Derivation of equation M.1:

$$P\left(\Delta x > 0 \mid \Delta x^{'\{n\}}\right) = \frac{1}{1 + \prod_{i=1}^{n} \frac{P(\Delta x_i^{'} \mid \Delta x < 0)}{P(\Delta x_i^{'} \mid \Delta x > 0)}}$$

$$\frac{P(\Delta x_i^{'} \mid \Delta x < 0)}{P(\Delta x_i^{'} \mid \Delta x > 0)} = \frac{\frac{P(\Delta x < 0 \mid \Delta x_i^{'})P(\Delta x_i^{'})}{P(\Delta x < 0)}}{\frac{P(\Delta x > 0 \mid \Delta x_i^{'})P(\Delta x_i^{'})}{P(\Delta x > 0)}} \underset{assumptions(i)}{=} \frac{P(\Delta x < 0 \mid \Delta x_i^{'})}{P(\Delta x > 0 \mid \Delta x_i^{'})} = \frac{\int_{-\infty}^{0} P(\Delta x \mid \Delta x_i^{'}) d(\Delta x)}{\int_{0}^{\infty} P(\Delta x \mid \Delta x_i^{'}) d(\Delta x)} =$$

$$= \frac{\int_{-\infty}^{0} \frac{P(\Delta x_i^{'} \mid \Delta x) P(\Delta x)}{P(\Delta x_i^{'})} d(\Delta x)}{\int_{0}^{\infty} \frac{P(\Delta x_i^{'} \mid \Delta x) P(\Delta x)}{P(\Delta x_i^{'})} d(\Delta x)} \underset{assumptions(i)}{=} \frac{\int_{-\infty}^{0} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x)}{\int_{0}^{\infty} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x)}$$

$$P\left(\Delta x > 0 \mid \Delta x^{'\{n\}}\right) = \frac{1}{1 + \prod_{i=1}^{n} \frac{P(\Delta x_i^{'} \mid \Delta x < 0)}{P(\Delta x_i^{'} \mid \Delta x > 0)}} = \frac{1}{1 + \prod_{i=1}^{n} \frac{\int_{-\infty}^{0} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x)}{\int_{0}^{\infty} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x)}} \qquad \text{(Eq. M.1)}$$

Derivation of equation M.2:

$$\int_{-\infty}^{0} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{0} e^{-\frac{\left(\Delta x_i^{'} - \Delta x\right)^2}{2\sigma^2}} d(\Delta x)$$

By replacing the integration variable x->t, where $t = \frac{\left(\Delta x_i^{'} - \Delta x\right)}{\sqrt{2}\sigma}$ we get the following:

$$\int_{-\infty}^{0} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x) = \frac{1}{\sigma\sqrt{2\pi}} \sqrt{2}\sigma \int_{\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}}^{\infty} e^{-t^2} dt = \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}} \frac{\sqrt{\pi}}{2} erfc\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right) = \frac{1}{2} erfc\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right)$$

In the same way:

$$\int_{0}^{\infty} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x) = \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}} \int_{-\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}}^{\infty} e^{-t^2} dt = \frac{1}{\sqrt{\pi}} \frac{\sqrt{\pi}}{2} \left(\frac{2}{\sqrt{\pi}} \int_{-\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}}^{0} e^{-t^2} dt + \frac{2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} dt \right) = \frac{1}{2}\left(erf\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right) + 1 \right)$$

$$\Rightarrow \frac{\int_{-\infty}^{0} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x)}{\int_{0}^{\infty} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x)} = \frac{\frac{1}{2} erfc\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right)}{\frac{1}{2}\left(erf\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right) + 1 \right)} = \frac{1 - erf\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right)}{1 + erf\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right)} = \frac{1 - erf\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right)}{1 + erf\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right)}$$

$$\Rightarrow P\left(\Delta x > 0 \mid \Delta x^{'\{n\}}\right) = \frac{1}{1 + \prod_{i=1}^{n} \frac{\int_{-\infty}^{0} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x)}{\int_{0}^{\infty} P(\Delta x_i^{'} \mid \Delta x) d(\Delta x)}} = \frac{1}{1 + \prod_{i=1}^{n} \frac{1 - erf\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right)}{1 + erf\left(\frac{\Delta x_i^{'}}{\sqrt{2}\sigma}\right)}} \quad \text{(Eq. M.2)}$$