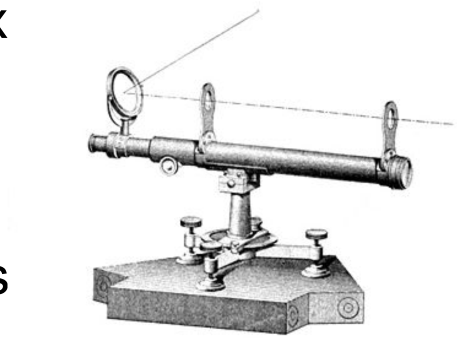
Algebra, number theory, mathematical analysis, statistics, Differential Geometry, Geodesy, Gravitation, Electricity and Electromagnetism, Astronomy, Optics.

In basic school the pupils were assigned to calculate the sum of numbers from 1 to 100. 7 years old Gauss realized the last +first terms equal the one before last+ second terms Etc., and gave the answer after a few seconds, inventing the sum of arithmetic series.

In Göttingen Gauss solved a problem posed by Euclid 2000 year before him, that equal edge polygon with number of edges that equals a Fermat number, $F_n=2^{2n}+1$ can be built with a ruler and a compass.
In the coming years he investigated number theory.

1799 Gauss studies in calculus and algebra are published in a book. includes The elementary theorem of Algebra: n-th order polynomial equation with complex number coefficients has n complex solutions.

When the Asteroid Ceres was discovered, attempts were made when and where he will be seen again, but since its orbit was close to the horizon only 3 degrees of his orbit was seen.
1801 Gauss calculated the entire orbit using conic sections, developed Fast Fourier Transform (preceding Fourier analysis at 1807 and the fast algorithm published at 1965 by Cooley & Tukey from I.B.M.) and predicted its return in a year with precision that impressed the science community, and gained him a chair in astronomy at Göttingen University, where he served till his death. Least Squares Approximation method that he developed for fitting the measurements became a most common basic method in ex



1818 Gauss was employed by the Kingdom of Hannover in geodesic survey. He invented the Heliotrope for these measurements, a precise way to send far a sun ray. For his Surveys he developed mathematical methods in Statistics, Complex analysis, Theory of functions, transformations between surfaces, Non-Euclidean geometry and more.

After 1828 Gauss consternated on Physics: crystallography and packing of spheres, Electricity and Magnetism (with Weber), including Gauss law in electrostatics (relating the flux from a closed surface with the charges enclosed inside), Telegraph (Between the telescope and Göttingen university). He organized mapping of the earth magnetic field, calculated the exact position of the magnetic pole and developed the electric and magnetic potential field equations.
In optics he calculated the linear (small angles) approximation to ray tracing called today The Gaussian approximation, and then calculated aberrations as higher order corrections.
Gauss gave mathematical description to insurance policies and pension rates, and dealt with financial speculations that greatly increased his income.
He implanted the love of topology to his students **Riemann, Möbius and Listing**.
He hated Napoleon, although Napoleon admired Gauss.
Gauss had well edited notebooks of all his work, but they were not published since he was a perfectionist and demanded always better polishing from himself. It is claimed that if all his ideas were published in time, instead of waited to be published by his followers, mathematics would be 50 years advanced.
Gauss hated to leave home, and only once traveled to Berlin at the invitation of Humbold, who admired Gauss. The book "**Measuring the world**" by Daniel Kehlmann describe the lives of Gauss and Humboldt, with opposite approaches to science.

1808 Gauss published optimal fit of of a line to a set of data points: Linear Regression. Was independently found at 1806 by Legendre.

Try to best fit n coefficients β_i of a linear function: $\sum_{j=1}^{n} X_{ij}\beta_j = y_i,\ (i = 1, 2, \ldots, m),$

For m>n measurements each n values of x and the corresponding y, so that the sum of square deviations is minimized.

$$S(\boldsymbol{\beta}) = \sum_{i=1}^{m} \left| y_i - \sum_{j=1}^{n} X_{ij}\beta_j \right|^2 = \left\| \mathbf{y} - \mathbf{X}\boldsymbol{\beta} \right\|^2.$$

The solution: at the minimum, all derivatives by the coefficients b_i should be zero. This gives n equations with n unknowns, and their solution in matrix notation is:

$$\hat{\boldsymbol{\beta}} = (X^T X)^{-1} X^T \boldsymbol{y}.$$

The errors of the coefficients are:

$$\mathrm{var}(\hat{\beta}_j) \approx S/(m-n) \left(\left[X^T X \right]^{-1} \right)_{jj}$$

Example: fit a straight line y=ax+b to a set of measurements x_j, y_j

$S = \Sigma\,(bx+a-y)^2$ $\quad dS/da = 2\Sigma(bx_j+a-y_j) = 0 \quad 2b\Sigma x_j + 2a\Sigma 1 = 2\Sigma y_j$

$dS/db = 2\Sigma(bx_j+a-y_j)x_j = 0 \quad 2b\Sigma x_j^2 + 2a\Sigma x_j = 2\Sigma y_j x_j$

We call the sums:

$\Sigma 1 = n \quad 1/n\Sigma x_j = X \quad 1/n\Sigma x_j^2 = Q \quad 1/n\Sigma y_j = Y \quad 1/n\,\Sigma y_j x = R$

And get

$b = (XY-R)/det \quad a = (XR-YQ)/det \qquad det = X^2 - Q$

1818 Gauss publishes papers about electromagnetic potential theory, and about "Gaussians".

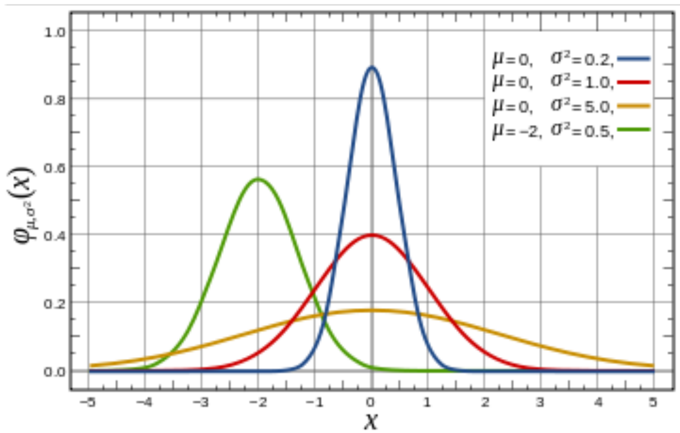
What is so special about Gaussians?

Many distributions approach a Gaussian when the number of samples is large.

Convolution of Gaussians is also a Gaussian.

Gaussian is a solution to differential equations.

$$f(x) = a \exp\left(-\frac{(x-b)^2}{2c^2}\right)$$



Karl Wilhelm Theodor Weierstrass (1786-1866)
modern analysis, Wierstrass function, M-test
Weierstrass defined the properties of continuous functions and uniform convergence of series. **Bolzano** worked before on converging limit, **Cauchy** developed the "delta-epsilon" criteria of convergence, but did not define uniform continuity.

Bolzano-Wierstrass theorem: Every bounded infinite sequence in $\mathbf{R}_n$ (points in n-dimensional Euclidean space with metric or distance function, D_{nm}, with zero distance to itself $D_{mm}=0$, and distances follow the triangular law $D_{nm}<=D_{nk}+D_{km}$) has a convergent subsequence (to a convergence point) OR: a subset of $\mathbf{R}_n$ is sequentially compact (for any epsilon distance there is another point) if and only if it is closed and bounded.

This theorem is equivalent to:

Heine–Borel theorem: For a subset *S* of Euclidean space $\mathbf{R}_n$, the following two statements are equivalent: 1. *S* is closed and bounded 2. *S* is compact (that is, every open cover of *S* has a finite subcover).

and to:

Cantor lema: if two series, on descending and the other ascending, approach each other without crossing but with diminishing distances, then both converge to the same point.

1861 Weierstrass showed that a continuous function is not necessarily derivable at all points in a range. It was not accepted, and was published by his students only at **1872**

Bernard Placidus Johann Nepomuk Bolzano (1781-1840) Czech mathematical analysis

See **Bolzano-Wierstrass theorem** in previous slide.

Bolzano published this theorem at 1817 as a lemma in a proof of The intermediate value theorem. Weiesrtrass wrote it independently As a theorem a few years later.
The intuitive idea behind the theorem is that a bound infinite series cannot "run away" too far, and at least some must be at diminishing distance. The proof is a constructive way to find these converging points.

What is the importance of these theorems?
It established a rigorous definition to continuity of functions.

Bolzano was first to produce rigorous approach to define The behavior of continuous functions, using the "delta epsilon" concept: for any delta you give, there is an epsilon that …

1851 Three papers by Bolzano were published after his death "paradox research" where he present examples for one-one correspondence between the elements of a set and a subset (e.g. between all integers and odd numbers).

We present the proof for a bound series in one dimensions.
Let $\{x_n\}$ (n=1, ∞) be a bound series. Then there must be a value M>0 so that $-M<x_n<M$ for all x_n. We now split the range into two halves. Since the series is infinite, there must be infinite number of points at least in one of the halves. We chose this half and cut it into two halves again, and chose the one with infinite number of points, and so on.
The series of cuts obey:

1. All are closed
2. Every one is enclosed in the previous ones.
3. The lengths approach zero (length of k-th cut equals $2M/2^k$)
4. All selected ones contain infinite number of points.

According to Cantor's lemma, the series converge.

Siméon Denis Poisson (1781-1840) France

Probability: Poisson random variable, Poisson approximation, Poisson process

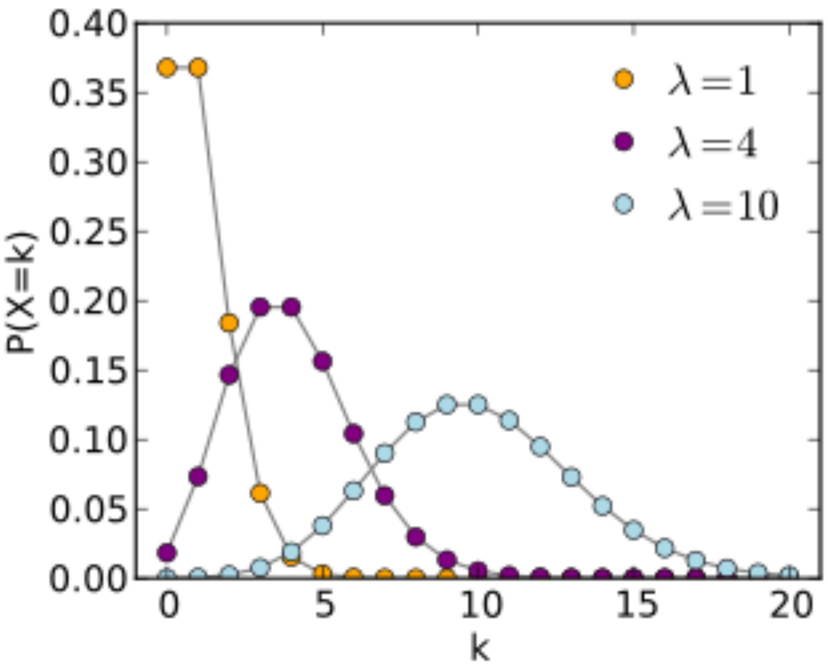
Poisson equation, the electric potential around charges, ρ

$$\nabla^2 \phi = -4\pi\rho$$

$$\nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = -\frac{\rho_e}{\varepsilon \varepsilon_0} \; .$$

Poisson distribution, f(k, λ); a discrete distribution (the variables, k,λ, are integers) expressing the probability to have k heads in λ coin tossing. As noted above, for larger λ the Poisson distribution approached a Gaussian.

$$f(k;\lambda) = \Pr(X{=}k) = \frac{\lambda^k e^{-\lambda}}{k!},$$



George Green (1793-1841) England

Calculus.

1828 Green's theorem: The integration on the normal projection of a vector function over a closed Surface, C, equals the integration on the divergent of this function over the volume enclosed, D.

$$\oint_C (L\,dx + M\,dy) = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) dx\,dy$$

$$\iint_D (\nabla \cdot \mathbf{F})\,dA = \oint_C \mathbf{F} \cdot \hat{\mathbf{n}}\,ds,$$

Complements in integral equation the Poisson differential equation.

Equivalent to Stokes theorem: volume Integral on divergent equals surface integral on the gradient. This theorem is important in electromagnetic field theory, since by a closed surface integral we determine the amount of charges inside, but the volume integral cannot be performed due to the singularities at the position of charges.

Green Function: is the function, G(x,s) that when Û is operating on it (e.g. Û is a differential equation) gives a delta function δ(s-x) (impulse):

Û G(x,s)=δ(s-x)

If we know G, we can solve Û g(x)=f(x) for arbitrary f(x) by convolution of G(x,s):

g(x)=∫ f(x-s)G(x,s) ds

Julius Wilhelm Richard Dedekind (1831-1916) Germany
number theory, Dedekind cuts (intersections), Dedekind circles, Piano-Dedekind axioms.

1872 A Dedekind cut in rational numbers is a separation into two non-empty groups, A & B, such that every element in group A is smaller than all elements of group B, and so that the elements of A do not have a maximum, but B has a minimum.
Example, "rational cuts": if r is a rational number, and we chose A={x|x<r} & B={x|x≥r} then A|B is the cut corresponding to r. A is the open interval (−∞, r), and B is the interval [r, +∞).

Dedekind cuts give rigorous definition to limits, and are one of the two methods to define the field of real numbers that are independent on geometrical axioms.

At that year **George Cantor** proposed construction based on "Cauchy series":
A series with converging elements: For any positive distance, ε , there is a place in the series that for all consecutive elements the distance between them is smaller than ε.
Every converging series is a Cauchy series. The inverse depends on the properties of the space.

Peter Gustav Diricklet (1805-1859) Germany
combinatorics, analytical number theory, **Diricklet's theorem**
Diricklet's function, series, units law
Problem of prime density.

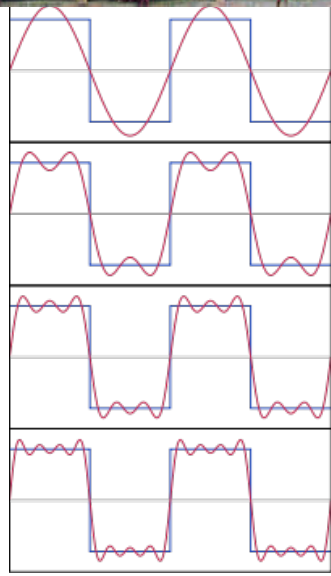
<ins>Dirickle's principle (dove's nest)</ins>: if there are n doves and m<n cages in the dovecote, there will be at least one cage with at least two doves.

Other examples: in a group of 13 people, at least two were born in the same month.

If we pull out 3 socks from a drawer with socks in two Colors, at least two will have the same color.

If in a party everyone shook hands with others, there are At least two that shook the same number of hands.

Every irrational number can be approximated by a rational Number to any precision.



Diricklet proved that every function can be expanded in terms of Fourier series.

PROBABILITY

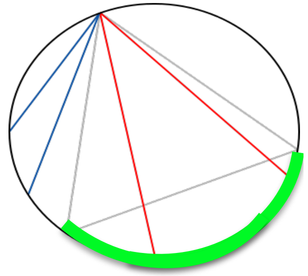
Joseph Bertrand (1802-1829) France

probability, number theory

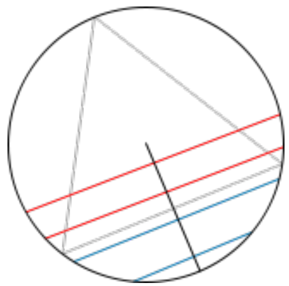
Bertrand conjecture: for any prime p exist integer n that: n < p < 2n-2

Bertrand-Tchebysheff's theorem: n < p < 2n

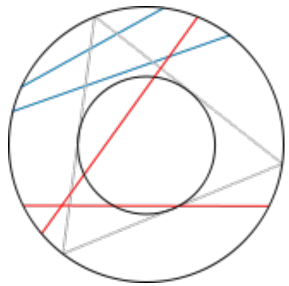
Bertrand paradox: what is the probability that a chord in a circle Will be longer than an equal edge triangle enclosed in the circle ? Here are three different solutions:



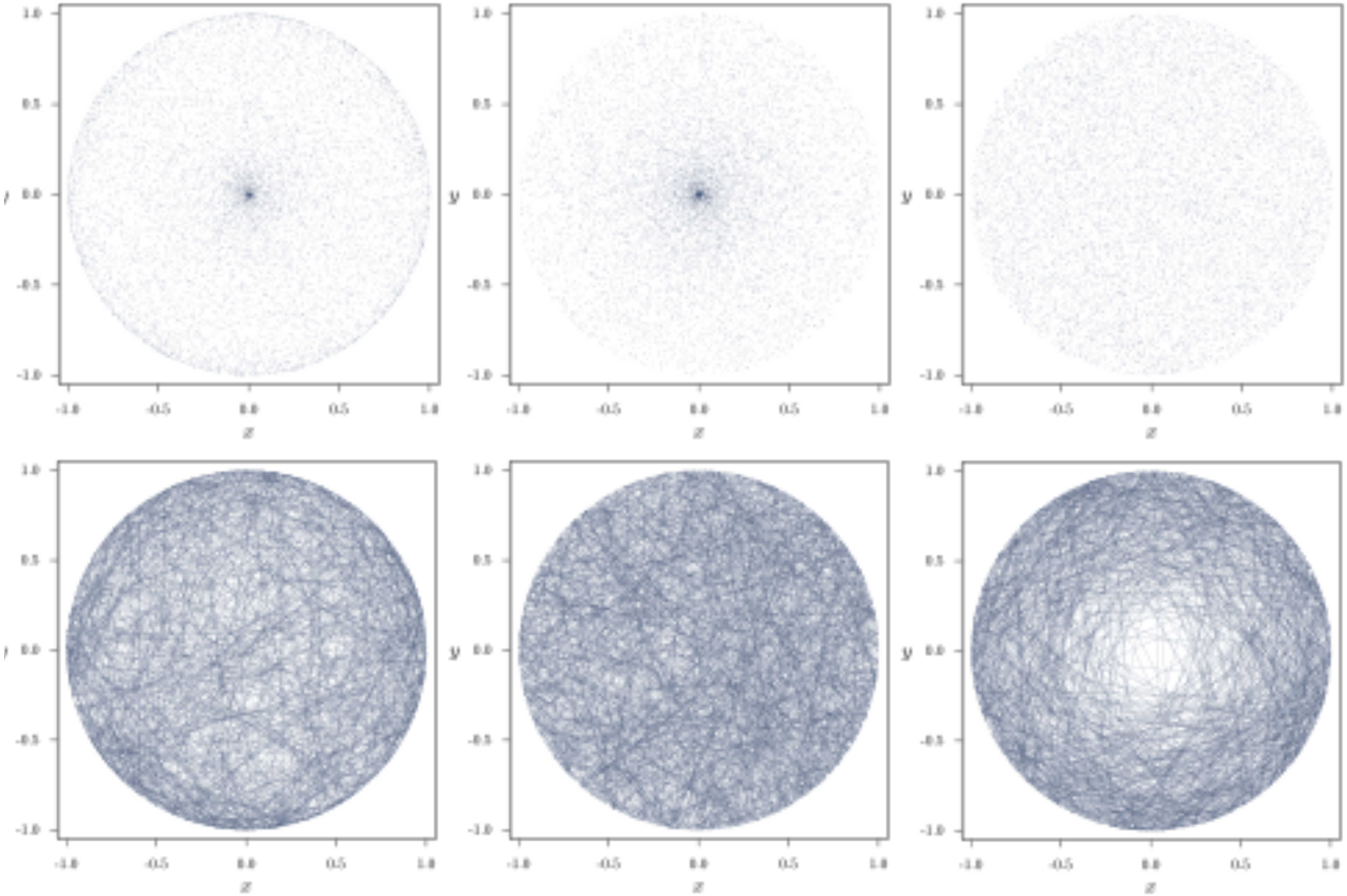
1. Draw a chord from the triangle vertex. The probability that it will be longer than one edge is proportional to the length of the green arc Divided by the circle perimeter, which is 1/3.



2. Draw a radius perpendicular to one edge. The probability is the length of the line out of the triangle area, which is 1/2.



3. Chose a point inside the circle, and draw a chord that the point is at its center. If it crosses the inscribed circle within the triangle, Its length is longer than the triangle edge. From the ratio of the circles area the probability is 1/4.



Top plates: the distribution of the chord centers according to the three solutions.
Bottom: the chords. The difference between the distributions account for the paradox.

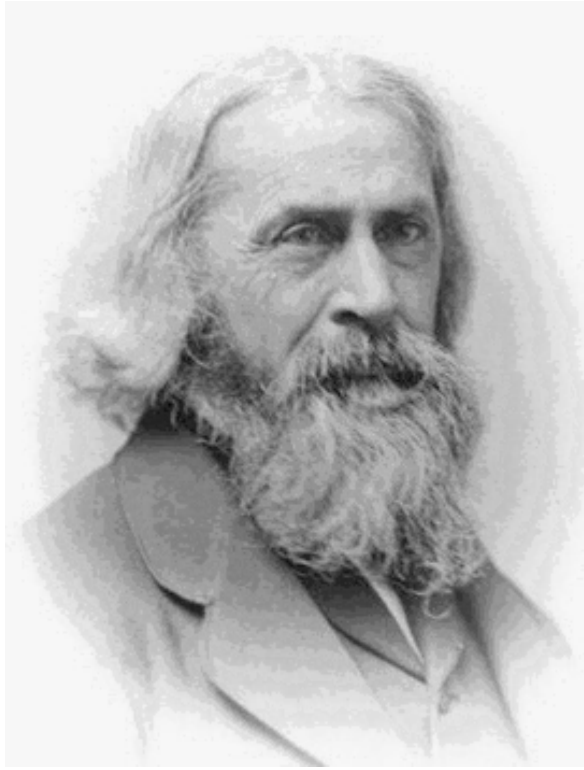
PROBABILITIES

1. Probability is the ratio between the “preferred event” and the total number of events.
2. The basic assumption: all events have equal probability.
3. For independent events the probability that two will happen is the product of the probabilities for each to happen: **Pr(A,B)=Pr(A)*Pr(B)**
4. For independent events: Probability for B to happen after A is probability of A times the probability that A and B will happen:

   Pr(A then B)=Pr(A)*Pr(A&B)
5. Probability for A after B is the probability that A & B will happen, divided by the probability for B: **Pr(A after B)=Pr(A&B)/Pr(B)**
6. Bayesian probability: Where event $A_j \in \{A_1,A_2, \ldots A_n\}$ exhaust the list of possible causes for even B, Pr(B)= $\Pr(A_1,A_2, \ldots A_n)$, then

$$\Pr(A_i|B) = \Pr(A_i)\frac{\Pr(B|A_i)}{\sum_j \Pr(A_j)\Pr(B|A_j)}.$$

Benjamin Peirce (1809 –1880) America



1867-1874 Supervised US coast survey.

Peirce's Criterion: Statistical rejection of outliers.
Called ROBUST STATISTICS

Iterative method: Calculate average and standard deviation, and reject measurements deviating more than two or three standard deviations from the average. Then recalculate average and standard deviation of the remaining measurements. The convergence is fast – negligible changes after two three iterations.

Other methods: replace average and standard deviations by median and absolute deviates. These statistical measures are less affected by outliers.

Robust statistics is of great importance in statistic applications e.g. to medical experiments, where outliers for reasons independent on the defined test can bend the results. The regulations of the FDA (Federal Drug Administration) provide rules for excluding “special” patients from drug tests.

Jean-Charles, chevalier de Borda (1733 –1799**)** French sails man

Boda count - a method of choosing head and members of a council, where every voter gives a ranged list of chosen delegates, and the selection is based on ranks.

This is the way <u>Eurovision</u> selects their songs rating based on three graded choices from each country. The method is applied in academic councils, but was more difficult to deal with general elections in pre-digital age. There are reasons for not implementing this method in modern elections, where most people would not have more than one preference…

ALGEBRA

Wilhelm Jordan (1842-1899) Germany **Geodesist**

Linear Algebra   Gauss-Jordan elimination method

Jordan was an engineer who worked in measurements. to best fit parameters using least squares he had to invert matrices, and he improved the stability of the inversion by Gauss elimination by <u>exchanging rows and columns so that normalization was always with the largest element.</u>

Lets calculate the best parameters β in a linear combination of m functions ϕ for measurements y_k at points x_i:

$$f(x, \beta) = \sum_{j=1}^{m} \beta_j \phi_j(x),$$

The matrix of the functions values at the measurement points:

$$X_{ij} = \frac{\partial f(x_i, \boldsymbol{\beta})}{\partial \beta_j} = \phi_j(x_i),$$

And the solution is the inverse matrix multiplied by the measurements vector:

$$\hat{\boldsymbol{\beta}} = (X^T X)^{-1} X^T \boldsymbol{y}.$$

Pseudo code for

Gauss-Jordan elimination method

For matrix inversion

```
i := 1
j := 1
while (i ≤ m and j ≤ n) do
  Find pivot in column j, starting in row i:
  maxi := i
  for k := i+1 to m do
    if abs(A[k,j]) > abs(A[maxi,j]) then
      maxi := k
    end if
  end for
  if A[maxi,j] ≠ 0 then
    swap rows i and maxi, but do not change the value of i
    Now A[i,j] will contain the old value of A[maxi,j].
    divide each entry in row i by A[i,j]
    Now A[i,j] will have the value 1.
    for u := i+1 to m do
      subtract A[u,j] * row i from row u
      Now A[u,j] will be 0,
       since A[u,j] - A[i,j] * A[u,j] = A[u,j] - 1 * A[u,j] = 0.
    end for
    i := i + 1
  end if
  j := j + 1
end while
```

GEOMETRY

Hermann Grassmann 1809-1877

Invented vector analysis, and inner and outer vector multiplications:

Inner (or scalar) product: $A.B = \Sigma A_i * B_i$

Outer (or vector) product: $AxB = (A_2*B_3 - A_3*B_2 , A_3*B_1 - A_1*B_3 , A_1*B_2 - A_2*B_1)$ for 3D.

Grassmann received little recognition during his life, until Cauchy published his work under his own name.

Arthur Cayley 1821-1895

A lawyer and amateur mathematician, who unified Euclidean, projective and metric geometries. Presented algebraic invariants and matrices that were applied in Quantum mechanics.

Bernhard Riemann 1826-1866

A student of Gauss. Developed the geometry of Riemann surfaces.
Geodesic coordinate system, with curvature tensor, in n-dimensions.

Felix Klein 1849-1925

Group theory – a way to organize symmetries and invariants under geometrical transformations.
Invented the "Klein bottle" – the 3D analogue of Möbius ring: a closed surface with one side only (see following slides).

David Hilbert 1862-1943

Invariant theory.
1899 Reformulated geometry based on 21 axioms, and presented 23 unsolved problems as a challenge for future mathematicians.

Oswald Veblen 1880-1960

Developed a set of axioms for geometry and projections with **John Young**
Published with his student **Henry Whitehead** topology and differential geometry, where he defined differentiable manifold.

Donald Coxeter 1907-2003

Regular polytopes in n-dimensions (Coxeter group).
Non-Euclidean geometry, group theory, combinatorics.

ANALYTICAL GEOMETRY

David Hilbert I (1862-1943) Germany

algebraic number theory, Euclid geometry

Hilbert space, 23 problems, dictionary, basis theorem, zeros theorem.

Studies in Konigsberg university, where he met Minkovski.

Paul Albert Gordan wrote about his paper on invariants (quantities such as energy and momentum, that are preserved) that it is “theology not mathematics”.

1895 Moved to Göttengen by the advice of **Felix Klein**. This was a world center for mathematics, and he stayed there till his death. He evidence the “cleaning up” of Jews from the university under the Nazis. He answered the Nazi science minister’s question “ “how is science in Göttengen?” that it is “non-existing”.

His most known contribution is <u>Hilbert spaces</u>, that were a basis for quantum mechanics equations. They define a vector field over a complete field, (e.g. real numbers) in a finite or infinite dimensions, with inner (or scalar) product that defines norm (equivalent to distance between points in space).

Otto Hölder (1859-1937) Germany. Astronomer

Hölder's inequality:

If p,q ∈ [1, ∞]

and 1/p + 1/q = 1

then for measure function defined on space S:

$$\|fg\|_1 \leq \|f\|_p \|g\|_q.$$

Example: for p=q=2 and measure function over the integers *f* = $\sum_{k=1}^{n} |v_k|^2.$

we get Cauchy-Schwartz inequality: $|\langle u, v\rangle| \leq \|u\| \cdot \|v\|$

Inequality:

or: $$\left|\sum_{i=1}^{n} u_i \bar{v}_i\right|^2 \leq \sum_{j=1}^{n} |u_j|^2 \sum_{k=1}^{n} |v_k|^2.$$

This is an extension of the triangle inequality for any space.

Otto Hölder (1859-1937) – cont.

Jordan-H**ö**lder theorem:
In group theory NORMAL SERIES of the group is a chain of sub-groups, each is normal sub-group of the one above:
The triangle denotes a normal sub-group.

$$1 = H_0 \triangleleft H_1 \triangleleft \cdots \triangleleft H_n = G,$$

The factors of the series are the ratio groups $\mathbf{G_i/G_{i+1}}$
Refinement of a series are all sub-groups $\mathbf{L_{ij}}$ of the above series or: $G_i \triangleright L \triangleright G_{i+1}$
and $G_i \neq L \neq G_{i+1}$ in this case $G = G_0 \triangleright \cdots \triangleright G_i \triangleright L \triangleright G_{i+1} \triangleright G_k$ is the refinement of the original series.
Composition series of a group is a normal series ending at {e} that cannot be refined.
Therefore a normal series is a composition series if and only if it ends with {e} and all its factors are simple groups.

Jordan-Hölder theorem says that the composition factors of a finite group G are constants up to isomorphism and order changes, and are independent on the composition series.
A soluble group is a group that has normal series with Abelian factors. Insoluble group always has composition series with factors that are simple non-Abelian group.

1887 Showed that Gamma function satisfies no algebraic differential equation. Γ(n)=(n-1)! $\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\, dx$

William Roan Hamilton (1805-1865) Ireland

graph theory, quaternion algebra, vector analysis

Hamiltonian , Hamiltonian path, Hamiltonian mechanics

A new formalism for classical mechanics laws and for electromagnetism.
Similar to the Lagrangian, the difference between kinetic and potential energy: L=T-V
that obey the principle of minimal action:

$$\delta \int_{t_1}^{t_2} L\,dt = 0.$$

and Lagrange–d'Alembert principle
of zero work at equilibrium against virtual displacements

$$\sum_i (\mathbf{F}_i - m_i \mathbf{a}_i) \cdot \delta \mathbf{r}_i = 0$$

he defined the Hamiltonian as a function of coordinates and momentum

$$H = H\left(q_1, \cdots, q_N; \frac{\partial S}{\partial q_1}, \cdots, \frac{\partial S}{\partial q_N}; t\right) \qquad S = S(q_1, q_2 \cdots q_N, t)$$

that obeys the equation of motion:

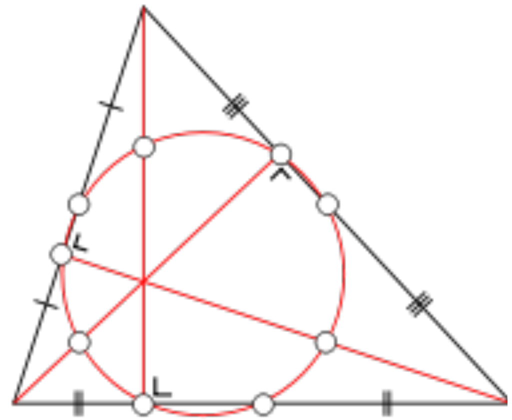
$$H + \frac{\partial S}{\partial t} = 0$$

The importance of this formalism is that it was applied to
Quantum Mechanics, to solve the wave function of electrons
In a potential energy field.

Charles Hermit (1815-1897) France

geometry, number theory   Hermit constant,

Hermit interpolation: extending Newton's interpolation, fitting a polynomial to n-values at n points, Hermit fitted the values of a polynomial function to the values and the derivatives (till m-th derivative) of a set of measurements.



O'lery Tarcam (1845-1922) France

number theory, geometry Tarcam's circle, Tarcam's points,

<u>proved Feuerbach's theorem on 9 point circle</u>

These nine points are:

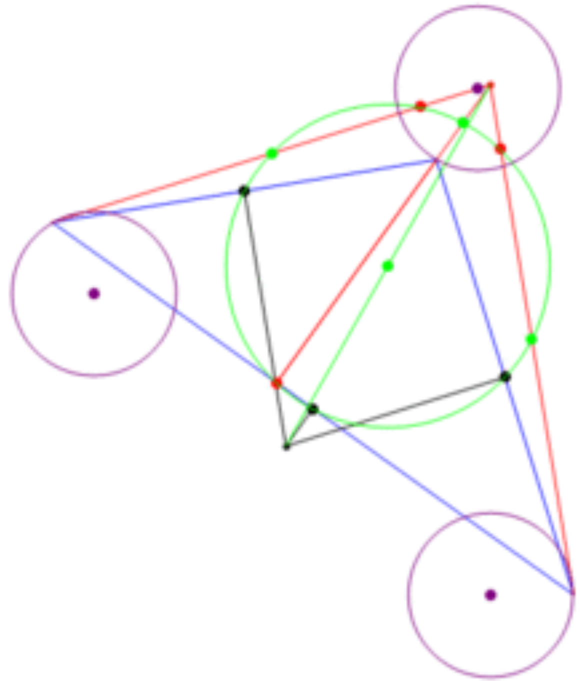
The **midpoint** of each side of the triangle

The **foo**t of each altitude

The **midpoint** of the line segment from each vertex of the triangle to the orthocenter (where the three altitudes meet; these line segments lie on their respective altitudes).

Feuerbach's theorem states that the nine-point circle of any triangle is tangent internally to the incircle and tangent externally to the three excircles.

Even if the orthocenter and circumcenter fall outside of the triangle, the construction still works.



William Henry Young (1802-1829) England

number theory

<u>Young's theorem</u>: If a function and its derivatives are continuous, the partial derivatives are commutative: $\frac{\partial^2 f}{\partial x_i\, \partial x_j} = \frac{\partial^2 f}{\partial x_j\, \partial x_i}.$

<u>Young's inequality</u>: p and q are positive real that obey 1/p+1/q=1 then: $ab \le \frac{a^p}{p} + \frac{b^q}{q}.$
Equality only if $\mathbf{a^p=b^q}$

If: $\frac{1}{p} + \frac{1}{q} = \frac{1}{r} + 1$ and we define: $\|f\|_p = \left(\int |f(x)|^p\, dx \right)^{1/p}$

then: $\|f * g\|_r \le \|f\|_p \|g\|_q.$ (* assigns a convolution)

This is the triangle inequality at an infinite dimensional space.

Karl Gustav Jacob Jacobi (1804-1851) Germany

calculus Elliptical functions.

Jacobean determinant: $J == \partial(f_1,\ldots,f_m)/\partial(x_1,\ldots,x_n) =$

$$\mathbf{J} = \frac{d\mathbf{f}}{d\mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{f}}{\partial x_1} & \cdots & \dfrac{\partial \mathbf{f}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \cdots & \dfrac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial f_m}{\partial x_1} & \cdots & \dfrac{\partial f_m}{\partial x_n} \end{bmatrix}$$

This gives the linear approximation at the vicinity of a point for multi-parametric functions.

Jacobean comes out at substitution of integration variables, the multidimensional extension of: $\int f(x)dx = \int f(y)\, dx/dy\, dy$

For a single variable.

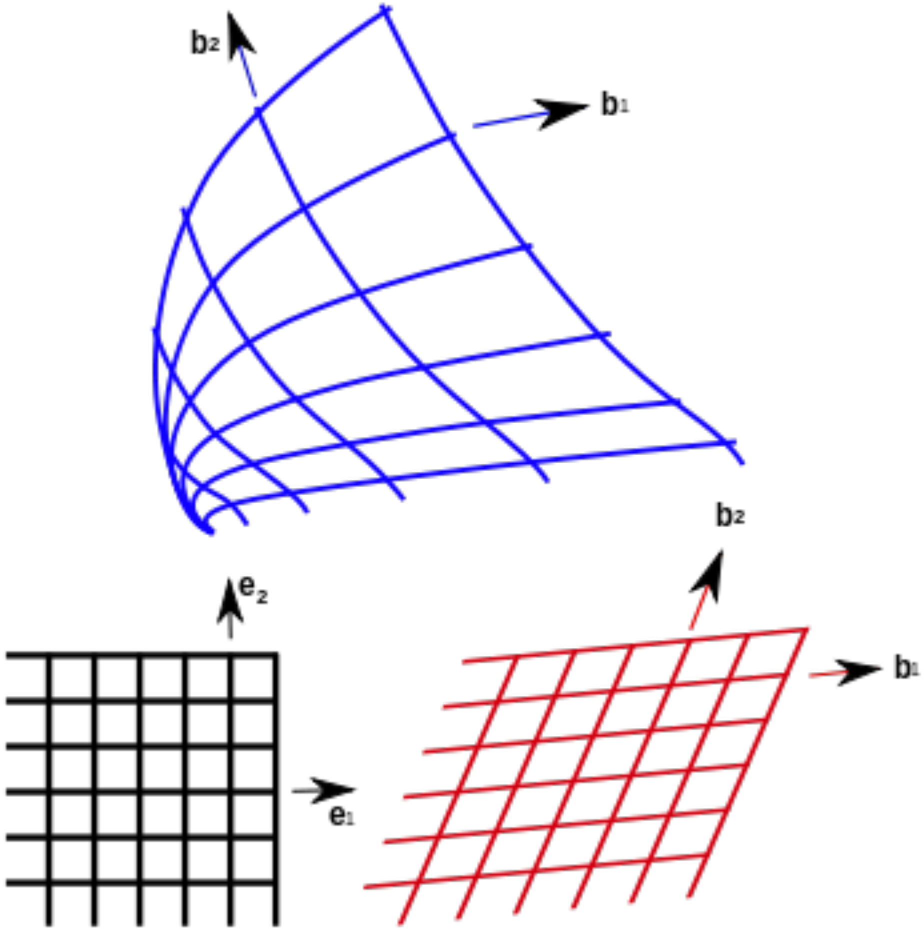
Gabriel Léon Jean Baptiste Lamé (1795-1870) France

number theory, calculus

Lame’s curve, Lamé constants, elasticity.

curvilinear coordinates:

local transformation Cartesian coordinates



Eduard Lukas (1842-1891) France

number theory, Fibonacci's numbers.
Lukas series, Lukas-LeHamer test, Lukas-Karmeikle numbers,

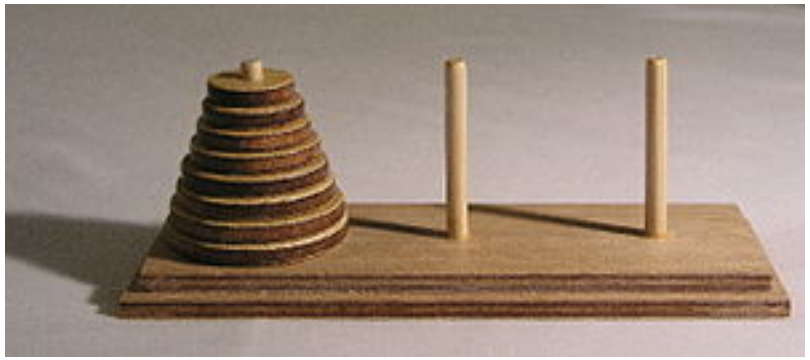
<u>Cannon ball problem</u>: a pile of balls enclosed in a pyramid.
Proposed a challenge: prove that the only solution to the Diophantic equation:

$$\sum_{n=1}^{N} n^2 = M^2$$

is N=24, M=70.

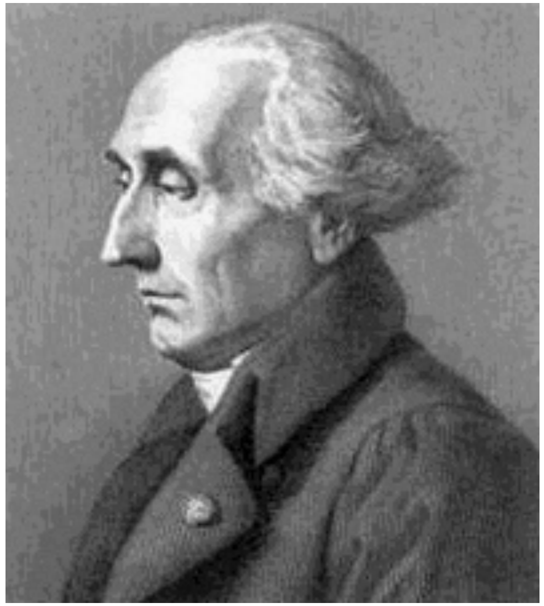
This was proven at 1918, and is significant for String Theory in physics.

Lukas showed that $(2^{127}-1)$ is a prime.



Tower of Hanoi puzzle
Iterative solution

Joseph Lewis Lagrange (1736-1813) Italy-Berlin-Paris
see 18-th century



<u>Euler-Lagrange equation</u>:

$$L_x(t, q(t), q'(t)) - \frac{\mathrm{d}}{\mathrm{d}t} L_v(t, q(t), q'(t)) = 0.$$

In physics the Lagrangian, $L = T - V$, obeys the dynamic equation:

$$\frac{\partial L}{\partial q} = \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}}$$

<u>Lagrange multipliers</u>: turns a minimization problem of a function $\mathbf{f(x_i)}$ of n variables with k constraints, $\mathbf{g_k(x_i)=0}$ to a minimization problem of $\mathbf{H=f+\lambda_k g_k}$ with k+n variables without constraints.
Example: find the maximum of xy on a circle $x^2+y^2=1$

$$\frac{\partial h(x,y)}{\partial x} = y + \lambda \frac{\partial\left(x^2+y^2-1\right)}{\partial x} = 0 \rightarrow y = -2x\lambda \rightarrow \lambda = -\frac{y}{2x} \quad y = \pm\sqrt{\frac{1}{2}}$$

$$\frac{\partial h(x,y)}{\partial y} = x + \lambda \frac{\partial\left(x^2+y^2-1\right)}{\partial y} = 0 \rightarrow x = -2y\lambda \rightarrow x^2 = y^2 \quad x = \pm\sqrt{\frac{1}{2}}$$

$$\frac{\partial h(x,y)}{\partial \lambda} = x^2 + y^2 - 1 = 0$$

Adrian Murray Legendre (1752-1833) France

<u>Legendre polynomials</u>: The radial solution for differential equations with spherical symmetry. Appeared later as the quantum mechanical solution for the electron wave function in Hydrogen atom

1806 Developed, independent of Gauss, best fit based on minimal square deviations, And was first to publish this method.

Joseph Liouville (1809-1882) France

number theory
Sturm-Liouville theorem, Liouville’s equation, function, formula

Liouville's theorem: Every complete function (that is expandable by Taylor series) that is bound must be a constant.

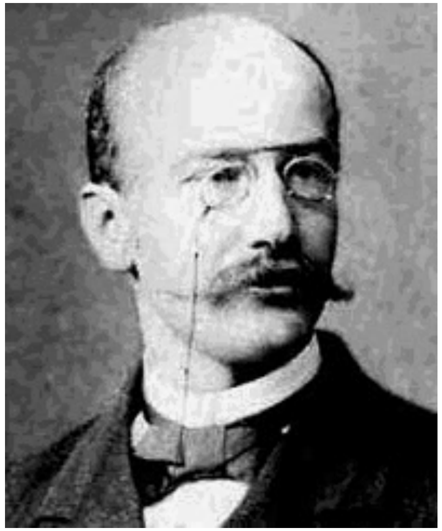
1815 Liouville shows that there are an infinite number of transcendental numbers.
Example: Liouville’s number

$$\sum_{k=1}^{\infty} 10^{-k!} = 0.110001000000000000000001000....$$

Karl F**erdinand Lindman (1862-1939)** Germany
number theory, Lindman-Wierstrass theorem

1882 Showed that π is not a solution of a polynomial equation with real coefficients.
Extended irrational numbers to transcendental numbers.
Proved that it is impossible to “square the circle”: use only a ruler and a compass to builds a square with area equals a circle.
1873 Hermit showed that **e** is a transcendental number.

George Cantor showed that the magnitude of rational numbers is $\aleph_0$ and the magnitude of transcendental numbers is $2^{\aleph_0} = \aleph$

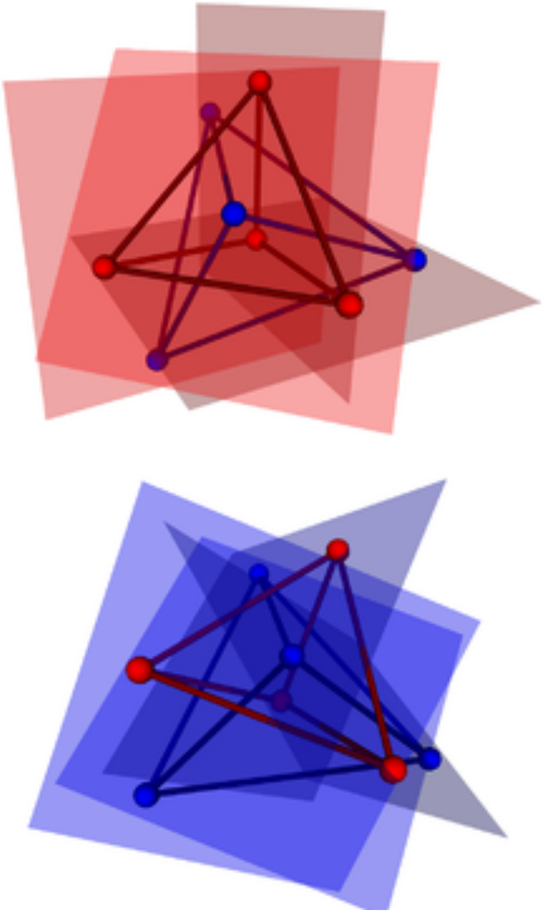


Augustus Ferdinand Möbius (1790-1868) Germany

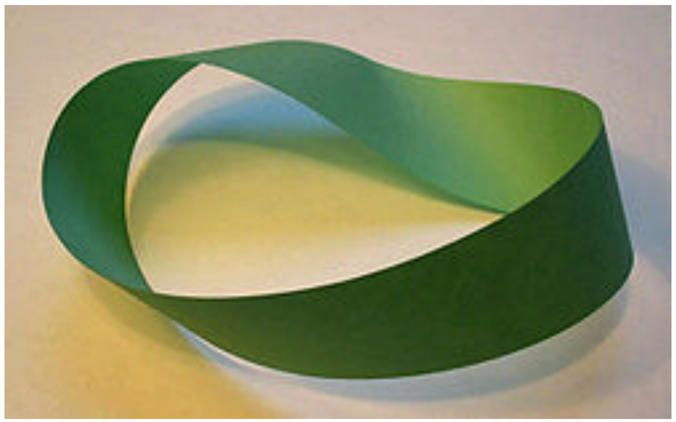
number theory, topology. Möbius inversion formula

1858 Studies one-sided surfaces and published Möbius ring.

Möbius configuration, formed by two mutually inscribed tetrahedral



Möbius ring



Christian Felix Klein (1858-1932) Germany

group theory, geometry   Klein bottle, group, geometry, surface

1872 Studied permutations of geometrical groups.

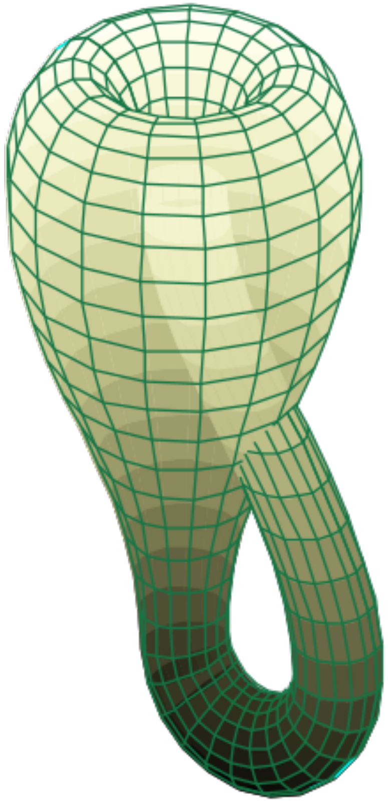
Bolyai-Lobachevsky three kinds of geometries: a line has two infinitely distant points.

Riemann: the points are imaginary, Euclid.

In this research he proposed the Klein bottle, the 3D extension of Möbius ring, and the Klein Four-group: the smallest group that is not cyclic. Below the multiplication table for the group. a,b are mirror reflection in x and y, c is rotation in 90 degrees.

The green cross with unequal arms is the geometrical object presenting the group.

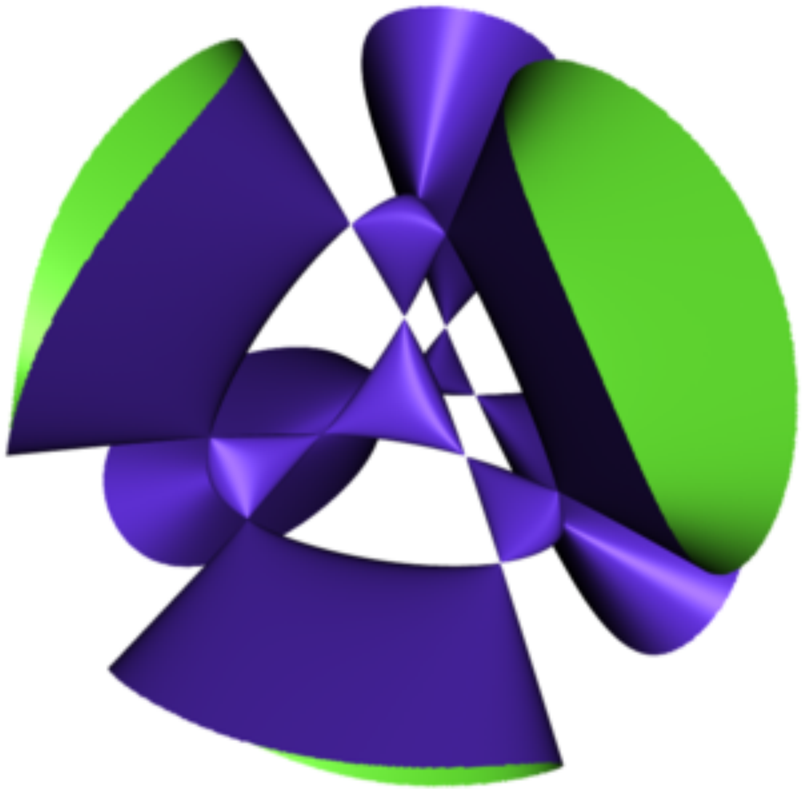
*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

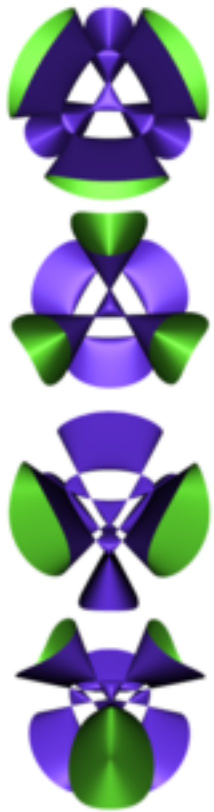


Ernst Eduard Kummer (1810-1892) Germany

number theory. Kummer function, series,

Kummer surface: results from taking the quotient of a two-dimensional abelian variety by the cyclic group {1, −1}



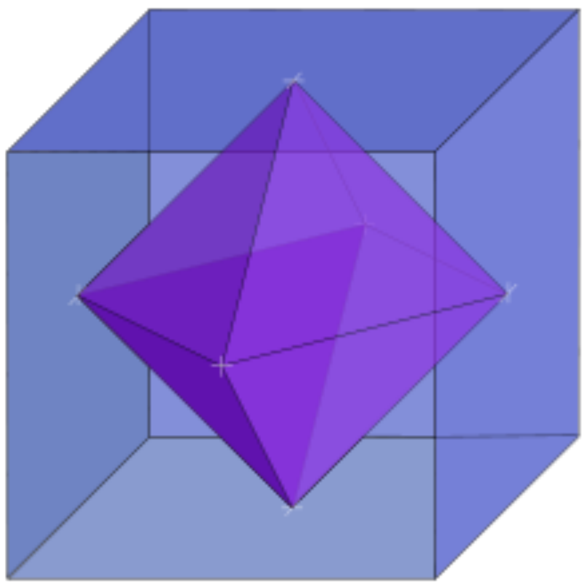


Eugene Charle Katalan (1814-1894) Belgium

number theoryɣ Katalan numbers, constants, problem, equation

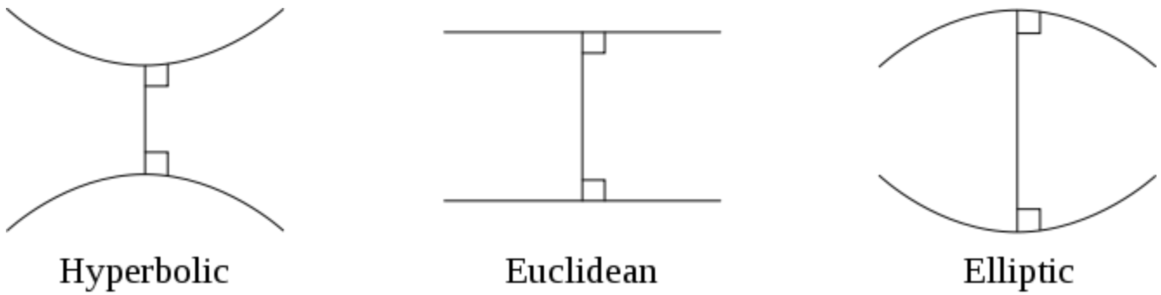
<u>Katalan conjecture</u>: the only solution for consecutive powers of integers $x^a - y^b = 1$
for $x,y>0$ and $a,b>1$ is: $x=3$, $a=2$, $y=2$, $b=3$

<u>Katalan solids</u>**:** or <u>Archimedean dual:</u>
The vertices of the polyhedron are at the center of faces
of the dual polyhedron



1826, Nikolai Ivanovich Lobachevsky (1792-1856) Russia

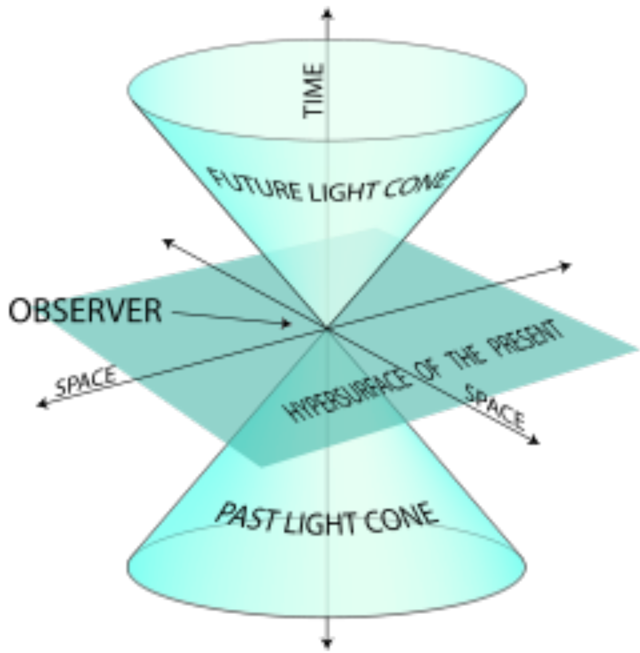
Lobachevsky developed Hyperbolic (non-Euclid) geometry, that has more than one parallels crossing a point.
Important for general relativity.



Herman Minkovski (1864-1909) Germany

number theory
Minkovski's theorem, inequality

Minkovski was Einstein's teacher.
His 4-dimensional space-time was the geometric presentation of the special relativity.



Marin Mersenne (1588-1648) France

Mersen (1856-1922) ????

number theory, father of acoustics

Mersen’s numbers (primes)

Emmy Noether (1882-1935) גרמניה

class theory, commutative algebra
Noeter’s normalization theorem, Noeter’s group

Fired from her academic position in Göttingen by the Nazis (was the first woman and Jew to have one), and moved to the US.

Noether established the link between symmetry of physical state equations and conservation rules.
Noether’s first theorem: every differentiable symmetry of the action of a physical system has a corresponding conservation law.
Examples: Conservation of momentum – derived from symmetry to translation.
Conservation of angular momentum – derived from symmetry to rotation.
Conservation of energy – derived from symmetry to translation in time.

The conservation laws brought to predict undetected products of radioactive decay, like the Neutrino, and define the quantum numbers characterizing elementary particles, such as charge, Baryon number and for photons – parity, strangeness and iso-spin.

George Gabriel Stokes (1819-1903) Ireland

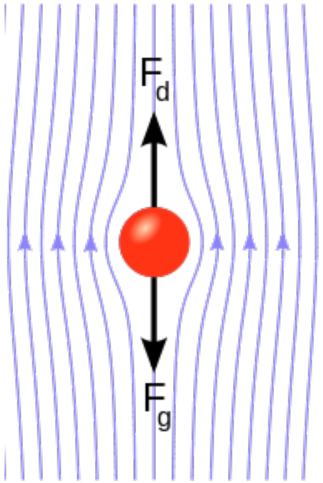
Stokes theorem: links integral over volume Ω of the derivative ∂ω of scalar function, ω, with the surface integral over the enclosing surface Ω∂ on ω

$$\int_{\partial\Omega} \omega = \int_{\Omega} \mathrm{d}\omega.$$

Kelvin–Stokes theorem: links between integral over volume Σ on curlF= $\breve{\nabla}$ F of a vector function **F**, and the integral over the enclosing surface ∂Σ of the component perpendicular to the surface **F**,

$$\iint_{\Sigma} \nabla \times \mathbf{F} \cdot \mathrm{d}\mathbf{\Sigma} = \oint_{\partial\Sigma} \mathbf{F} \cdot \mathrm{d}\mathbf{r}.$$

These relations provide a way to calculate the electric and magnetic fields at any point in space, given a distribution of charges or electric currents, despite the irregularities of these fields at the sited of the charges.



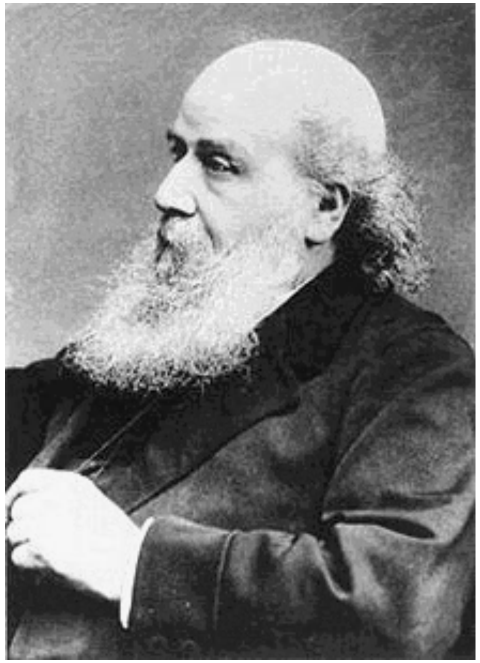
Stokes law: see in Physics,
a ball falling in viscous liquid

Thomas Jean Stiltis (1866-1894) Netherland

mathematical analysis

Riemann-Stiltis integral, Lebesgue-Stiltis integral,

James Joseph Sylvester (1814-1897) England



Linear algebra, geometry. Sylvester's theorem

Solved the Frobenius coin problem:
What is the highest value one cannot pay with two coin types.
e.g. with coins of 2 & 5 one cannot pay 3, but can pay
any higher value.

Sylvester's inequality for ranks of matrices of dimension n:
(rank is the highest dimension of the vector space
spanned by the matrix rows).

$$\operatorname{rank}(A) + \operatorname{rank}(B) - n \leq \operatorname{rank}(AB).$$

$$\operatorname{rank}(AB) + \operatorname{rank}(BC) \leq \operatorname{rank}(B) + \operatorname{rank}(ABC).$$

Necessary and sufficient condition for Hermitian matrix to be positively defined is that all its leading principal minors are positive.
(leading minor of order k is the determinant of the sub-matrix of k top rows and k left columns).

Function of matrices, F(A), expressed in terms of its EigenValues and Vectors:

$$f(A) = \sum_{i=1}^{k} f(\lambda_i)\ A_i\ ,$$

Sylvester's determinant theorem: $\det(I_p + AB) = \det(I_n + BA),$

Dimensions of A are pxn and B are nxp, I is the unit matrix.

Jules-Henry Poincaré (1854-1912) France polymath

A student of Hermit, who studied and worked also as a mining engineer.
Intervened for Dreyfus in his trial, claiming his charges have no real support.
Studied the geometry of the solutions to differential equations.
<u>Poincaré's conjecture</u>: three-dimensional topological sheet with simple connectivity (closed, compact, no boundaries or holes), is homeomorphic to a sphere.
Proved by **Gregory Perlman** 2002-6.
<u>Poincaré's group</u>: transformation group from space-time vectors (xyzt) in Minkovsky's space while saving metrics (distance) $ds^2=dt^2-dx^2-dy^2-dz^2$
<u>3-body problem</u>: showed chaotic solutions to sets of deterministic equations. E.g.:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma y - \sigma x,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \rho x - xz - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$

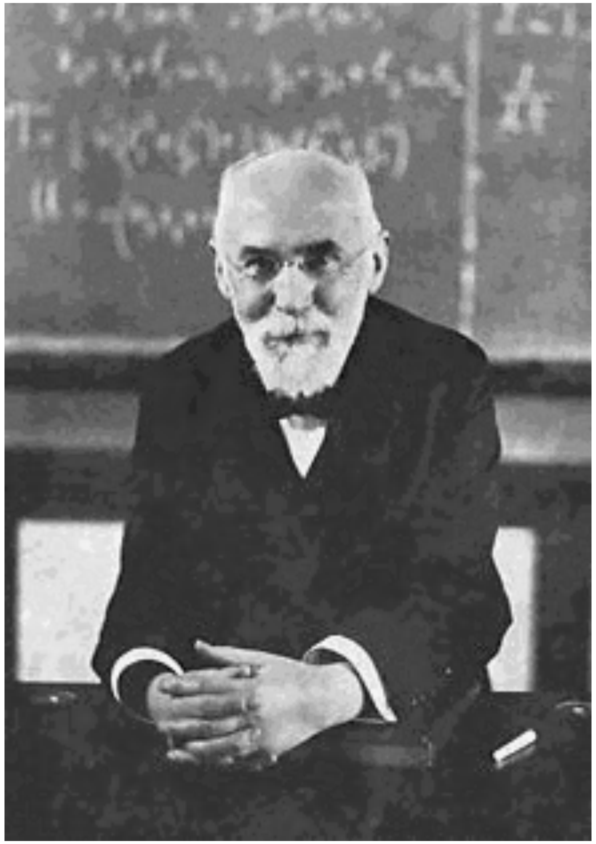
Formulated the relativity principle: physical laws are identical in inertial systems (moving at constant relative speed).
1879 Showed how to apply automorphic functions as uniform coordinates on algebraic curves.

Hendrik Antoon Lorentz (1853–1928) Netherland physicist

1902 Nobel prize with Pieter Zeeman for the theoretical explanation of Zeeman effect: the splitting of electron energy levels in atoms due to magnetic field. Developed the 4-dimensional space-time transformation that Einstein applied to special relativity.

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix},$$

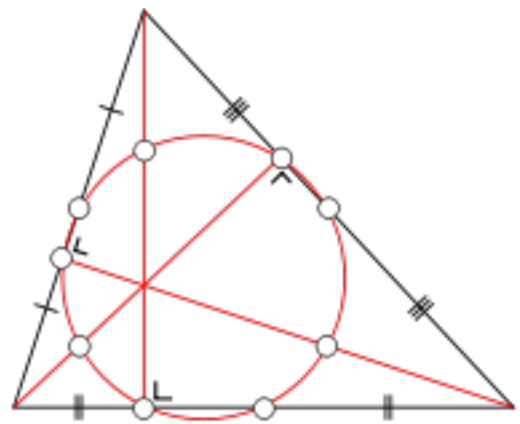
$$\begin{aligned} t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \\ x &= \gamma\left(x' + vt'\right) \\ y &= y' \\ z &= z', \end{aligned}$$



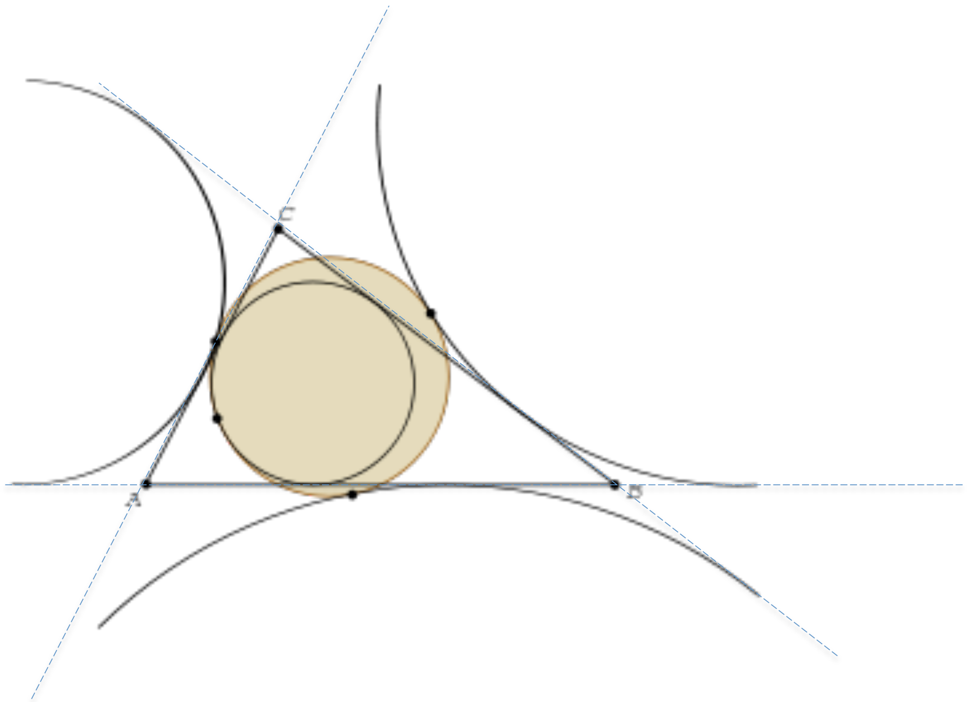
Karl Wilhelm Feuerbach (1800-1834) Germany

geometry

See **Olery Tarcam** 9-points on a circle problem:
For every triangle the following 9 points are on one circle:
3 edge centers
3 bases of center perpendiculars
3 center points between the vertices and the perpendiculars meeting point.



<u>Feuerbach's theorem</u>: the 9-point circle is internally tangential To the three circles enclosed by a triangle edge and the continuation of the other two edges.

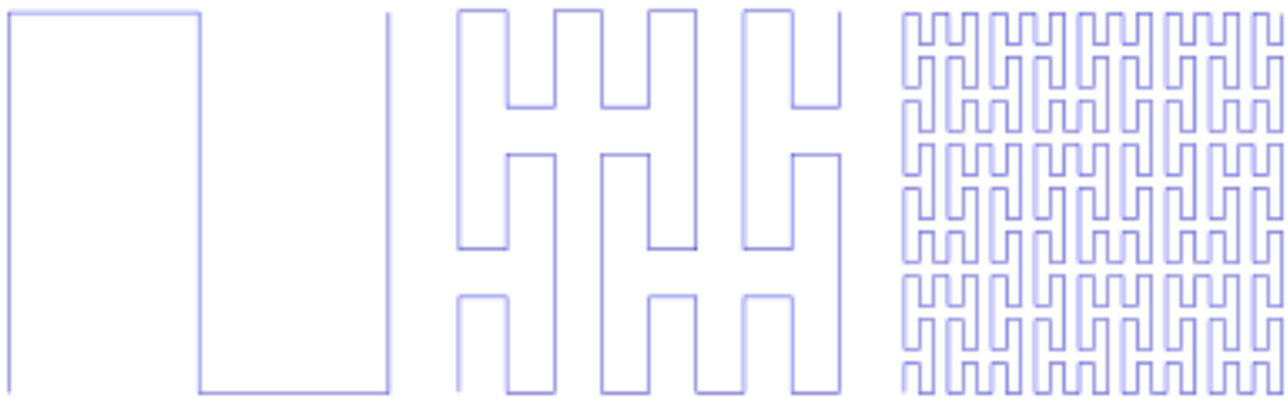


Giuseppe Peano (1858-1932) Italy

group theory, logics
Peano's projective plane,
1889 Peano's axioms defining the natural numbers, a foundation for extensions to negative numbers, rationales …

1. Zero is a number.
2. If a is a number, its consecutive is also a number.
3. Zero is not consecutive to any number.
4. Two numbers whose consecutives are identical are themselves identical.
5. (Induction axiom) If a group includes zero and all consecutives, it includes all numbers.

The first three iterations of Peano's curve that at the limit fills space.



Carl Pearson (1857-1936) England

Probability, Pearson’s match

Pearson’s contributions made the foundations of many statistical methods we apply today:

1. Correlation coefficient and the relation to linear regression – (maybe first defined by **Francis Galton**).
2. Moments of distributions – was borrowed from physics.
3. Continuous distribution functions – defined by up to 4^{th} order moments.
4. c^2 Test
   to define closeness of two distributions $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$
   or a distribution and a model. Assumes normal distributions, later extended to Mahalanobis distances.
5. P-value, estimating quality of model fitting (continued **Laplace’s** significance test for excess males in a population of half a million newborns). Estimated the probability that the model assumptions are wrong, or the complementary volume in χ^2 radius. Pearson (and later **Jerzy Neyman and Egon Pearson**) established a quantitative basis to statistical confirmation of model validity (1^{st} order errors, or false positives).
6. Principal component analysis – multiparametric linear fit of distributions.

Carl Pearson (1857-1936) cont.

Despite his important contributions, there is a dark aspect in Pearson's work that must be noted: The race doctrine was common among biologists, who attempted to establish a scientific foundation to their theories. Pearson proposed a "racial correlation coefficient" based on skull dimensions measurements. He was not alone. Humboldt, the German naturalist, during his south American explorations, risked himself and stole native skulls from burial grounds in order to make measurements and reveal the origin of species and the relation to European populations.

CORRELATION COEFFICIENTS

$$\mathrm{cov}(X,Y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - E(X))(y_i - E(Y)).$$

covariance

$\bar{x}$ ==E(x)=Σ(x_i)/n

average

$$\sigma_X = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

standard deviation

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y}$$

correlation coefficient

They estimate statistically if when one variable increases the second one increase, decrease or random. r=1 means perfect linear correlation between the two variables, r=-1 means anti-correlation.

linear regression $y_i=\beta x_i+\alpha$

The best fitted line to a set of n points **(x_i, y_i)** and the relation to the correlation coefficient (or the covariance) -

$$\beta = \frac{(\Sigma y)(\Sigma x^2) - (\Sigma x)(\Sigma x y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$\alpha = \frac{n(\Sigma x y) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$\beta = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$= \frac{\mathrm{Cov}[x, y]}{\mathrm{Var}[x]}$$

$$= \rho_{xy} * \sigma_y/\sigma_x$$

$$\alpha = \bar{y} - \hat{\beta}\,\bar{x},$$

MOMENTS

N-th order moment of a function f(x) around x=c:

$$\mu_n = \int_{-\infty}^{\infty} (x-c)^n \, f(x)\, dx.$$

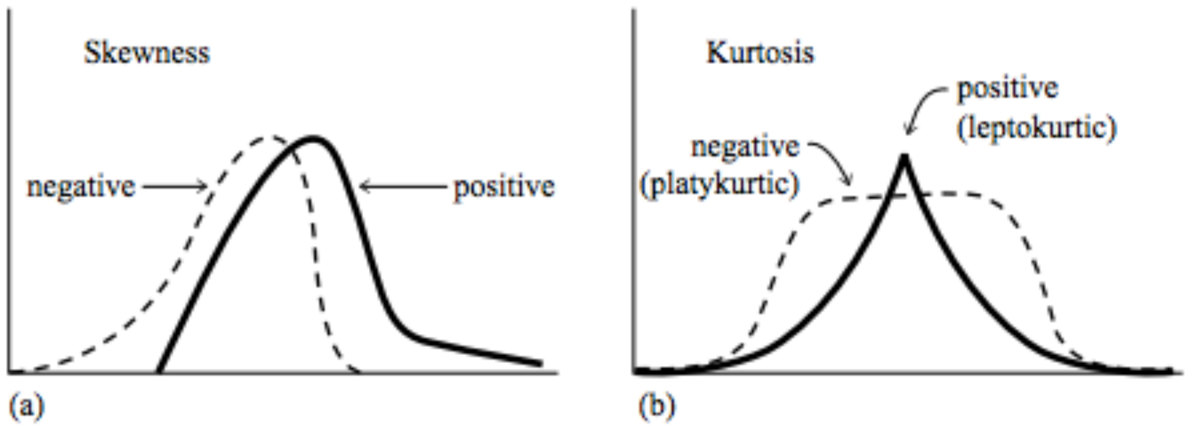
The first moment is the average:

$\bar{x}$ = 1/n Σ x_i

The second moment around the average is the standard deviation squared normalize by the number of degrees of freedom, n/(n-1) :

$$\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

The third moment around the average is called Skewness, and estimate single-sided deflection of the distribution.
The fourth order moment called Kurtosis, estimates the weight of the distribution tails.



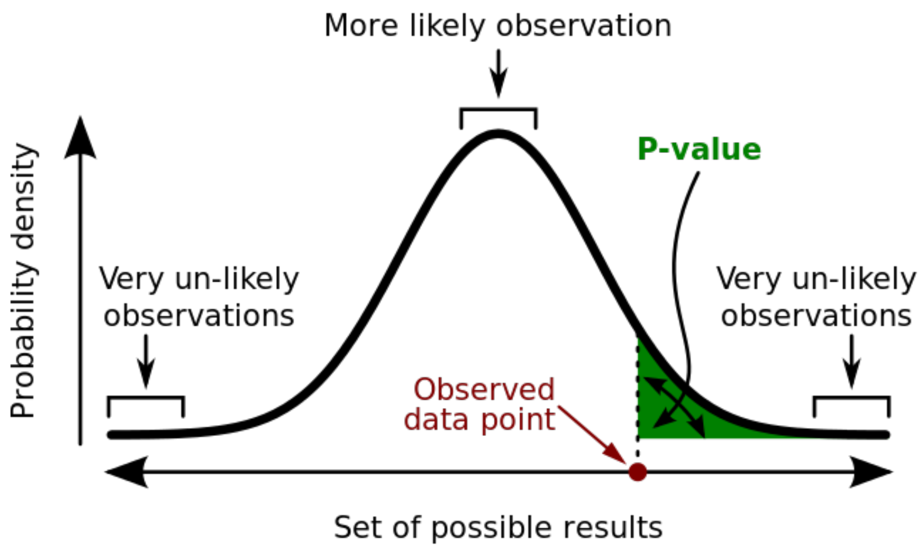
p-value

Assuming a statistical model ("hypothesis", for example normal distribution of the measurements) p-value estimates the probability to obtain a value more extreme than a given value. The smaller the p-value is, the more significant statistically is the value as presenting the measurements. P-value is one-sided or two-sided.

For example, if a set of measurements has an average and standard deviation values, C & S, and the distribution is assumed to be Gaussian, the p-value for C+x is given by the integral over the distribution tail from C+x to infinity, which is called the "error function" erf(x).

Example: The probability to get 5 times heads in 5 coin tossing yield p-value=$(1/2)^5$=0.03 (one-sided). We can chose a significance value of 0.05 to deduce that obtaining 5 heads excludes the assumption that the coin is not faked, or the coin is faked.

If we apply two-sided test, p-value=$2*(1/2)^5$=0.06 therefore the result of 5 heads does not exclude a fake coin. We need more tossing results to improve our conclusions.



p-value cont.

If after 20 tossing experiments we get 14 heads, we sum the probabilities for 15 till 20 heads to obtain the right-tailed p-value:

$$\begin{aligned}&\text{Prob}(14\text{ heads}) + \text{Prob}(15\text{ heads}) + \cdots + \text{Prob}(20\text{ heads})\\&= \frac{1}{2^{20}}\left[\binom{20}{14} + \binom{20}{15} + \cdots + \binom{20}{20}\right] = \frac{60{,}460}{1{,}048{,}576} \approx 0.058\end{aligned}$$

A significance test of 5% (p-value<0.05) the result of 14 heads in 20 tossing can (marginally) happen even if the coin is not fake.

Type 1 errors = false positives

Reject an null hypothesis that is actually true

Type 2 errors = false negatives

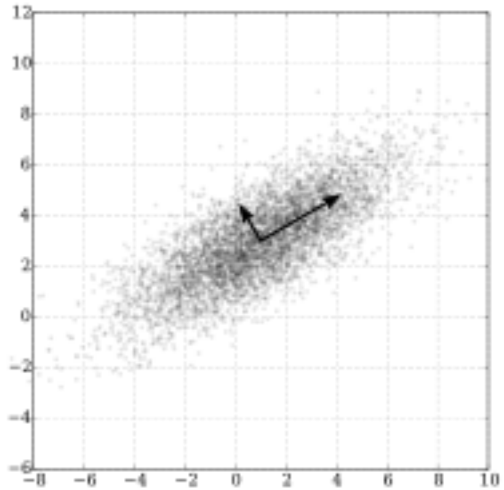
Do not reject a null hypothesis that is actually false

Principal component analysis

A commonly applied analysis for multidimensional spaces, rotating the parameter coordinate system so that the new coordinated (that are linear combinations of the original parameters) present uncorrelated variables. The first coordinate has the highest variability, the second (which is perpendicular to the first) display the second in size variability, etc.

Data bases (e.g. of population) keep for every sample many parameters, although some may be somewhat dependent on others (e.g. height and weight). Principal component analysis provides a way to reduce parameter space dimensions, for example, by displaying the first two components (see plot).

The different sampled parameters may have drastically different scales, therefore Euclidean distances are meaningless. For this reason one defines **Mahalanobis distances**, where every coordinate is normalized by its variability, creating a distribution within a hyper sphere (rather than hyper-ellipsoids)



Pafnuty Tchebyshev [Tchebitcheff] **(1821-1894)** Russia

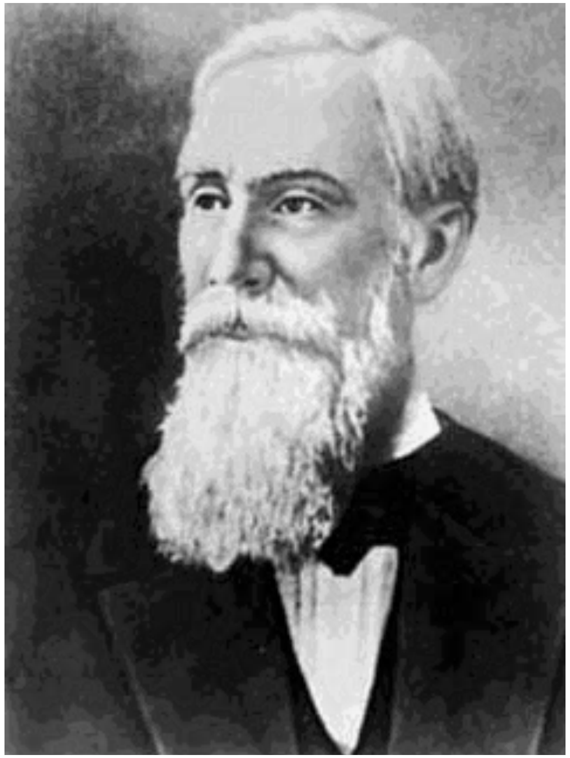
probability, number theory

Bertrand-Tchebitcheff theorem,

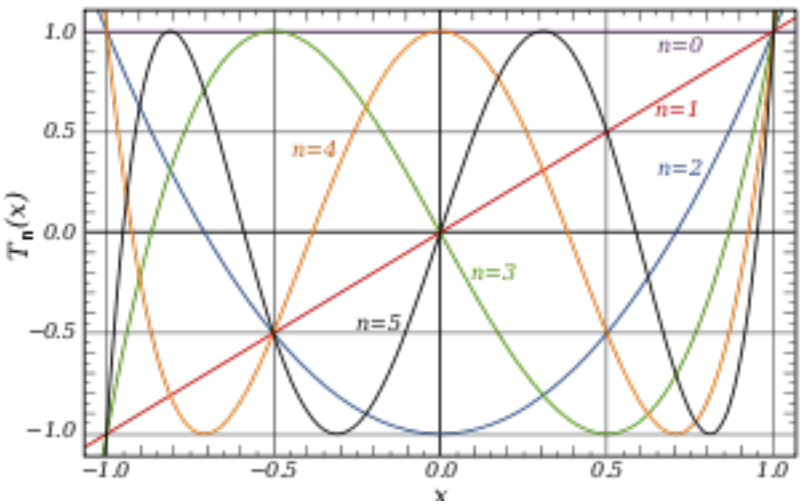
Tchebytcheff's inequality: If x is a random variable with standard deviation σ>0 then the probability that x will be no farther than $a\sigma$ from the average is:

$$\Pr(|X-\mathbf{E}(X)|\geq a\,\sigma)\leq\frac{1}{a^2}.$$

This is used to prove the weak law of large numbers: for every a, small as desired, the average of n tries for large enough n will be close to <x>.



Chebyshev polynomials: Orthogonal polynomials in the range of **(-1,1)**



Ernest Zermelo (1871-1953) Germany

Set theory, games theory

Zermelo-Fraenkel: Axioms that define theory of sets.

Zermelo's theorem: For every finite game of two players with full knowledge, no luck and always one wins, there exist a strategy that guarantees winning.

Augustine Lewis Cauchy (1789-1857) France

Probability Cauchy-Riemann equations, Cauchy theorem, test, equality, random variate, Rigorous calculus and complex analysis, functions with singularities,
Elasticity, stress tensor, Convergence [delta epsilon definition].

Fled away from France during the French revolution, and returned during Napoleon III.

<u>Cauchy integral theorem</u>: integral along a simple (no ties) close contour is zero. Extended to singular functions:

$$\oint_\gamma f(z)\,dz = 0.$$

The integral equals the sum of residues inside the contour, where the residue at a singular point z is:

$$\frac{1}{2\pi i}\oint_C f(z)dz = \sum_{k=1}^{n} \operatorname*{Res}_{z=a_k} f(z),$$

$$\operatorname*{Res}_{z=a} f(z) = \lim_{z\to a}(z-a)f(z),$$

Hence also independence of the integral on the contour path:

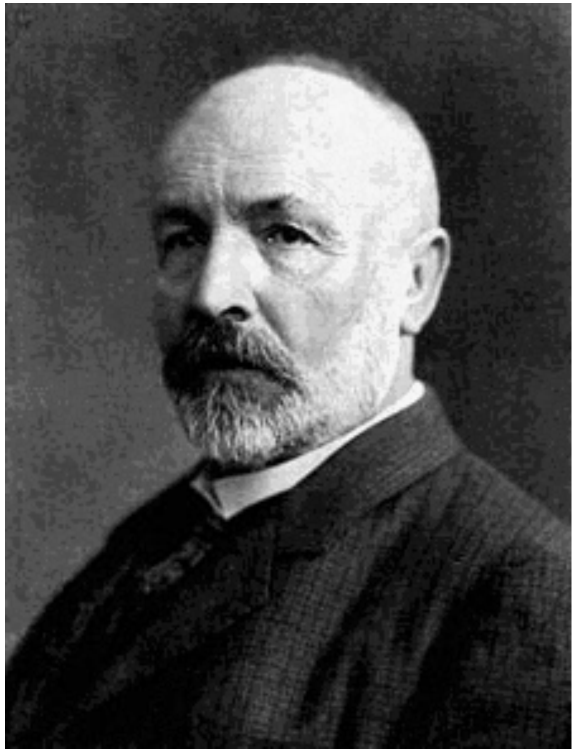
$$f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}dz,$$

Cauchy's work detached between analytical functions and their graphic presentation, and gave a rigorous foundation to function theory. The important application to physics is that electromagnetic field functions are singular, and contour integrals provide calculations of charges, energies and work without hitting the singular points.

George Cantor (1846-1918) Germany

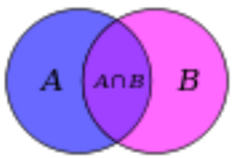
calculus, set theory
Cantor theorem, lemma, function
Set theory: one-to-one correspondence,
more real numbers than integers, infinity of infinities.

1869 Cantor published a proof (solving an apparent paradox) that infinite group is not larger than a part of it.
Cantor proved one-to-one correspondence of integers with even numbers, and with rational numbers (countable sets), and showed that real numbers are not countable.



real numbers natural (countable) numbers

$\aleph$

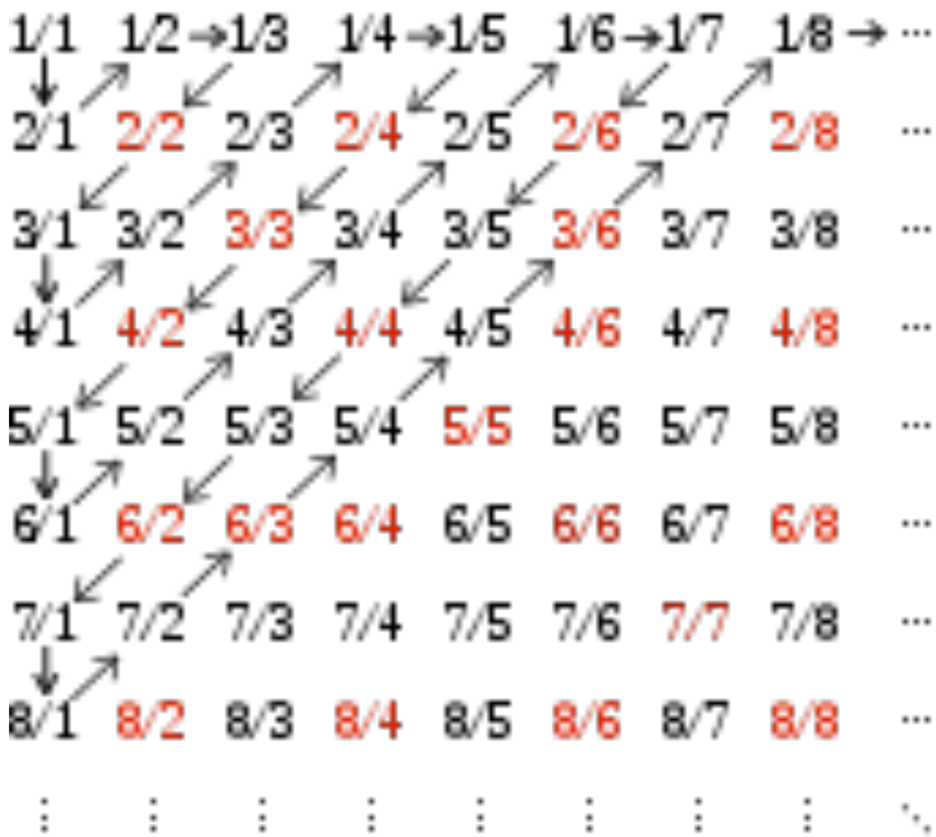


$\aleph_0$

Cantor **1845-1918** group theory

$\aleph_0$ is the notation of a countable set, e.g. 1,2,3,…

Although there are an infinite rational numbers between 1 and 2, they are countable: here is Cantors constructive proof:



On the other hand irrational numbers are not countable: proof by negating the claim they can be counted:

Lets assume we paired all real numbers between 0 and 1 to integers. The table of correspondence will start like this:

For real that end after a finite number of digits we add infinite number of zeroes.

$1 \leftrightarrow 0.112452...$

$2 \leftrightarrow 0.743212...$

$3 \leftrightarrow 0.213945...$

$4 \leftrightarrow 0.432912...$

$5 \leftrightarrow 0.394854...$

...

We create now a new number consisting of the first digit that is different than the first digit of the number in our table, second digit different from the second number etc. For example in the above table:

0.85863...

We are sure this number is not in our table, since it has at least one digit different than any number in the table. Here is the beginning of such comparison.

0.11245...	0.74321...	0.21394...	0.43291...	0.39485...
0.85863...	0.85863...	0.85863...	0.85863...	0.85863...

But this defies our assumption, that every real number was “counted”. The meaning is that “strength” or “cardinality” of reals is stronger than integers, and is uncountable. Cantor assumed there is no set with cardinality between integers and real (continuity hypothesis), but this was not proved yet.

Alan Joseph Canningham (1842-1928) England

number theory

Canningham's numbers, $b^n \pm 1$ when b cannot be expressed as k^m (n,k,m integers).

The question is if Canningham numbers are primes. $2^{2^n}+1$

Reminiscence of the question if Fermat numbers are primes:

and Mersen numbers, that some are primes: 2^n-1

Leopold Kronecker (1823-1891) Germany

logics, number theory

Kronecker’s delta function

$$\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j, \\ 1 & \text{if } i = j. \end{cases}$$

$$\sum_{i=-\infty}^{\infty} a_i \delta_{ij} = a_j.$$

Dirac delta function

$$\int_{-\infty}^{\infty} \delta(x-y) f(x) dx = f(y),$$

Robert Carmichael (1879-1967) US

number theory, relativity

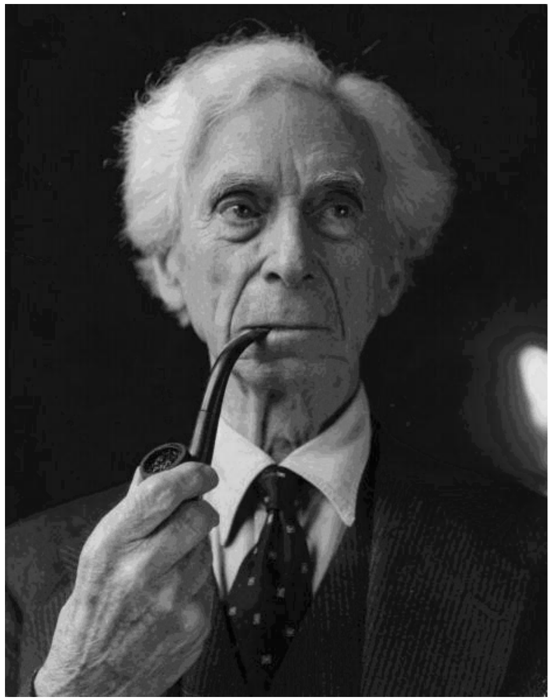
Lukas-Carmichael numbers (and conjecture)**:** $\mathbf{b^n=b \bmod(n)}$

Finding these numbers and discovering their properties engage mathematicians till today.

Bertrand Russell (1872-1970) England

group theory, logics and language
Russell paradox helped redefine the axioms of Cantor's set theory
Nobel prize in literature.

Russell was an active pacifist during the first world war, but was "converted" by Hitler's rise.

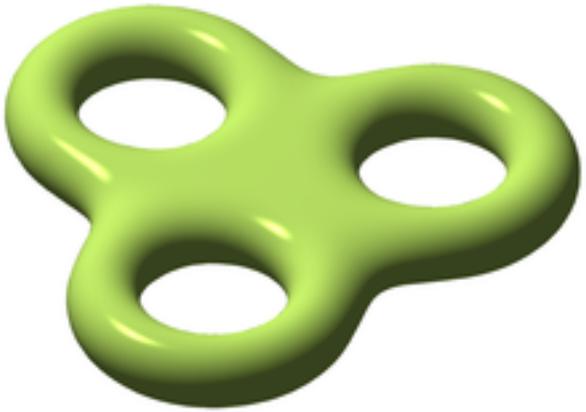


Gustav Roch (1839-1866) Germany

complex analysis, analytical geometry

Riemann-Roch theorem

Links Riemann's surfaces "genus" (number of holes) and the analytical properties.

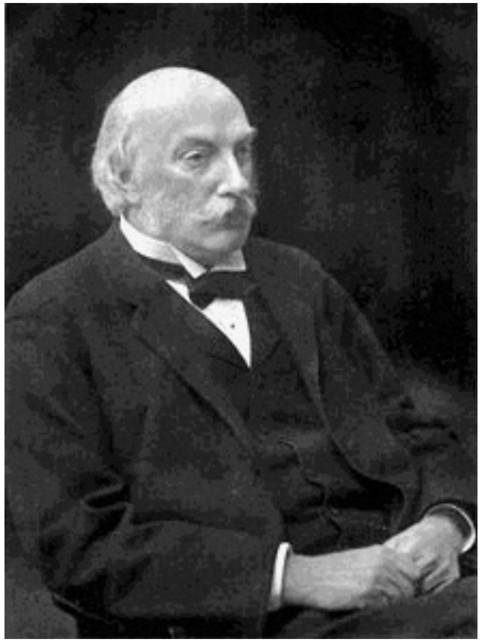


John William Strutt, 3rd Baron Rayleigh (1842-1919) England

probability

Rayleigh ‘s density function:

Most contributions (and 1904 Nobel prize) in physics.



George Bernhard Riemann (1826-1866) Germany

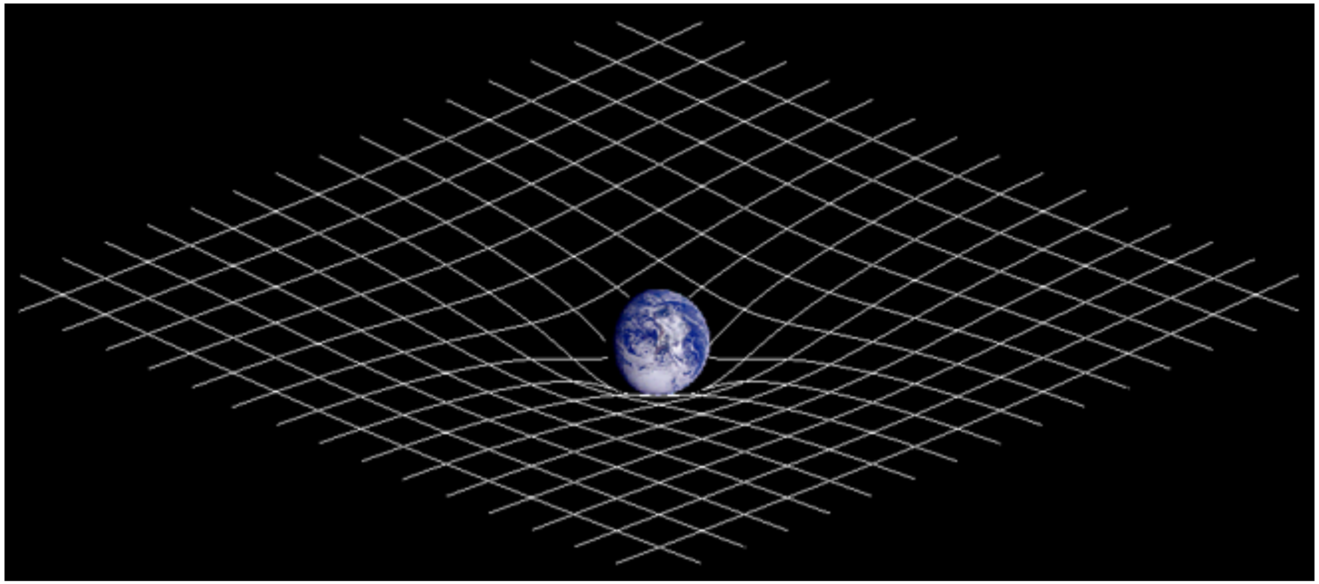
calculus, number theory

Riemann's hypothesis, zeta function, integral, sum

Cauchy-Riemann equations general relativity

1854 Riemann space – is a space with matrix that is different from one point to another. Replaces the Euclidean 5^{th} axiom (only one parallel to a line at every point).

1859 Extended Gauss differential geometry to n-dimensional spaces. Formed the basis for general relativity.



Srinivasa Ramanujan Iyengar (1887-1920) India
number theory, mathematical analysis
Ramanujan's conjecture on divisions, Tau function

What is the value of: $\sqrt{1+2\sqrt{1+3\sqrt{1+\cdots}}}.$

He set the equation:

$$x+n+a=\sqrt{ax+(n+a)^2+x\sqrt{a(x+n)+(n+a)^2+(x+n)\sqrt{\cdots}}}$$

For x=2, n=1, a=0 the answer for the above value is 3.

Ramanujan also studies Bernoulli's numbers and found interesting properties, series presentations of π, and more.

Karl Hermann Amandus Schwarz (1843-1921) Germany

complex analysis

Cauchy-Schwarz inequality:

$$|\langle x,y\rangle|^2 \leq \langle x,x\rangle \cdot \langle y,y\rangle,$$

or (triangle inequality)

$$|\langle x,y\rangle| \leq \|x\| \cdot \|y\|.$$

or

$$\left|\sum_{i=1}^{n} x_i \bar{y}_i\right|^2 \leq \sum_{j=1}^{n} |x_j|^2 \sum_{k=1}^{n} |y_k|^2.$$

and

$$\mathrm{Var}\,(Y) \geq \frac{\mathrm{Cov}\,(Y,X)\,\mathrm{Cov}\,(Y,X)}{\mathrm{Var}\,(X)}.$$

Schwartz lemma: If z<1 defines open disk in the complex plane, and f(z) has derivatives everywhere in the disk than if: $|f(z)| \leq |z|$ than: $|f'(0)| \leq 1$

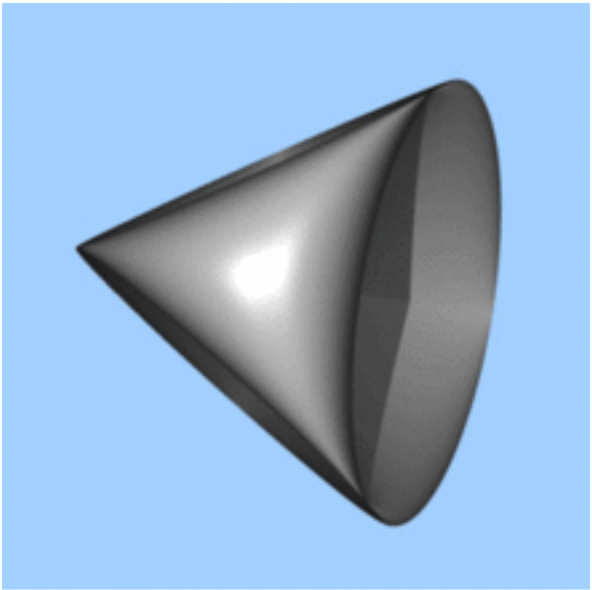
Although stronger theorems (e.g. Riemann's mapping theorem) were proved, this lemma expresses the extreme rigidity (with limitations) of complex functions with derivatives.

Jacob Steiner (1796-1863) Swiss

geometry, mathematical analysis

Steiner's problem, Minkovsky-Steiner equation

Steiner's surface: $x^2y^2 + y^2z^2 + z^2x^2 - r^2xyz = 0$

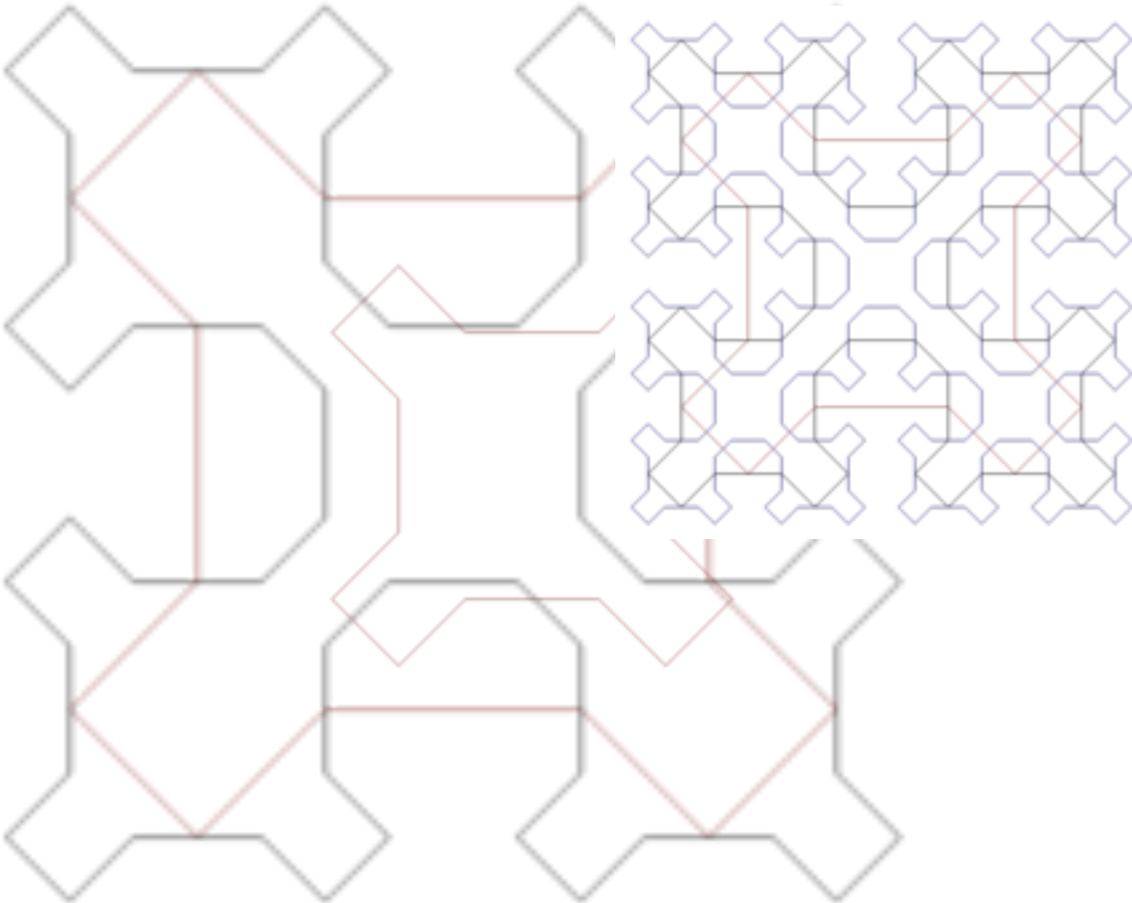


Waclaw Sierpinski (1882-1969) Poland

set theory, number theory, topology
Sierpinski’s triangle, carpet
Sierpinski’s curves of orders ,2 & 3:

$$l_n = \frac{2}{3}(1+\sqrt{2})2^n - \frac{1}{3}(2-\sqrt{2})\frac{1}{2^n}$$

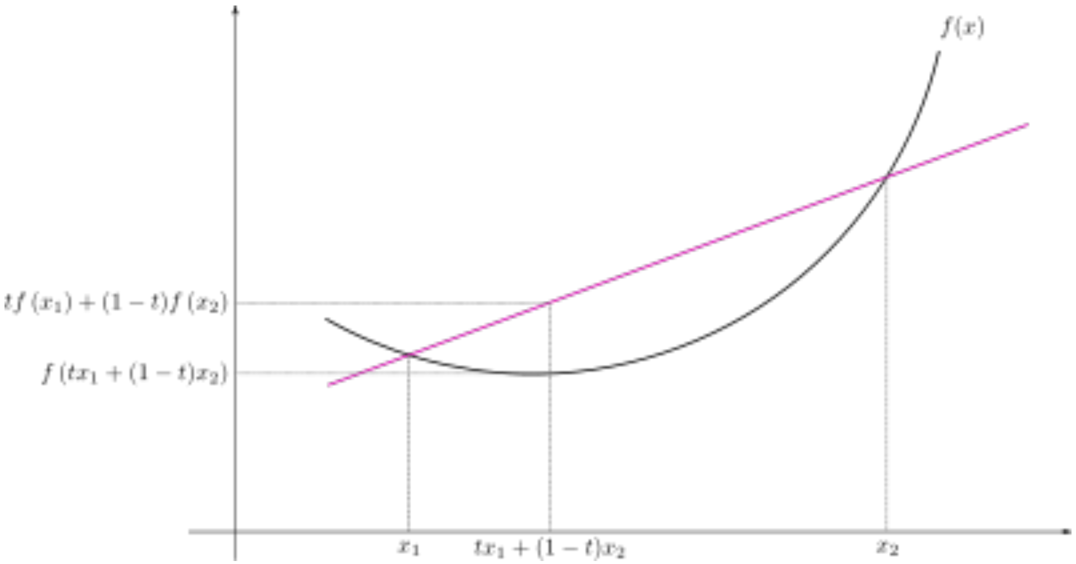
Fractals with converging area of 5/12



Johan Jensen 1859-1925 Denmark

Number theory

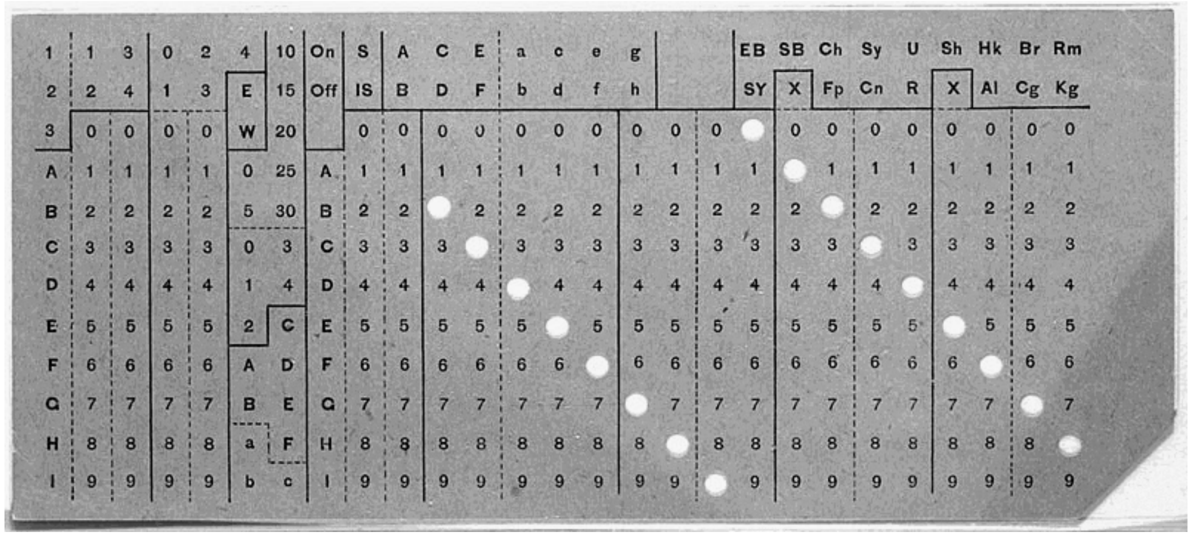
Jensen's inequality: relates the value of a convex function of an integral to the integral of the convex function:

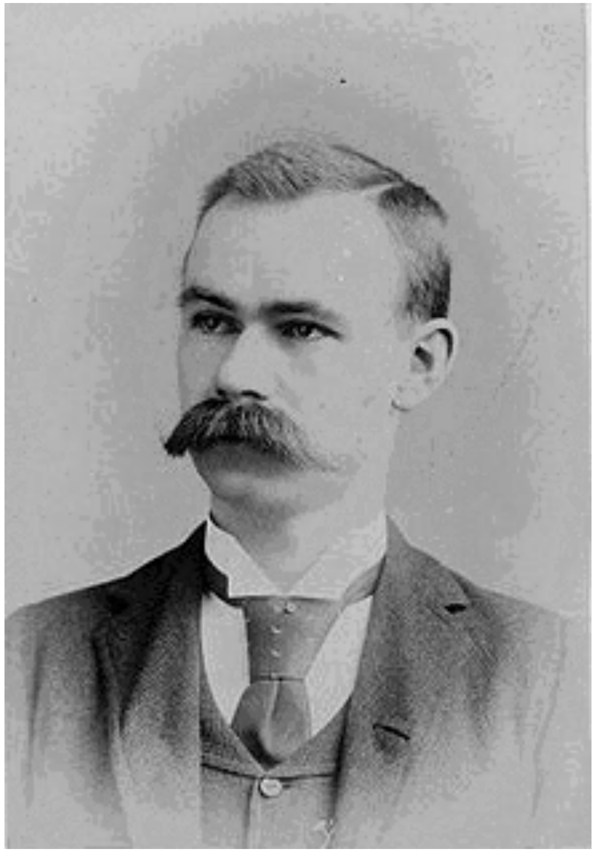


Extended the theorem that a line intersecting a convex curve
Is above any point between the intersections.

Herman Hollerith US

1890 Invents the punch cards and associated sorting and cataloguing machine. Applied in population surveys, and expanded use in commercial data bases. He started a company that later merged and founded IBM.





APPENDIX:

LIST OF SCIENTISTS
IN THE ROMANTIC AGE

Adrain
Germain
Barlow
Poinsot
Gauss
Hecht
Brisson
Spence
Wronski
Gompertz
Crelle
Somerville
Poisson
Bolzano
Plana
Claude Mathieu
Brianchon
Kulik
Bessel
Dupin
Navier
Binet

Arago
Horner
William Hamilton
Fresnel
Poncelet
Ohm
Cauchy
Möbius
Peacock
Savart
Petit
Babbage
Herschel
Coriolis
Lobachevsky
Olivier
Hopkins
Chasles
Green
Dandelin
Taurinus
Holmboe

Louis Richard
Lamé
Morin
Carlyle
Léger
Quetelet
Steiner
Sadi Carnot
Brashman
Bienaymé
Duhamel
Saint-Venant
Savary
Finck
von Staudt
Gudermann
Bobillier
Franz Neumann
Calpeyron
Gräffe
Talbot
Feuerbach

Clausen
Plücker
Airy
Cournot
Ostrogradski
Plateau
Abel
Bolyai
Libri
Sturm
Bellavitis
Doppler
Weber
Verhulst
Jacobi
Bunyakovsky
Jerrard
Sang
Dirichlet
Hamilton
Minding
Kirkman

De Morgan
Petzval
Listing
Dupré
Grassmann
Lioville
Benjamin Peirce
Pratt
Menabrea
MacCullagh
Kummer
Le Verrier
Hesse
Galois
Shanks
Göpel
Anstice
Duncan Gregory
Pierre Laurent
Schläfli
Catalan
Sylvester

Wantzel
Somov
Boole
Weierstrass
Lovelace
Frenet
Delaunay
Rosenhain
Wolf
Ferrel
Borchardt
Briot
Genocci
Joachimsthal
Adams
Arnhold
Stokes
Serret
Bouquet
Salmon
Bonnet
Puiseux

Rankine
Jonquiéres
Todhunter
Heine
Chebyshev
Cayley
Helmholtz
Seidel
Clausius
Galton
Lissajous
Bertrand
Hermite
Hoüel
Eisenstein
Schlömilch
Betti
Amsler
Kronecker
Davidov
Codazzi
Kirchhoff

Thomson
Brioschi
Dase
Balmer
Carl Bjerknes
Spottiswoode
Vashchenko
Faó di Bruno
John Walker ??
Battaglini
Meissel
Riemann
Smith
Christian Wiener
Merrifield
Henry Watson
Roberts
Peterson
Bruno
Schroeter
Moritz Cantor
Christoffel

Hirst
Cremona
Routh
Tait
Mannheim
Dedekind
Maxwell
du Bois-Reymond
Dodgson
Carl Neumann
Lipschitz
Bour
Sylow
Clebsch
Fuchs
Laguerre
Venn
Newcomb
Josef Stefan
Emile Mathieu
Jevons
Méray
Beltrami

Casorati
Amringe
Weingarten
Gordan
Bachmann
Lexis
Tilly
Bugaev
Königsberger
Jordan
Hill
Reye
Thiele
Gibbs
Hankel
Adolph Mayer
Zeuthen
Barbier
Peterson
Charles Peirce
Saicci
Mertens

Neuberg
Lemoine
Thomae
Henrici
Abbe
McClintock
Rudolf Sturm
Loyd
Hermann Laurent
Schröder
Sokhotsky
Lucas
Stolz
Heinrich Weber
Darboux
Rosanes
Reynolds
Brill
Rayleigh
Lie
D’Ovidio
Schwartz
Geiser

Tarry
Pasch
Paul Tannery
Niven
Boltzmann
Lueroth
Mansion
Muir
Max Noether
Halphen
Wangerin
Tisserand
Cantor
Clifford
Brocard
Darwin
Dini
Edgeworth
Schoute
Mittag-Leffler
Poretsky
Bertini

Zhukovsky
Zolotarev
Killing
Greenhill
Castigliano
Suter
Jules Tannery
Korteweg
Schubert
Netto
Weyr
Eötvös
Bruns
Glaisher
Frege
Andreev
Gegenbauer
Sonin
Klein
Kempe
Frobenius
Lamb

Julius König
Woodward
Kovalevskaya
Heaviside
Gram
Ball
Pringsheim
Chrystal
Dickstein
Schottky
FitzGerald
Elliott
Le Paige
Lindemann
Burnside
Upton
Ricci-Curbastro
Pincherle
Schönflies
Lorentz
Maschke
Halsted
Poincaré

Veronese
MacMahon
Rydberg
Brillouin
Mellin
Sleszynski
Juel
Rohn
Boys
Appell
Guccia
Biamchi
Markov
Emile Picard
Rudio
Meyer
Runge
Hobson
von Dyck
Stieltjes
Pearson
Bolza
Lyapunov

Lamor
Dudeney
Koenigs
Planck
Goursat
Forsyth
Johnson
Peano
Scott
Henry Fine
Georges Humbert
Cajori
Cesáro
Hurwitz
Shatunovsky
Jensen
Meshchersky
Hölder
Bukreev
Lerch
Hollerith
D‘Arcy Thompson

Volterra
Pieri
Stott
Weldon
Morley
Whitehead
Mathews
Duhem
Bendixon
Burali-Forti
Heawood
Molin
Cole
Heath
Burkhardt
Hensel
Engel
White
Slaught
Hilbert
Eliakim Moore
Macaulay

Vilhelm Bjerknes
Kneser
Study
Campbell
Jules Richard
Stäckel
Thue
Love
Fields
Miller
Aleksei Krylov
Corrado Segre
Grave
Young
Painlevé
Padé
Adler
Zaremba
Richmond
Van Vleck
Vailati
Steklov

Wien
Osgood
Kürschák
Minkowski
Macdonald
Landsberg
Vessiot
Dixon
Castelnuovo
Kotelnikov
Bohl
Nielsen
Jacques Salomon Hadamard 1865–1963
Fiske
Wirtinger
Bortolotti
Cosserat
Fredholm
Baker
Valée Poussin
Tauber
Scheffers

Brown
Kolosov
Bôcher
Kutta
Arthur Dixon
Sintsov
Couturat
Voronoy
Bortkiewicz
Padoa
Sommerfeld
Lasker
Chisholm Young
Kagan
Chaplygin
Cartan
Hausdorff
Egorov
Snyder
Koch
Lindelöf
Carslaw

Enriques
Fano
Borel
Yule
Drach
Steinitz
Epstein
Zermelo
Galerkin
Heegaard
Sitter
Russell
Levytsky
Pfeiffer
Vacca
Blichfeldt
Levi-Civita
Loewy
Carathéodory
Coolidge
Schwarzschild
Whittaker
Baire

Dickson
Huntington
Schur
Bromwich
Max Abraham
Takagi
Lebesgue
Fischer
Vitali
Cantelli
Eisenhart
Schmidt
Esclangon
Montel
Bliss
Gosset
Wilczynski
Hedrick
Hardy
Landau
Jeans
Mason
Faber

Boggio
Felix Bernstein
Fatou
Grossman
Löwenheim
Fréchet
Dehn
Lukasiewicz
Erlang
Coble
Kellogg
Fubini
Einstein
Severi
Edwin Wilson
Hahn
Jourdain
Sommerville
Nikolai Krylov
Ehrenfest
Riesz
Fejér