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Beyond Charm

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1. Introduction

The discovery of charm has brought to an end a beautiful and exciting chapter in the development of particle physics. The next chapter promises to be equally exciting, although we have no way of guessing how long it will take or what name will be finally given to it. A good temporary name is, obviously, "beyond charm". These lecture notes are devoted to the first glimpses that we have had into this new chapter. Like all historical processes, the new chapter actually began before the previous one ended. Speculations about leptons, quarks and weak currents beyond the now "standard" four leptons, four quarks and V-A currents, have been advanced by many authors over the last few years. The fascinating problems of relating the quarks and leptons to each other, and of unifying the weak, electromagnetic and strong interactions, attracted some (not enough!) attention. These topics are the main subject of our discussion.

We start by reviewing many "truths" and few "dogmas" which we have learned in the years before 1974. Section 2 is devoted to a brief description of leptons, hadrons and their quark constituents, gluons, weak currents and strong interactions. We mention the experimental evidence for each "truth" and the motivation for each "dogma". We also stress many consequences as well as problems which will become crucial in our discussion of the "New Physics".

Section 3 is a very brief discussion of charm. Why was it predicted? How necessary is it? What is the experimental evidence for it? The section ends with a definition (hardly necessary, by now) of the "standard" Glashow-Iliopoulos-Maiani-Salam-Weinberg model and with a list of options for "beyond charm" model-architects.

Sections 4, 5, 6 include a detailed discussion of several theoretical and/or experimental reasons which encourage us to go "beyond charm".

The first of these (Section 4) is the possibility of incorporating a CP-violating interaction into a gauge theory of the weak interactions. This necessitates additional quarks and/or additional weak currents and/or additional Higgs particles, beyond the minimum required by the "standard model". Since several different reasons point toward additional quarks, we emphasize a CP-violating theory based on six quarks.

Section 5 deals with three unrelated subjects which converge into one conclusion: More leptons and more quarks are needed. The three subjects are the triangle anomalies and the conditions for their removal in a renormalizable gauge theory; the observed value of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ above charm threshold, and its implication for the existence of additional fundamental fermions; the $e^\pm\mu^\mp$ events observed in e^+e^- collisions and the possibility that they reflect the existence of a heavy lepton.

Section 6 is devoted to the possible existence of V+A currents. We discuss the so-called y-anomaly and the increasing charged current ratio of $\sigma(\bar{\nu}N)/\sigma(\nu N)$, and the observed violation of parity in neutral current processes. We review the phenomenological necessity for introducing additional (V+A) weak currents and additional quarks. "Old-fashioned" nonleptonic weak processes such as $K \to 2\pi$, $K \to 3\pi$

provide us with interesting constraints in this context. Some theoretical aspects of "vector-like" theories are also discussed.

In Section 7 we move into even more speculative grounds. Leptons and quarks are fundamental, $J = \frac{1}{2}$, pointlike objects which respond in the same way to weak currents. They differ, of course, by their strong interaction properties. One cannot escape the feeling that a deep relation between leptons and quarks exists. We review several of the motivations that led to the exploration of such relations and set the stage for a search for a unifying gauge group of the weak, electromagnetic and strong interactions.

Section 8 is devoted to a discussion of such "grand unification schemes". Two specific models, the Georgi-Glashow SU(5) scheme and the E(7) scheme of Gursey and his collaborators are singled out as instructive examples of a "minimalistic" and a "maximalistic" model. The shortcomings of such models and their various features are discussed.

The same logic that led us to name the present section "Introduction", tells us that the last section must be devoted to "What Next?". Whether our "What Next?" of today will become the "Introduction" to one of the next Les Houches summerschools, only time will tell.

2. Leptons, Quarks and their Currents: Truths and Dogmas

2.1 Why bother?

All present models of the particles and their interactions are based on a large body of facts, hypotheses and speculations which mostly stem from the last two decades of experimental and theoretical work. Many of these basic ingredients are well-known and do not need reviewing. However, the specific pieces of evidence for a given "truth" or "dogma" are not always appreciated, and we often find physicists who accept (or use) such principles without realizing how strong (or how weak) is the evidence for them. We therefore believe that a brief review of the basic principles and facts might be appropriate, especially if it emphasizes the experimental evidence for each item as well as the open possibilities for future extensions and modifications. That is why we bother.

2.2 Leptons

Four leptons are well-established: The electron and its neutrino, the muon and its neutrino. They have $J = \frac{1}{2}$ and they are "pointlike" in the sense of a minimal coupling in weak and electromagnetic processes. Only the left-handed helicity states of the leptons participate in the known charged weak currents. The two neutrinos are consistent with being massless. In an SU(2) x U(1) Weinberg-Salam gauge theory of the weak and electromagnetic interactions(1) the left handed leptons form two SU(2) doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$$

All the above statments are well-established. However, the world of leptons leaves a surprisingly large area of speculations, extensions and modifications:

(i) The neutrinos might have a mass. The ν_μ may even have a mass of the order of magnitude of the electron mass. Right-handed neutrinos might exist.

(ii) The neutral weak leptonic current need not be a pure V-A current. At the time of this writing even a pure vector neutral weak leptonic current is still possible, although unlikely. V+A components are certainly possible.

(iii) Additional leptons may exist (see Section 5.3). No reason was ever provided for the muon, and nothing prevents the existence of more charged leptons, provided that their masses are above 1.5 GeV or more. Many additional neutrinos may, of course, exist.

(iv) V+A charged currents are possible, provided that they do not connect the four known leptons to each other. In other words, any one of the four leptons might combine with a new lepton in a charged right handed weak coupling.

(v) All known interactions of the electron and the muon are identical, and yet - something must be different. In the conventional gauge theory the only difference is (naturally) in the coupling to the Higgs particles. The amazing agreement between experiment and the QED prediction for the muon g-factor puts severe limits on any new proposed interaction for the muon.

In summary: everything we know (except for the SPEAR eμ events[2]; see Section 5.3) is consistent with two left handed

SU(2)-doublets of $J = \frac{1}{2}$ pointlike leptons. More leptons and more currents are possible.

2.3 Hadrons Contain Pointlike Constituents

Deep inelastic electron and neutrino experiments (in the spacelike region) and e^+e^- experiments (in the timelike region) exhibit scaling properties.

The structure functions W_1 and νW_2 for ep and en scattering are approximately described by a function of $x = q^2/2M\nu$; the total neutrino (charged current) cross section is linearly rising[3] and the structure functions are consistent with scaling; the ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ is constant both below and above the charm threshold[4] (i.e. for $2.5 < W < 3.5$ and $4.5 < W < 7.5$ GeV); the inclusive distribution for $e^+ + e^- \rightarrow \pi^\pm$+anything obeys approximate scaling for regions in which $R \sim$ constant[4].

To the extent that scaling is exact, it indicates that the proton and neutron (in the stationary target experiments) and the pion (in e^+e^- collisions) behave as if they contain charged pointlike constituents. No such constituents were ever seen, of course. However, if such constituents were to exist, their interactions might produce the observed scaling behaviour.

The connection between scaling and pointlike constituents is not very convincing. However, all "explanations" of scaling (light cone approach, asymptotic freedom) represent more sophisticated (and possibly more correct) descriptions of essentially the same picture. No other explanation for scaling is known.

Significant violations of scaling have been reported in the last two years. Three types of violations should be mentioned:

(i) A q^2-dependence of ep and μp structure functions, at fixed x, was observed both at SLAC[5] and at the Fermi Laboratory[6]. The deviations are not dramatic, but they seem to exist.

(ii) The ratio ep/en for the νW_2 structure function seems to decrease significantly[5] as a function of q^2, at $x \sim 0.5$. This is unlikely to be due to new particles, and it demands an explanation (see Section 2.9).

(iii) The ratio $\sigma(\bar{\nu}N)/\sigma(\nu N)$ for charged current reactions increases significantly from 0.35-0.4 to 0.6-0.7, over the Fermilab energy region[7]. This is probably related to the so-called y-anomaly[8] (see Section 6). This violation of scaling may or may not be related to the production of new particles and to the existence of new currents.

It is crucial to identify the source of this violation of scaling in neutrino processes. If it has the same source as the other violations listed above, it would perhaps change our understanding of the scaling phenomenon itself, but it would not imply the existence of new quarks and currents. If, however, the neutrino scaling-violation turns out to be larger and more significant, it may indeed require physics "beyond charm". This is an extremely important issue, and we return to it in Section 6.

2.4 Hadrons are Made of Quarks. Ordinary Hadrons Contain Three Quark Flavors

All hadrons behave as if they are made of quarks. All mesons are $q\bar{q}$ states. All baryons are qqq states. All hadrons known before Nov. 11, 1974, can be accounted for by three flavors of quarks: u, d, s forming an SU(3) triplet[9]. The experimental evidence for this hypothesis is overwhelming. It consists of the simple statement that all known mesons fit in SU(3) octets and singlets and all known baryons fit in decimets, octets and singlets. With well over a hundred hadrons, this cannot be an accident. Searches for mesons and baryons with exotic quantum number (of the 1st kind, i.e. charge, isospin, strangeness, SU(3)) failed again and again.

With the discovery of charm, new kinds of exotic particles are defined. These include states such as $C = S = 1$ neutral mesons (F^0), charmed ($C = +1$) $Q = 2$ or $Q = -1$ mesons, $C = -S = 1$ states, etc. It is important to search for such states, and to find out whether they exist or not.

2.5 The Quark Charges are $Q_u = +\frac{2}{3}$, $Q_d = -\frac{1}{3}$, $Q_s = -\frac{1}{3}$

The structure of SU(3) dictates that the electric charges of the u, d, s quarks are, respectively, Q, Q-1, Q-1. By studying the mesons and their strong interactions, there is no way to determine the value of Q. It can be determined, however, in several different ways:

(i) With quark charges Q, Q-1 the baryon charges are predicted to be 3Q, 3Q-1, 3Q-2, 3Q-3. Experimentally, all baryons have charges +2, +1, 0, -1. Consequently: $Q = \frac{2}{3}$.

(ii) The coupling of the photon to the quark is measured by the direct photon-vector meson couplings. Since:

$$\rho = \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}) \ ; \ \omega = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d}) \qquad \phi = s\bar{s}$$

we have:

$$\Gamma(\rho \to e^+e^-):\Gamma(\omega \to e^+e^-):\Gamma(\phi \to e^+e^-) = 1:(2Q-1)^2:2(Q-1)^2$$

Experimentally[10], the ratios are consistent with $1:\frac{1}{9}:\frac{2}{9}$, as expected for $Q = \frac{2}{3}$.

(iii) The ratio between the structure functions for deep inelastic electron scattering and neutrino scattering provides us with a measurement of Q. Within the usual Parton model assumptions we have[11]:

$$\frac{F_2(ep) + F_2(en)}{F_2(\nu N) + F_2(\bar{\nu}N)} = \frac{1}{2}\left[Q^2+(Q-1)^2\right] .$$

For $Q = \frac{2}{3}$ we predict a ratio of $\frac{5}{18}$. The experimental value[12] is in good agreement with this value.

We find it very impressive that three completely different sets of experimental facts lead to the same conclusion: $Q_u = \frac{2}{3}$, $Q_d = -\frac{1}{3}$, $Q_s = -\frac{1}{3}$.

2.6 Quarks have $J = \frac{1}{2}$

Two independent sources tell us that quarks are $J = \frac{1}{2}$ object:

(i) The observed spectrum of mesons and baryons agrees with the expected spectrum for $J = \frac{1}{2}$ quarks. The low lying mesons are predicted to have $J^{PC} = 0^{-+}, 1^{--}$ followed by $J^{PC} = 0^{++}, 1^{++}, 1^{+-}, 2^{++}$.

No $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}$ mesons are allowed (exotic mesons of the 2nd kind). Experimentally, none of the forbidden J^{PC} values seem to exist while all the "recommended" J^{PC} values correspond to observed particles. The baryon spectrum follows an SU(6) pattern based on three flavors of $J = \frac{1}{2}$ quarks. The lowest lying baryons are in a positive parity, L = 0, 56-multiplet. The next states fill a negative parity, L = 1, 70-multiplet. Both the meson spectrum and the baryon spectrum *indicate not only that quarks have* $J = \frac{1}{2}$, but also that we do not have parity doublets of quarks (i.e. if the u-quark is defined to have positive parity, there is no negative parity quark with the same internal quantum numbers, etc.).

(ii) Deep inelastic ep scattering experiments as well as e^+e^- experiments indicate that quarks have $J = \frac{1}{2}$. In both cases the virtual photon could couple to the quarks through its longitudinal and/or transverse components. In both cases, one can define two *independent measurable cross sections:* σ_L and σ_T. In both cases $J = \frac{1}{2}$ quarks couple only to σ_T while $J = 0$ partons would couple only to σ_L. In both cases experiments yield[3] $\sigma_L/\sigma_T \sim 0$, implying $J = \frac{1}{2}$ quarks. Note that ep scattering experiments probe the constituents of the proton while the e^+e^- reactions probe mainly the constituents of the pion. In both cases the constituents are found to have $J = \frac{1}{2}$.

It is, again, remarkable that two unrelated sets of experiments such as the spectroscopy data and the deep inelastic data lead independently to the same conclusion concerning the properties of the

2.7 Quarks Come in Three Colors

We believe that quarks come in three colors (or more formally: transform like a triplet of a *color-SU(3) group which commutes with* ordinary SU(3) and represents an exact symmetry). The color hypothesis[13] is clearly one of the most controversial ideas in particle physics. Three experimental arguments support it, all of them indirect:

(i) The wave function of the three quarks in the low-lying baryons seems to be completely symmetric in the spin, space and SU(3) degrees of freedom. The fully symmetric 56(L = 0) multiplet of SU(6) is the lowest-lying baryon representation. A fully symmetric wave-function of $J = \frac{1}{2}$ quarks contradicts the usual sacred connection between spin and statistics. If we postulate, however, that quarks come in three colors and that all hadrons are color singlets, we find that the 3q-wavefunction must be fully antisymmetric in its color degrees of freedom and the correct spin-statistics connection is restored.

(ii) The decay $\pi^0 \to 2\gamma$ is forbidden in the soft pion limit, except for the contribution of the anomalous triangle diagram[14] (fig. 1). The contribution of this diagram involves a summation over quark colors[15] (if any). The measurement of the π^0 lifetime may then serve as an (indirect) measurement of the number of colors. The observed lifetime is consistent with the existence of three colors (and is, of course, too short by a factor 9 in comparison with a model with colorless quarks).

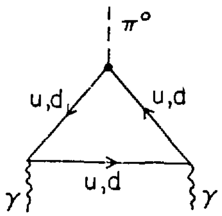

Figure 1: The anomaly diagram in $\pi^0 \to 2\gamma$

(iii) The value of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ measures the sum of squared quark-charges. Below the charm threshold it presumably reflects the u, d, s flavors. In the absence of color we expect:

$$R = \sum_{i\,=\,u,d,s} Q_i^2 = \frac{2}{3}$$

If each quark can be produced in three different colors we expect $R = 2$. Experimentally[16], in the region $2.5 < W < 3.5$ GeV, we have:

$$R \sim 2.5 \pm 0.5$$

consistent with the existence of three colors and inconsistent with colorless quarks.

In addition to these three arguments we might mention one other attractive feature of the color hypothesis. If we assume (without proof or justification) that all hadrons are colorless, we immediately understand why three quarks produce a hadron while two or four quarks do not combine to create any observed object. This does not solve anything, since we did not explain why all hadrons are colorless. However, the single assumption of colorless hadrons seems to fit into several different puzzles and that is encouraging.

The existence of the color degree of freedom is therefore more of a "dogma" than a "truth". We believe in it and find it attractive, but we should remember that on one hand color has not been experimentally observed, while on the other hand no convincing explanation exists for the confinement of all colored objects. This confinement problem is, of course, one of the most interesting open problems of particle physics.

For completeness we must mention those models in which color is not confined[17], heavy hadrons may be colored, the electromagnetic current carries a colored component and the quarks have integer charges (although the average charge of the three colored u-quarks is still $\frac{2}{3}$, etc.). We are convinced that, at present, there is no evidence for colored hadrons. We believe (without proof) that they do not exist. However, they may exist and may be discovered one day.

Assuming that the color idea is correct, an extremely puzzling question still remains: Why SU(3)? What fundamental principle selects SU(3) as the color group, rather than SU(2) or some other group? We will return to this question very briefly in Section 8.

2.8 Color is the "Strong Charge". It is Mediated by Gluons.

Quarks differ from leptons by the fact that they possess color. Quarks are allegedly confined by their color property. It is therefore natural to suspect that the color property is what provides the quark with its strong interaction. Color is the "strong charge".

The quark-quark interaction is presumably mediated by a vector particle which couples to the color degree of freedom - the gluon. Since color is generated by the nonabelian SU(3) group, the gluons themselves possess color and transform like a color octet. Since color is presumably exactly conserved, the eight gluons are probably massless. If color is always confined, gluons will never be seen.

The hadron is colorless, it has no "strong charge" and in the zeroth approximation it has no strong interaction (in the same way that a neutral atom has no electromagnetic interaction in such an approximation). The strong interactions among hadrons are presumably residual effects of the gluon exchange forces among the quarks (in the same way that interactions between neutral atoms are residual electromagnetic effects).

Thus we have an interesting analogy:

color ↔ electric charge

gluon ↔ photon

colored quark ↔ charged particle (e^-,p)

hadron ↔ atom

q-q interaction (gluon exchange) ↔ e^--p interaction (photon exchange)

hadron-hadron interaction ↔ atom-atom interaction

However, like all analogies, it holds only up to a point:

gluons are colored ↔ photons are not charged

hadrons are always colorless ↔ atoms are not always neutral (ions)

colored particles are confined ↔ charged particles are not confined

A popular, unproven, conjecture states that the last three statements are related to each other. In other words, the nonabelian nature of the theory which forces us to have colored gluons, is also responsible for the confinement property. This remains to be understood.

2.9 Probing the Hadron

Hadrons contain pointlike quarks which provide them with their quantum numbers. These are the qqq triplets in the baryon and the $q\bar{q}$ pairs in the meson. These are the "valence quarks". In addition, *a hadron probed by a high momentum weak or electromagnetic current may* appear to contain additional quark-antiquark pairs. Such pairs produce the so-called $q\bar{q}$ "sea" which carries no internal quantum numbers, but contains $J = \frac{1}{2}$ pointlike charged objects. In addition to the "valence quarks" and the "sea quarks" the hadron should, of course, contain gluons. The gluons carry no internal quantum numbers (except color) and they do not respond to the weak and electromagnetic interactions.

Deep inelastic electron and neutrino experiments teach us that:

(i) The high momentum components of the hadron (near $x \to 1$) are usually associated with valence quarks. Near $x \to 1$ we find the largest ratio for $F_2(ep)/F_2(en)$, reflecting the different quantum numbers of the proton and neutron.

(ii) Near $x \sim 0$ the $q\bar{q}$ "sea" is dominant. In fact, it seems that as $x \to 0$, $F_2(x) \to$ const. This would mean that the $q\bar{q}$ "sea" is infinite. The ratio $F_2(ep)/F_2(en)$ approaches one as $x \to 0$, consistent with the expectation from a neutral $q\bar{q}$ "sea".

(iii) Most of the total momentum of the quarks is carried by the valence quarks. The low energy charged current ratio $\sigma(\bar{\nu}N)/\sigma(\nu N) \sim \frac{1}{3}$

indicates that approximately 90% of the quark momentum is deposited with the three valence quarks in the nucleon.

(iv) The total quark momentum ("valence" and "sea" quarks) accounts for approximately 50% of the hadron momentum. The other 50% are presumably carried by the gluons. This is deduced from a sum rule for the mean squared charge of the constituents. The sum rule is:

$$\int_0^1 F_2(ep)dx = \langle Q^2\rangle_p$$

For valence quarks $\langle Q^2\rangle_p = \frac{1}{3}$; for the "sea": $\langle Q^2\rangle = \frac{2}{9}$ (assuming equal numbers of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$). Since most of the quark momentum is carried by the valence quarks, we therefore expect:

$$\int_0^1 F_2(ep)dx \sim 0.3 - 0.33$$

The measured experimental value is[5] approximately 0.16, implying that only half of the overall momentum of the proton is associated with the quarks while the other half is associated with neutral constitutents which do not interact with the electromagnetic current. These are, presumably, the gluons.

A similar conclusion is reached when we study the same sum rule for en scattering.

The overall picture leads us to the conclusion that, at least at low energies (SLAC, CERN) the $q\bar{q}$ "sea" carries 5%-10% of the nucleon's momentum while the rest is divided more or less equally between the valence quarks on one hand and the gluons on the other hand.

The observed deviation from scaling at Fermilab energies may imply that the share of the momentum carried by the $q\bar{q}$ "sea" is increasing with energy. Alternatively, such violation may be due to new currents and new quarks. A careful evaluation of the relative contributions of the "valence" and "sea" quarks as a function of energy will be extremely important.

2.10 Weak Currents and Quarks

All observed weak processes involving the three "traditional" quarks u, d, s can be described by the following assertions:

(i) The charged weak quark-current is a V-A current.

(ii) The weak currents "select" linear combinations of the d, s quarks such that:

$$d' = d\cos\theta + s\sin\theta$$

$$s' = -d\sin\theta + s\cos\theta$$

and $\theta \sim 15^{\circ}$ is the Cabibbo angle.

(iii) The quarks (u,d') transform as a doublet under the Salam-Weinberg SU(2) x U(1) gauge group.

This theory is incomplete in three aspects:

(a) No explanation is provided for the absence of strangeness changing neutral currents. Charm cures that.

(b) The CP violating weak interaction is an additional independent interaction. This is not cured by charm and will be discussed in Section 4.

(c) No convincing explanation is provided for the strong enhancement of $\Delta I = \frac{1}{2}$ nonleptonic transitions. Charm and/or other additional quarks may be related to this problem.

The conventional weak currents mentioned above do not exclude the possibility of significant V+A charged currents, as long as they do not have a substantial matrix element between the three quarks u, d, s. Such currents could connect any one of these three quarks to a new type of quark.

2.11 Summary

The overall pre-charm picture is that of a satisfactory phenomenological picture based on a gauge theory for weak and electromagnetic interactions and on a theory of colored gluons for the strong interactions. All matter is made out of three types of tri-colored quarks, four leptons and several vector gauge particles mediating all interactions.

The only two experimental difficulties prior to November 1974 were the absence of strangeness changing neutral currents and the peculiar high energy behaviour of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$.

3. Charm and the Standard Model

3.1 Prehistory: Charm May Exist

The earliest motivation for introducing an additive quantum number beyond Strangeness were based on some kind of lepton-hadron analogy[18]. As soon as the fourth lepton (ν_μ) was discovered, speculations concerning a fourth "fundamental baryon" beyond p, n, Λ, were entertained. The brilliant confirmation of SU(3) symmetry (with the 1964 discovery of Ω^-) immediately led to several proposals of an extended SU(4) symmetry[18] involving one more quantum number - "charm".

All of these attempts were based either on a "Why not?" philosophy or on an aesthetic analogy between leptons and hadrons. There was no compelling theoretical reason for the new quantum number and no experimental need for it.

The essential ingredients which were predicted at that time[18] were the existence of a new spectroscopy of charmed particles and (implicitly)the weak interaction connection between charmed hadrons and strange hadrons.

3.2 History: Charm Must Exist

The history of charm begins when prehistory ends: in 1970. The GIM paper[19] established, for the first time, a real reason for charm. GIM showed[19] that a simple and reasonable (gauge) theory of weak interactions must have neutral currents and that the absence of strangeness changing neutral currents can be reconciled with the presence of strangeness conserving neutral currents only if a fourth quark is added. The c-quark then belongs to a weak SU(2) doublet (c,s'). Charmed particles must exist and they mostly decay into strange particles. Neutral weak currents conserve all additive quantum numbers (charge, strangeness, charm).

The importance of the GIM paper is in providing the first serious theoretical framework as well as the first experimental reason for charm. It transformed the prediction from a pure speculation into a necessity (within a specific theoretical framework).

The second reason for the existence of the charmed quark came two years later. It was noted[20] that the cancellation of the divergences of the triangle anomalies in a gauge theory of V-A currents, requires the existence of a fourth quark. We will discuss the question of anomalies in more detail in Section 5.1. Here it suffices to say that the anomaly argument was entirely independent of the GIM argument, and both were necessary within the simplest gauge theory framework. The anomaly argument also provided, for the first time, a constraint connecting the world of leptons and the world of quarks.

Thus we reach November 11, 1974 equipped with four "motivations" for charm:

1. Why Not?
2. Four leptons exist. It would be nice to have four quarks.
3. $|\Delta S| = 1$ neutral currents are not observed.
4. Triangle anomalies must cancel.

3.3 Charm Exists

The value of $R = \sigma(e^+e^-\to hadrons)/\sigma(e^+e^-\to\mu^+\mu^-)$ goes through a clear threshold around $E_{c.m.} \sim 4$ GeV [16].

The ψ and ψ' particles[21] behave like bound states of a new quark and its antiquark.

Two muon events in neutrino reactions[22] prove the existence of a new additive quantum number which is conserved by the strong interactions and is not conserved by the weak interactions.

All of these discoveries were announced within a two months period (Nov. 1974 - Jan. 1975). Each one of them, by itself, has provided an indirect proof that a new quark and a new quantum number exist. Additional indications came from other neutrino data such as the candidate for charmed baryon found at Brookhaven[23] and the so-called y-anomaly[8].

However, the final definite proof came only in May-June 1976 when the D^o and D^+ charmed mesons were identified[24] and their decays to final states involving K-mesons were established.

By now there cannot be any doubt that charm is found and that it possesses all the essential ingredients predicted by GIM[19] and by later authors[25].

Many open problems concerning charm spectroscopy still remain, of course, but most of them are of secondary importance. We will now discuss some of these questions.

3.4 Spectroscopy of the ψ-Family: A Few Problems

The spectrum of the ψ-family includes[26]:

(i) Two well established $J^{PC} = 1^{--}$ states ψ and ψ';

(ii) Five $C = +1$ states at 2.85, 3.41, 3.45, 3.50, 3.55 GeV with different degrees of experimental reliability.

(iii) A complicated structure including several broad $J^{PC} = 1^{--}$ states as well as possible thresholds and interference patterns in the range $E_{c.m.} = 3.8$-4.5 GeV.

The gross features of this spectrum are in remarkable agreement with those expected from a $c\bar{c}$ system[27]. All states below the charmed meson threshold ($E_{c.m.} = 2m(D) = 3.73$ GeV) are narrow and their decays are suppressed by the Zweig-Iizuka rule[28]. All states above the $D\bar{D}$ threshold are broad (say, $\Gamma > 10$ MeV).

The detailed properties of the ψ-spectrum have been discussed by many authors[29]. Here we focus our attention on a few difficulties which require more experimental work and theoretical analysis:

(a) The state χ(2.85) provides us with several puzzles. Its most natural assignment is as the $1\,^1S_0$ companion of ψ(3.1). However, the ψ-χ mass splitting of 250 MeV is substantially larger than all theoretical estimates[27]. We believe that the most reasonable "explanation" for this large mass difference is the assumption that χ(2.85) contains a small but significant component of non-charmed quarks[30] (a few percent), while ψ(3.1) is a much more pure $c\bar{c}$ state. The possible mixing of "light" quarks into the χ(2.85) wave function implies that the η and η' mesons contain $c\bar{c}$

components. This may explain the unusually large decay widths for $\psi \to \eta'\gamma$, $\psi \to \eta\gamma$, $\psi' \to \psi\eta$ [30].

Another problem concerning x(2.85) is the small decay rate for $\psi \to \gamma x$. Experimentally[26], it seems to be smaller than 3 keV. The theoretical prediction is around 20 keV. The discrepancy is p not so serious in view of the ambiguities in the theoretical estimate. However, if the actual $\psi \to \gamma x$ width is even smaller than the present upper limit, a serious problem may develop.

Experimentally, we have[26]:

$$\frac{\Gamma(\psi \to \gamma x)}{\Gamma(\psi \to all)} \cdot \frac{\Gamma(x \to \gamma\gamma)}{\Gamma(x \to all)} \sim 2\cdot 10^{-1}$$

Since:

$$\frac{\Gamma(\psi \to \gamma x)}{\Gamma(\psi \to all)} < 5\%$$

We conclude:

$$\frac{\Gamma(x \to \gamma\gamma)}{\Gamma(x \to all)} > 0.4\%$$

Various estimates of $\Gamma(x \to \gamma\gamma)$ range between 1-10 keV. Accepting these, we then find:

$$\Gamma(x \to all) < 2 \text{ MeV}$$

Such a small total width is barely consistent with estimates based on the two-gluon decay picture of a $^1S_0$ state[27]. It is probably too small if we assume that x(2.85) contains a few percent mixture of light quarks[30].

The overall picture is therefore that the ψ-x splitting is too large, the $\psi \to \gamma x$ rate is too small and the total x-width is too small. Better experiments are needed in order to sharpen these statements. If they do become sharper, we may have a serious problem.

(b) Four $C = +1$ states are observed[26] between $\psi'(3.68)$ and $\psi(3.1)$. Four $C = +1$ states are predicted by the simple Charmonium picture[27] - three P-states ($^3P_0$, $^3P_1$, $^3P_2$) and one excited S-state (2^1S_0). If we identify the four observed states at 3.41, 3.45, 3.50, 3.55 GeV with the four predicted states, only one assignment is possible:

$$\chi(3.41) \equiv {}^3P_o;\ \chi(3.45) \equiv 2^1S_o;\ \chi(3.50) \equiv {}^3P_1;\ \chi(3.55) \equiv {}^3P_2$$

This assignment immediately leads to serious trouble[31] concerning the identification of $\chi(3.45)$ as a 2^1S_o state. The $\chi(3.45)$ - $\psi'(3.68)$ splitting is, again, much larger than the expected ^1S_o - $^3S_1$ splitting and the absolute decay width of $\psi' \to \gamma\chi(3.45)$ is, again, somewhat too small. These difficulties are very similar to those mentioned above in our discussion of $x(2.85)$. The third difficulty concerning $\chi(3.45)$ is also similar to the third difficulty of $x(2.85)$, but it is quantitatively much more serious. Experimentally we have[26]:

$$\frac{\Gamma(\chi(3.45) \to \gamma\psi)}{\Gamma(\chi(3.45) \to \text{all})} > 25\%$$

A reasonable estimate for $\Gamma(\chi(3.45) \to \gamma\psi)$ gives approximately 1 keV. Consequently, the total width of $\chi(3.45)$ is a few keV. This is totally unacceptable for an S-state. The expected total width should probably be a few MeV, and it certainly cannot be a few keV.

A possible solution to the problem may be the association of $\chi(3.50)$ or $\chi(3.45)$ with a $^1D_2$ state(32) and the assumption that the 2^1S_0 has not yet been discovered. Better data are needed to resolve this issue.

(c) The energy range $3.7 < E_{c.m.} < 4.5$ GeV shows many peaks of different shapes and widths. Theoretically, the following structures are expected in this region:

(i) The first and second $^3D_1$ states as well as the 3^3S_1 and, possibly, 4^3S_1 states. Mixing between $^3D_1$-states and $^3S_1$-states is allowed, and is not easily estimated.

(ii) Thresholds for $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$, F^+F^-, $F\bar{F}^*$, etc. Among these, the $D^*\bar{D}^*$ and (probably) $F^*\bar{F}^*$ thresholds should be prominent, in view of the relatively large production cross sections which are expected.

(iii) Interference effects between resonant states, "cusp" effects and various other complications could arise from an accidental proximity or from a more fundamental relation between different vector particles and/or new threshold.

At present, experiment indicates a small structure around 3.85 GeV ($D\bar{D}^*$ threshold?), another structure at 3.95 GeV (a ψ''?), a sharp edge at 4.03 GeV ($D^*\bar{D}^*$ threshold?), a broad bump around 4.10-4.15 GeV (ψ'''?) and a clear bump at 4.41 GeV (another ψ?). Additional structures are possible and more accurate data are needed.

According to the Charmonium picture, the total number of vector particles in the 3.7-4.5 GeV region should be three or, at most, four. It is particularly important to verify that no additional vector particles exist in the same region. The existence of such states might provide an indication for the existence of additional quarks, beyond the charmed quark[33].

The overall picture of the ψ-spectrum is in remarkable accord with the qualitative features of the Charmonium scheme[27]. Whether all the details fall into place, time will tell.

3.5 Spectroscopy of Charmed Mesons and Baryons: Brief Remarks

We already know that:

(a) D^o exists[24]. Its mass is 1865 MeV. It decays into $K^-\pi^+$, $K^-\pi^+\pi^-\pi^+$ and probably $K^o\pi^-\pi^+$. Its branching ratio into each of these modes is a few percent. Consequently, its total branching ratio into K+anything is large. It is definitely much larger than the 5% expected for a Cabibbo-suppressed decay. Whether it is well above 50%, as expected for the GIM mechanism, we will know soon.

(b) D^+ exists[24]. Its mass is 1875 MeV. It decays into $K^-\pi^+\pi^+$ and probably $K\pi$, $K\pi\pi\pi$, etc.

(c) D^* exists[24]. Its mass is approximately 2010 MeV.

(d) A charmed baryon ($\Lambda_c^+ \equiv cud$) probably exists[34] around 2250 MeV.

(e) Semileptonic decays of charmed particles are seen in e^+e^- collisions[35]. Semileptonic decays of (the same?) charmed particles were earlier seen in neutrino reactions[22].

(f) An unusually large number of K-mesons (too many?) are associated with semileptonic decays of charmed particles in neutrino reactions[36] and, possibly, also in e^+e^- collisions[37].

All of these facts are consistent with the general expectation of the charm scheme. What remains to be done in the very near future is:

(i) Find the F^+. Its mass should be somewhere around 2 GeV.

(ii) Search for "exotic" charmed particles such as F^0, D^{++}, D^- (with C = +1), etc. Such states should not exist if only one new quark (with charge +2/3) is present.

(iii) Discover other low-lying charmed baryons, such as Σ_c^{++}, Σ_c^+, Σ_c^0 .

(iv) Determine the spin of D, D^*. Establish parity violation in D-decays.

(v) Establish the K/π ratio in nonleptonic and semileptonic decays of charmed particles.

(vi) Study the space-time structure of semileptonic D-decays. Is it pure V-A?

Needless to say, all of these points represent the tip of the charm-spectroscopy iceberg. However, it is the tip of the iceberg which is most interesting.

3.6 The Standard Model

We are now in a position to define the "standard model" in which we must all believe, and from which we embark on our "beyond charm" excursion.

The standard model assumes the existence of four leptons and four quarks. Their left handed components transform as doublets under an SU(2) x U(1) gauge algebra of the weak and electromagnetic interactions. The doublets are:

$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}u \\ d'\end{pmatrix}_L \quad \begin{pmatrix}c \\ s'\end{pmatrix}_L$$

where

$$\begin{pmatrix}d' \\ s'\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix} \begin{pmatrix}d \\ s\end{pmatrix} .$$

The right-handed components of the same eight fundamental fermions transform as SU(2) singlets. Hence, the charged weak currents are V-V currents.

All quarks are color triplets. All leptons are colorless.

The free parameters of the theory are the quark and lepton masses, the Cabibbo angle and the Weinberg angle.

This is the standard model based on the Weinberg-Salam SU(2) x U(1) gauge group(1) and the Glashow-Iliopoulos-Maiani scheme(19).

The model must include Higgs mesons (which we do not specify) as well as colored gluons and a theory of the strong interactions (Quantum Chromo-Dynamics?).

The model does not allow much freedom as far as the weak and electromagnetic interactions of the four quarks and four leptons are concerned. However, there is a lot of freedom to add ingreidents such as additional quarks, additional leptons, V+A currents (not connecting the original 4+4 fermions), new interactions, etc. Such additional ingredients may be required by theoretical considerations or by experimental facts. The next three sections are devoted to them.

4. CP-Violation: New Quarks or New Currents?

4.1 How to Break CP in a Gauge Theory?

The charged weak current of the "standard model" can be written as:

$$J = (\bar{u}\ \bar{c})\ \gamma_\mu(1+\gamma_5)\ A\begin{pmatrix} d \\ s \end{pmatrix}$$

where A is a unitary 2 x 2 matrix. In principle, such a unitary matrix can be fully parametrized in terms of four real parameters. However, three of these parameters can be "absorbed" into the definitions of the quark states u, d, c, s. In other words, we can redefine u an $ue^{i\phi}$ without suffering any observable consequences. Four quark states can absorb only three phase parameters - one for each quark except for one overall phase. We therefore remain with an A-matrix which is fully determined by one real parameter. A is then necessarily an orthogonal matrix and the single parameter can be chosen as the Cabibbo angle θ :

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

All weak transition matrix elements involving the four quarks and the four vector gauge particles (W^+, W^-, Z^0, γ) will be relatively real in such a theory. Consequently, CP is necessarily conserved in a gauge theory based on the "standard model".

One might suggest that the interaction responsible for CP violation is not an integral part of the gauge theory of weak and electromagnetic interactions. In that case, all the fundamental questions which were solved by the introduction of gauge theories must be reopened. It is not clear, for instance, that a gauge theory with an external CP-violating piece, remains renormalizable, etc.

It would be much more attractive to be able to account for CP-violation within the framework of the gauge theory, in a fashion that preserves all the beautiful features of the theory. This could be achieved if the transition matrix elements would contain a complex phase which cannot be eliminated by redefining the physical states, and which is, therefore, experimentally observable.

There are, at least, three ways to achieve this within the standard framework of gauge theories (but not within the simplest version of the "Standard Model"). We now discuss them briefly.

4.2 More Higgs Particles

We may remain with the "Standard Model" (four quarks, V-A currents) but introduce the complex phase parameter into the interactions of the Higgs particles(38). So far, we have refrained from specifying the properties or even the number of Higgs particles. For any given set of quarks, leptons and currents there is a "minimal" set of Higgs particles which are necessary. In the case of the simplest $SU(2) \times U(1)$ Weinberg-Salam model at least four Higgs particles are needed. Three of them are "eaten up" by the three massive vector gauge bosons and one remains as a physical particle. It is clear, however, that we can also introduce a larger set of Higgs particles. Such an assumption is neither elegant nor necessary, but it is perfectly consistent with all the requirements of the theory. Weinberg(38) has recently pointed out that the (otherwise ugly) possibility of doubling the number of Higgs particles, enables us to introduce an arbitrary relative phase parameter between the interactions of different Higgs particles. Such a phase will produce CP-violation through the interference of diagrams involving different virtual Higgs particles. The attractive part of this scheme is its ability to incorporate CP-violation into the "Standard Model" without introducing new quarks or new currents. The unattractive feature is the explicit dependence on the properties of the Higgs particles, and the required non-minimal set of such particles.

4.3 More Currents

If we do not appeal to the Higgs particles, we may still introduce CP-violation into the four-quark model. This can be done by the introduction of additional weak currents, beyond the V-A currents of the "Standard Model". The idea is simple: The 2 x 2 matrix A has "lost" three of its arbitrary parameters through a redefinition of the quark states. This could be done only if the four quarks participate only in the V-A charged weak current of Section 4.1. If, however, the same quarks also participate in a V+A current, we do not have the freedom to absorb the phase parameters of the additional current into the redefined quark states. In other words, a relative phase between the V-A transitions and the V+A matrix elements cannot be, in general, eliminated.

This method of violating CP was first suggested by Mohapatra[39] several years ago. It was, since then, discussed by many authors[40]. It departs from the "Standard Model" by the introduction of new currents and, consequently, by the assignment of some of the right handed quarks into doublets of $SU(2) \times U(1)$.

4.4 More Quarks

The third possibility of introducing CP-violation into the framework of gauge theory is to increase the number of quarks while remaining with V-A currents and with a minimal set of Higgs particles.

Let us consider a model with N SU(2) doublets of left-handed quarks (All right-handed quarks are assumed to be in SU(2) singlets. No V+A currents.). The charged weak current would be similar to that of Section 4.1 except that the A-matrix will now be a unitary N x N matrix (N = 2 for the Standard Model).

A unitary N x N matrix is characterized, in general, by N^2 real numbers. Of these, 2N-1 can be absorbed into the redefined quark states (we have 2N quarks). We remain with $(N-1)^2$ real parameters. An orthogonal N x N matrix requires $\frac{1}{2}N(N-1)$ real parameters (generalized Euler angles). Consequently, our $(N-1)^2$ real parameters can be chosen as:

(i) $\frac{1}{2}N(N-1)$ real rotation angles

(ii) $\frac{1}{2}(N-1)(N-2)$ phase parameters

For the "Standard Model" (N=2) we, obviously, have one rotation angle (the Cabibbo angle)* and no phase parameters. Hence - no CP-violation.

The next simplest case is a six-quark model with N = 3 . Here we have three generalized Cabibbo angles and one phase. Hence - the theory does not conserve CP. Needless to say, with still larger numbers of quarks, the number of phase parameters increases rapidly.

We, therefore, see that the minimal number of quarks needed in order to violate CP in a pure V-A theory is six. This was first observed by Kobayashi and Maskawa(41).

4.5 A Six-Quark Model for CP-Violation

We consider an extension of the "Standard Model" involving three, rather than two, left handed doublets:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

The t and b quarks have electric charges $+\frac{2}{3}$, $-\frac{1}{3}$, respectively(42). The d', s' and b' states are linear combinations of d, s, b defined by a 3 x 3 unitary matrix A which can be chosen as(41):

$$A = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & -s_2s_3e^{i\delta}+c_1c_2c_3 & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & -c_2s_3e^{i\delta}-c_1s_2c_3 & -c_1s_2s_3+c_2c_3e^{i\delta} \end{pmatrix}$$

Here $c_i \equiv \cos\theta_i$, $s_i = \sin\theta_i$; θ_1 is the usual Cabibbo angle; θ_2 and θ_3 are additional Cabibbo-like angles; δ is a phase parameter, responsible for CP-violation. The charged weak current is

$$J^- = (\bar{u}\ \bar{c}\ \bar{t})\ \gamma_\mu(1 + \gamma_5)\ A\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

It is clear that all CP-violation effects in such a theory will be proportional to $\sin\delta$.

If we restrict our attention to CP-violating phenomena involving the three "light" quarks (u, d, s), we immediately see that all such CP-violating amplitudes are also proportional to s_3 (since in the A_{22}

and A_{32} elements of the A-matrix, $e^{i\delta}$ is always accompanied by s_3).

What can we say about θ_3? The angle θ_3 and the parameter s_3 must be small. They can be estimated by observing that the original Cabibbo theory agrees well with both strangeness conserving and strangeness changing weak processes. The value of $A_{11} = \cos\theta_1$ can be determined by comparing neutron and muon beta-decay. The value of $A_{12} = \sin\theta_1 \cos\theta_3$ is deduced from K-decays and hyperon decays. Experimentally[43] $A_{11}^2 + A_{12}^2 = 1.001 \pm 0.004$. Hence: $A_{13}^2 \leq 0.003$ or:

$$\sin\theta_1 \sin\theta_3 \leq 0.055$$

Since[43] $\sin\theta_1 \sim 0.23$, we conclude $\sin\theta_3 \leq 0.24$. In other words, the angle θ_3 is at least as small as the Cabibbo angle, and possibly much smaller. Also, to a good approximation : $\cos\theta_1 \sim \cos\theta_3 \sim 1$. We have no similar strong bounds on θ_2 .

As long as we are interested only in CP-violating processes involving the u, d, s quarks, we can also show that all CP-violating amplitude must vanish in the limit $m_c = m_t$. The proof is simple: if $m_c = m_t$ and if the c and t quarks do not appear in the initial or final state of the considered transition, we may always choose one linear combination of c and t which decouples from both the d and s quarks. Consequently, no interference between amplitudes of different phase is possible. We therefore conclude that all CP-violating transitions involving only u, d, s quarks must be proportional to $m_c^2 - m_t^2$.

The overall conclusion of our discussion in this section is that a V-A six-quark model allows CP-violation, and that all CP-violating amplitudes among states containing only u, d, s quarks must be proportional to:

$$(m_c^2 - m_t^2)\sin\theta_3 \sin\delta$$

This holds for all CP-violating K-decays as well as for the electric dipole moment of the neutron.

Why is CP-violation a small effect? The present theory does not answer this question. The parameters θ_3 and δ may be extremely small, but they do not have to be. All we know is: $0 \le \sin\delta \le 1$, $0 \le \sin\theta_3 \le 0.24$. This is, of course, consistent with, but does not explain, the magnitude of CP-violation. The c-t mass difference may be a very small parameter. In that case the ψ-family should represent combinations of $c\bar{c}$ and $t\bar{t}$ states. This is an intriguing possibility which is not yet ruled out.

4.6 CP-Violation in $K \to 2\pi$ and the Six Quark Model

Following the discussion of the previous section we may now proceed to calculate the parameters of the CP-violating amplitudes in $K^o \to 2\pi$.

This was first done by Pakvasa and Sugawara[44] and, independently, by Maiani[45]. The standard formalism starts from the mass matrix of the neutral K-system with matrix elements $M_{k\ell} - \frac{1}{2}i\Gamma_{k\ell}$.

$$M_{k\ell} = M_o + (k|H_w|\ell) + P\sum_x \frac{(k|H_w|x)(x|H_w|\ell)}{M_o - E_x}$$

$$\Gamma_{k\ell} = 2\pi \sum_x (k|H_w|x)(x|H_w|\ell)\ \delta(E_x - M_o)$$

If CP is violated the M-matrix is not symmetric and its eigenvalues are proportional to:

$$(1-\varepsilon)K_o+(1+\varepsilon)\bar{K}_o; \quad (1+\varepsilon)K_o-(1-\varepsilon)\bar{K}_o$$

The ε-parameter which charactertizes the magnitude of CP violation in the K_o eigenstates is given by:

$$|\varepsilon| = \frac{\text{Im } M_{12}}{\sqrt{\Delta M^2 + \frac{1}{4}\Gamma_S^2}}$$

where $\Delta M = m(K_S)-m(K_L)$ and Γ_S is the width of K_S. The violation of CP in $K_L \to 2\pi$ may be due either to the mixture of opposite CP-values in the K_L-state (characterized by ε) or to CP-violation in the decay amplitude itself. The latter is characterized by the parameter ε', where:

$$\varepsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2-\delta_o)} \frac{\text{Im } A_2}{A_o}$$

δ_2, δ_o are the I = 2, 0 ππ S-wave phase shifts at $\sqrt{s} = M_K$ and A_2, A_o are the corresponding $K \to 2\pi$ amplitudes:

$$\langle K^o|(\pi\pi)_{I=0}\rangle = A_o e^{i\delta_o}$$

$$\langle K^o|(\pi\pi)_{I=2}\rangle = A_2 e^{i\delta_2}$$

The calculation of ε requires a calculation of $\frac{\text{Im } M_{12}}{\Delta M}$. The numerator is given by the diagram in figure 2, where x = c, t. The denominator is given by the same diagram, but is dominated by the x = u term. In the approximation $\cos\theta_1, \cos\theta_3 \sim 1$, and assuming:

$$\Delta \equiv \frac{m_t^2 - m_c^2}{m_c^2} < 1$$

we obtain[46]:

$$\left|\frac{\mathrm{Im}\, M_{12}}{\Delta M}\right| \sim \Delta \sin\delta \; s_2 c_2 s_3 \; \frac{1-\Delta(c_2^2 - s_2^2)}{1+\Delta s_2^2}$$

If $\Delta \gtrsim 1$ the expression is similar but involves many $\log(\Delta+1)$ terms[46]. Using this expression we realize that, as explained in Section 4.5, the order of magnitude of the ε-parameter is given by:

$$\varepsilon \sim \Delta \cdot \sin\delta \cdot s_3$$

and the small absolute magnitude of ε remains unexplained (but not inconsistent with the theory).

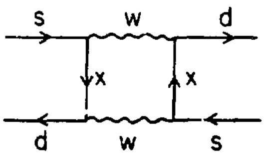

Figure 2: Contribution to $K^0 - \bar{K}^0$

The ε'parameter is experimentally consistent with zero, and is definitely smaller than ε. In the six-quark model this is actually predicted[44][46]. There are two classes of diagrams which contribute to ε' (figure 3). The diagram of figure 3a involves the conversion of a $c\bar{c}$ or $t\bar{t}$ system into $d\bar{d}$. This is strongly suppressed by the Zweig-Iizuka rule[28]. The suppression factor cannot be determined accurately but is probably of the order of 1%-10%. The second diagram (fig. 3b) involves a term of order M_q^2/M_W^2. For $m_t \lesssim$ 5-10 GeV this gives us another factor of 1%-10%. The estimates of ε' are given by[46]:

$$|\varepsilon'| \sim \varepsilon\left|\frac{A_2}{A_0}\right| \cdot O\left(\frac{m_q^2}{m_W^2}\right) \qquad \text{(fig. 3a)}$$

$$|\varepsilon'| \sim \left|\frac{A_2}{A_0}\right| \sin\delta \; s_2 c_2 s_3 \; (\xi_c - \xi_t) \qquad \text{(fig. 3b)}$$

where ξ_c, ξ_t are the $c\bar{c}$ and $t\bar{t}$ Zweig-Iizuka suppression factors, respectively. We therefore conclude that:

$$\left|\frac{\varepsilon'}{\varepsilon}\right| \sim \left|\frac{A_2}{A_0}\right| \; (1\%\text{-}10\%)$$

This prediction is consistent with the experimental situation and represents a nontrivial success of the model.

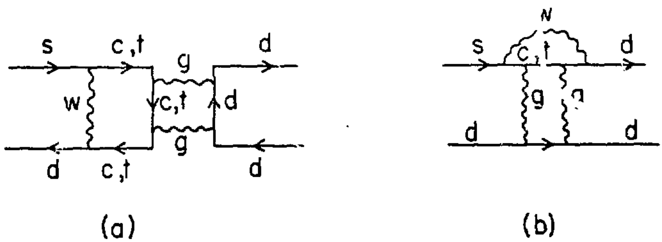

Figure 3: Diagrams contributing to ε'

Predictions of the model for other CP-violating K-decays have been discussed by Ellis et al.(46) In all cases the predictions of the model are experimentally indistinguishable from the predictions of the superweak theory.

§ 7 Neutron Electric Dipole Moment and the Six Quark Model

The electric dipole moment of the neutron is due, in the six quark model to the diagram of figure 4(45). It is easy to see that here, again, the CP-violating effect would disappear if either θ_3 or δ would vanish; it would also vanish if $m_c = m_t$ or $m_s = m_b$ (Since here the s-quark does not appear in the initial or final state). Maiani(45) and Ellis et al.(46) have discussed this process. The predicted dipole moment is:

$$\left|\frac{D}{e}\right|_n \sim \frac{G\alpha}{\pi^3} \sin\delta \; s_1^2 s_2 c_2 s_3 \frac{(m_t^2-m_c^2)\,(m_b^2-m_s^2)}{m_W^4} m_u$$

for $m_t \sim m_b \sim$ 5-10 GeV we find:

$$\left|\frac{D}{e}\right|_n \sim 10^{-30} \text{ cm}$$

This prediction is, again, not very different from the predictions of the superweak theory, and is, again, consistent with (but far below) the present experimental limit.

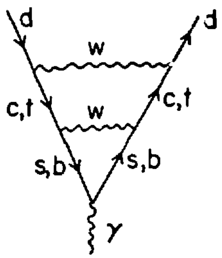

Figure 4: Diagram contributing to the neutron's electric dipole

4.8 Summary

The "standard model" with four quarks, V-A currents and a minimal set of Higgs particles, conserves CP. More quarks or additional currents or additional Higgs particles introduce CP-violation in a natural way. Except, possibly, for the Higgs particle scheme[38], no direct explanation is given to the "miliweak" magnitude of CP-violating amplitudes. However, all CP-violating gauge theories are consistent with the observed magnitude. There is no difficulty in predicting the small ε'/ε ratio. The six quark model does it in a natural and direct way. The four quark model with V+A currents can be arranged to give the same result[40]. All three CP-violating extensions of the standard model are, so far, consistent with the few available data.

Thus, the inclusion of CP-violation within the gauge theory framework must take us beyond the minimal standard model. It may lead us to a larger number of quarks, but this is not necessary. However, if a six quark model becomes a theoretical or an experimental necessity due to other reasons, the violation of CP will be an immediate consequence of the theory.

5. Anomalies, R-values and eμ Events: New Leptons and New Quarks?

5.1 Triangle Anomalies

The triangle anomaly diagram[14] (figure 5) is bad for gauge theories. It is linearly divergent and it leads to a modification of the usual Ward identity for the axial vector vertex, thus preventing the renormalization of the theory. The linearly divergent part of the diagram does not depend on the masses of the fermion lines which form the triangle. It depends only on the couplings of these fermions to the three external currents. Since these couplings are proportional to the various charges of the fermions, we may hope to create a situation in which the sum of all anomaly diagrams, summed over all possible fermion loops, will vanish. That would save the renormalizability of the theory.

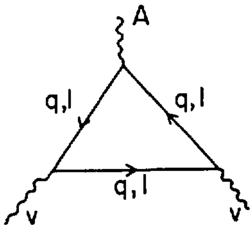

Figure 5: The triangle anomaly diagram

The cancellation of the "bad" part of the anomaly diagram is, therefore, a necessary condition in any gauge theory.

There are two basic cancellation mechanisms. One is the simple observation that in a pure vector theory (with no axial currents), no anomalies occur. Hence, if the weak current includes V-A and V+A pieces of equal strength, the full current would be a pure vector current and no anomalies will be present. Even if the V-A currents and the V+A currents connect different pairs of fermions, the same conclusion remains true as long as for each left handed SU(2) x U(1) multiplet of fermions there is a similar right handed multiplet containing fermions of the same electric charges. This simple observation follows from the mass independence of the "bad" part of the diagram.

This cancellation mechanism was first proposed by Georgi and Glashow[47]. It operates in all "vector-like" theories (see Section 6) and it does not require any connection between quarks and leptons or even between different multiplets of quarks.

The second mechanism looks, at first, more artificial. However, if true, it must have a profound influence on theories of quarks and leptons. If all weak currents are of the V-A variety, each fermion (quark or lepton) may contribute to an anomaly diagram. Let us consider the diagram for $A_3 \to \gamma+\gamma$ where A_3 is the third weak isospin component of the axial vector current. The "bad" part of the diagram is proportional to $Q_i^2 I_{3i}$ where Q_i, I_{3i} are, respectively, the electric charge and the third weak isospin component of the fermion f_i circulating in the triangle. The sum of all such anomaly diagrams becomes harmless if and only if:

$$\sum_i Q_i^2 I_{3i} = 0$$

where the summation is over all fundamental fermions (quarks and leptons). In the specific case in which all left handed fermions are in doublets or singlets of SU(2) x U(1), this condition can be simplified into:

$$\sum_i Q_i = 0$$

where the summation is now over all fermions in SU(2) doublets.

For each doublet of left handed quarks with $Q = +\frac{2}{3}, -\frac{1}{3}$ we have: $\sum Q_i = 1$ (counting all three colors). For each doublet of left handed leptons with $Q = 0, -1$ we have: $\sum Q_i = -1$. Hence, a model containing only left handed quarks or only left handed leptons of the usual electric charges, cannot be accepted[20]. Moreover, the pre-charm model of three quarks (u, d, s) and four leptons (ν_e, e, ν_μ, μ) is unacceptable. The number of left handed quark and lepton doublets in a V-A theory must be equal. This was the second argument which necessitated charm[20], and which we mentioned in Section 3.2.

Note that the quark-lepton cancellation may have far-reaching consequences. It is the first indication that we have for a definite connection between quarks and leptons. The existence of a given set of leptons dictates certain constraints on the world of quarks and vice versa. We will return to this subject in Section 7.

Returning to our cancellation mechanisms, we conclude that the vector-like mechanism as well as the quark-lepton mechanism are both possible, and any combination of them is obviously allowed.

In a pure V-A theory the cancellation of anomalies and the existence of four leptons implied the existence of the fourth (charmed) quark. In such a theory, the existence of an additional (heavy) lepton, would, again, lead to the necessary existence of additional quarks.

The anomalies, by themselves, do not take us beyond charm. However, once we find a new fermion (such as a heavy lepton), new currents or other new fermions are needed. This observation acquires immediate practical importance in the next two sections.

5.2 The value of R

The value of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ provides us with a measure of the sum of the squared charges of all the fundamental pointlike fermions which are produced at a given energy. In the range $4.5 \leq E_{c.m.} \leq 8$ GeV the value of R is approximately constant and is given by[4]

$$R \sim 5 - 5.5$$

with a 20% error in absolute normalization. The standard model predicts: $R = 3\frac{1}{3}$. Approximately 2 ± 1 units of R remain unexplained. They may be due to additional quarks and/or additional leptons. The absolute minimum would be one charged lepton or one quark with $Q = +\frac{2}{3}$. A heavy lepton could be of the sequential type, accompanied by its own neutrino and forming a third SU(2) doublet of left handed leptons. It could also be associated with the electron or the muon, forming an SU(2) triplet with e, ν_e or with μ, ν_μ. In both cases, the anomaly argument requires that either V+A currents or new quarks should exist.

The simplest possibility would, of course, be a pure V-A theory with a third doublet of leptons (ν_U, U^-). This would necessitate a third doublet of quarks (t,b) of the same type discussed in connection with CP-violation in Section 4. If the new lepton U^- has a mass around 2 GeV, it might be responsible for an extra unit of R. The t and b quarks may then appear only above present energies[29]. Note that the mass independence of the anomaly term prevents us from predicting the masses of the missing fermions. They can be arbitrarily heavy.

It is also possible that the observed extra units of R are entirely due to one or more new quarks (t and possibly others). In that case, again, the anomaly argument requires V+A currents or heavy leptons.

The overall conclusion is, therefore, that the measured value of R in the 4-8 GeV range necessitates fermions beyond the standard model. Futhermore, if we remain only with V-A currents, both new quarks <u>and</u> new leptons are necessary. Only one new fermion <u>must</u> be produced at present energies. The others may be postponed to higher energies.

5.3 The eμ events

Events of the type:

$$e^+ + e^- \rightarrow e^{\pm} + \mu^{\mp} + \text{no other observed particle}$$

have been detected at SPEAR[2]. By now, more than 100 events are available and they appear to represent the production and decay of a pair of new fermions. It is unlikely that these events are due to decays of charmed particles. The only "conventional" explanation which is consistent with the data is the production and decay of a new heavy lepton of mass around 2 GeV. Such a heavy lepton could be a sequential lepton U^- forming with its neutrino a (ν_U, U^-) left-handed SU(2) doublet. The observed events are, therefore, presumably due to:

$$e^+ + e^- \rightarrow U^+ + U^-$$

followed by:

$$U^+ \rightarrow e^+ + \nu_e + \bar{\nu}_U; \quad U^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_U$$

If this interpretation of the $e\mu$ events is correct, the large value of R is partly (perhaps fully) explained. In that case, however, the anomalies require another pair of quarks, possibly at higher energy.

No matter how we look at it, the value of R, the $e\mu$ events and the cancellation of anomalies tell us that substantial physics beyond charm is already found. This new physics must include at least two of the following ingredients which go beyond the standard model:

(i) New leptons beyond ν_e, e, ν_μ, μ.

(ii) New quarks beyond u, d, s, c .

(iii) New weak currents beyond the V-A current.

6. Neutrino Processes: New Quarks and New Currents?

6.1 The Ratio $\sigma(\bar{\nu})/\sigma(\nu)$, the y-Anomaly and V+A Currents

The simple minded parton model for deep inelastic neutrino processes, assuming V-A charged weak currents, and ignoring the "infinite sea" of $q\bar{q}$ pairs, predicts:

$$\frac{\sigma(\bar{\nu}N \to \mu^{+}+\text{anything})}{\sigma(\nu N \to \mu^{-}+\text{anything})} = \frac{1}{3}$$

Low energy results from CERN have confirmed this prediction[48], providing supporting evidence for the various assumptions involved. Early results from the Fermi laboratory have confirmed that at relatively low neutrino energies the $\bar{\nu}/\nu$ ratio is, indeed, approximately 1/3. However, data at higher energies revealed two new, related, striking results:

(i) The $\sigma(\bar{\nu})/\sigma(\nu)$ ratio increases dramatically as a function of energy. It reaches 0.6-0.7 at 150 GeV or so[7].

(ii) The y-distribution of the $\bar{\nu}$-events changes with energy[8]. More high-y events are observed at high energies and the y-distribution at these energies is inconsistent with the $(1-y)^2$ shape which is observed at low energies and predicted by the V-A current assumption.

Both of these experimental observations could be explained if, above a certain threshold (which must be around $E_\nu \sim 30$ GeV), new weak currents of the V+A type come into play. The association of these new currents with a threshold phenomenon hints that they are related to the production of a new quark. In fact, if we continue to neglect the "infinite $q\bar{q}$ sea", and allow only quarks with

$Q = \frac{2}{3}, -\frac{1}{3}$, we note that in the charged current $\bar{\nu}N$ reaction, the struck quark must be a u-quark. The transition:

$$\bar{\nu}+u \to \mu^{+}+x$$

must yield a state x with electric charge $-\frac{1}{3}$. For a V-A current, x would normally be the d-quark, and rarely (with probability $\sin^2\theta_c \sim 5\%$) an s-quark. If the u-quark is also involved with a V+A current, the produced quark x can be neither d nor s. Hence, a new quark is needed, with the quantum numbers of the b quark (see Section 4).

The theory must then include a right-handed SU(2) x U(1) doublet $(u, b)_R$, in addition to the usual left-handed doublets $(u, d')_L$, $(c, s')_L$. The mass of the b-quark could be anywhere in the 3-10 GeV range and the $\sigma(\bar{\nu})/\sigma(\nu)$ ratio should eventually reach 4/3.

It is, of course, possible that additional right handed and/or left handed doublets exist. In that case the asymptotic value of $\sigma(\bar{\nu})/\sigma(\nu)$ may be different. However, the cross sections:

$$\nu+d \to \mu^{-}+u \quad \text{(left handed)}$$

$$\bar{\nu}+u \to \mu^{+}+b \quad \text{(right handed)}$$

are asymptotically equal, thus guaranteeing that the ratio $\sigma(\bar{\nu})/\sigma(\nu)$ is much larger than 1/3.

We must emphasize, at this point, that the $\bar{\nu}N$ scattering data is, at present, the only experimental evidence for V+A currents. It suffers from two obvious drawbacks, one experimental and one theoretical:

(a) The data come from one experiment. No independent confirmation is available. The experiment is difficult. The determination of the y-dependence involves serious experimental corrections while the measurement of $\sigma(\bar{\nu})/\sigma(\nu)$ depends on difficult questions of absolute normalization. We have no reason to doubt the experimental result, but so much hangs on it, that we would feel more comfortable with an independent confirmation.

(b) The changing y-distribution, as well as the increasing $\sigma(\bar{\nu})/\sigma(\nu)$ ratio, could be due to a scaling violation which reflects the much increased significance of $q\bar{q}$ pairs at high energies. If at high energies, the $q\bar{q}$ pairs carry, say, 30% of the nucleon momentum, instead of 5% at low energies, we would expect a similar experimental effect, without any V+A currents or new quarks. Such a shift in the momentum distribution would represent an enormous violation of scaling. It would lead to strong scaling violation in deep inelastic ep and μp scattering at the same energy and q^2. On the other hand, V+A weak currents would have no relevant influence on ep and μp scattering.

Thus, before we convince ourselves that V+A currents are necessary, it is important to settle these two questions.

For the rest of this section, however, we will assume that the $\bar{\nu}$-data does tell us that V+A currents are necessary, and we will study the implications of this possibility.

6.2 Vectorlike Theories

The most satisfactory theoretical framework for introducing V+A weak currents is the hypothesis of a vectorlike theory[47]. A vectorlike theory is a theory which includes V-A and V+A currents of equal strength, and which, in the limit of massless fundamental fermions, becomes a pure vector theory.

The standard V-A current can be written as:

$$\bar{q}\ \gamma_\mu(1+\gamma_5)Aq$$

where q is a vector of quark states and A is a matrix of Cabibbo-like angles (and, possibly, phases). If, in addition, we have V+A currents we will have an extra term of the form:

$$\bar{q}\ \gamma_\mu(1-\gamma_5)Bq$$

where the matrix B is, in general, different from A. if the strengths of the two currents are identical and if A = B we clearly have a pure vector current of the form $\bar{q}\gamma_\mu Aq$. If $A \neq B$ but both matrices are unitary, we can find a unitary transformation U such that $B = UAU^\dagger$. We can then rewrite the current as:

$$\bar{q}'\ \gamma_\mu\ A\ q'$$

where

$$q' = [(1+\gamma_5) + (1-\gamma_5)U]q$$

The theory[47] is vector-like in the sense that it conserves parity and contains only a vector current in the limit of massless quarks. If we introduce the quark mass term, parity violation as well as axial vector terms are reinstated.

The vectorlike theory has several attractive features:

(i) There are no anomalies[47]. This follows from the absence of axial currents in the massless limit and from the mass independence of the linearly divergent term of the triangle graph.

(ii) The violation of parity is introduced in the same way as any other symmetry breaking. We have an "ideal" world with massless fermions in which strong, electromagnetic and weak interactions are all parity conserving vector interactions, obeying a gauge theory. We then have a mysterious mechanism of (presumably spontaneous) symmetry breaking, in which:

Fermions acquire masses

Cabibbo angles are determined

Parity violation is introduced

CP violation is introduced

It is clear that such a theory is very appealing, although we do not know how the symmetry breaking operates.

On the phenomenological level, a vectorlike theory of this type necessitates the introduction of, at least, six quarks. It leads to several interesting predictions. The next few sections are devoted to a discussion of these predictions.

6.3 Vector-Like Theories Require Six Quarks

We consider a charged weak current of the form:

$$J^{-} \propto \bar{q}_1 \gamma_\mu (1+\gamma_5) A q_2 \; + \; \bar{q}_1 \gamma_\mu (1-\gamma_5) B q_2$$

where q_1 is a vector of N quark states with charge $+\frac{2}{3}$ and q_2 is a vector of N quarks with charge $-\frac{1}{3}$. The matrices A, B are unitary N x N matrices. The known four quarks u, d, s, c dictate $N \geq 2$. Can we construct such a theory with N = 2 ? The answer is clearly no. The matrix B would connect the u-quark to a combination $d'' = d\cos\theta_R + s\sin\theta_R$, where θ_R is an unknown Cabibbo-like angle for right handed quarks, which determines the parameters of the B-matrix.

We know from neutron and hyperon beta decays that no substantial V+A terms contribute either to the $\bar{u}d$ transition or to the $\bar{u}s$ transition. Since, at least, one of these transitions should be large for any value of θ_R, we conclude that a four-quark vector-like model is excluded by experiment.

The next possible model is a six-quark model (N = 3). It is immediately clear that the B-matrix will connect the u-quark mostly to the b-quark, thus avoiding the V+A transitions $u \leftrightarrow d$ and $u \leftrightarrow s$. The right handed doublets would then be[49-52]:

$$\begin{pmatrix} u \\ b \end{pmatrix}_R \quad \begin{pmatrix} c \\ \tilde{s} \end{pmatrix}_R \quad \begin{pmatrix} t \\ \tilde{d} \end{pmatrix}_R$$

where $\tilde{s}$, $\tilde{d}$ are orthogonal linear combinations of s, d determined by yet another Cabibbo-like angle χ :

$$\tilde{d} = d \cos\chi + s \sin\chi$$

$$\tilde{s} = -d \sin\chi + s \cos\chi$$

The two limits $\chi \sim 0$ and $\chi \sim 90^\circ$ are quite interesting. If $\chi \sim 0$, the right handed pairs are (c, s) and (t, d) . We then have (c, s) pairs both for left-handed and right-handed quarks. The $c \leftrightarrow s$ transition is then, to a good approximation, a parity conserving, pure vector transition. On the other hand, if $\chi \sim 90^\circ$, the right-handed pairs are (c, d) and (t, s). Consequently, the decays of charmed particles should yield strange particles only in 40% of all decays, instead of the standard prediction of 70%-80%.

In the next section we argue that the $\chi \sim 0$ solution is actually favored.

6.4 The Complete Assignment of Right-Handed Doublets in a Six-Quark Vector-Like Model

Two independent arguments exclude the existence of a significant $\bar{c}d$ V+A transition.

The first argument is due to Wilczek et al.[51] and is based on the calculation of the K_S-K_L mass difference. In the standard model (with only V-A currents), the K_S-K_L mass difference can be computed in terms of the mass difference between the c-quark and the u-quark. In the limit of c-u degeneracy, the K^o-$\bar{K}^o$ transition vanishes. In the realistic case of different c, u masses the mass difference ΔM is proportional to:

$$\Delta M \propto (m_c^2 - m_u^2)\cos^2\theta_c \sin^2\theta_c$$

The correct value of ΔM is obtained[53] if we assume $m_c \sim 1.5$-2 GeV. This calculation was, in fact, used in order to predict the effective mass of the charmed quark before the discovery of charmed particles. Note that the mass difference is proportional to $\sin^2\theta_c$. This follows from the simple fact that each one of the intermediate quarks (u and c) couple to either s or d with a $\sin\theta_c$ coefficient.

If we now consider a vector-like theory, we immediately see that the diagram involving an intermediate c-quark will contribute a term proportional to $\cos^2\theta_c \sin^2\chi$ where $\cos\theta_c$ represents the left handed $\bar{c}s$ transition and $\sin\chi$ represents a right handed $\bar{c}d$ transition. For $\chi \sim 0$ this term is irrelevant and the original successful estimate of ΔM remains unchanged. For $\chi \sim 90^{\circ}$ we obtain an extra factor of $\sin^2\chi/\sin^2\theta_c \sim 20$. In fact, the right-handed coupling induces a few additional corrections and the overall estimate[51] is too large by a factor 100. This can be rectified only if we set $m_c \sim 100$-200 MeV , which is totally unacceptable. We, therefore, conclude that the $\chi \sim 0$ solution is favored.

Another argument against a right handed $\bar{c}d$ current is due to Golowich and Holstein[54]. They have studied the transformation properties of the weak Hamiltonian H_W for nonleptonic K-decays, under chiral $SU(2) \times SU(2)$. In a V-A theory, the $\Delta I = \frac{1}{2}, \frac{3}{2}$ pieces of H_W transform like the $(\frac{1}{2}, 0)$, $(\frac{3}{2}, 0)$ representations of $SU(2) \times SU(2)$. Consequently:

$$[H_W, Q+Q_5] = 0 \quad \text{and} \quad [H_W, Q_5] = - [H_W, Q]$$

where Q, Q_5 are, respectively, the vector and axial vector charges. In a vector-like theory with a substantial V+A current connecting $c \leftrightarrow d$ (namely $\chi \sim 90^{\circ}$), the $\Delta I = \frac{1}{2}$ piece of H_W will contain a term:

$$\bar{s}\gamma_\mu(1+\gamma_5)c \cdot \bar{c}\gamma_\mu(1-\gamma_5)d$$

Such a term belongs to the $(0, \frac{1}{2})$ representation of $SU(2) \times SU(2)$ and obeys:

$$[H_W^{1/2}, Q-Q_5] = 0 \quad \text{and} \quad [H_W^{1/2}, Q_5] = [H_W^{1/2}, Q] \, .$$

If the $\Delta I = \frac{1}{2}$ piece of H_W is dominated by this term, as suggested by several authors, we have the following situation:

(i) In the standard model:

$$[H_W^{3/2}, Q_5] = - [H_W^{3/2}, Q] \; ; \quad [H_W^{1/2}, Q_5] = - [H_W^{1/2}, Q]$$

(ii) In a model with a V+A current of the form $\bar{c}d$:

$$[H_W^{3/2}, Q_5] = - [H_W^{3/2}, Q] \quad ; \quad [H_W^{1/2}, Q_5] = [H_W^{1/2}, Q] \quad .$$

We can decide between the two models by measuring the relative sign of the $I = \frac{1}{2}$ and $I = \frac{3}{2}$ amplitudes in K-decays. These amplitudes are related by PCAC to the corresponding $[H_W, Q_5]$ commutators, and their measured values clearly favour(54) the case (i), again ruling out the $\chi \sim 90^0$ version of the vectorlike theory.

The general result of both arguments can be stated in the following way: No quark is allowed to have transitions of order 1, to both s and d. If a given quark (c or any other quark) is paired with s in a left-handed doublet, it cannot be paired with d in a right-handed doublet, and vice versa. In the framework of the six-quark vector-like model, this yields $\chi \sim 0$. However, the result is much more general, and it remains an important constraint in any theory.

The complete, unique, SU(2) x U(1) assignments in a six-quark vector-like theory are then(49-52):

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad ; \quad \begin{pmatrix} u \\ b \end{pmatrix}_R \quad \begin{pmatrix} c \\ s \end{pmatrix}_R \quad \begin{pmatrix} t \\ d \end{pmatrix}_R$$

where all small Cabibbo-like angles have been neglected. The A-matrix is approximately diagonal. The B-mntrix has elements of order unity in the second diagonal.

6.5 Phenomenological Predictions: The Rise and Fall of Vectorlike Theories

The six-quark vectorlike theory leads to several phenomenological predictions. Among them we mention:

(i) Above the b-threshold, a substantial cross section for $\bar{\nu}+u \rightarrow \mu^{+}+b$ should be observed, leading to a larger $\sigma(\bar{\nu})/\sigma(\nu)$ ratio and to a flat y-distribution. This is confirmed by experiment and constitutes the only evidence for V+A currents.

(ii) The $\bar{c}s$ charged current approximately conserves parity. This can be studied in charmed particle decays[55], but was not yet tested.

(iii) Many interesting phenomena are expected in the leptonic sector of the model[52]. Neutrinos are predicted to have masses; weak decay of the form $U^{-} \rightarrow e^{-}+\gamma$ are expected[56]; neutrino oscillations may exist [57]; etc. None of these pehnomena have been observed, but no contradiction with experiment have been established.

(iv) Last but not least: the neutral current is predicted to conserve parity. This follows from the simple fact that the GIM mechanism guarantees a diagonal neutral weak current, regardless of the parameters of the A and B matrices. Hence, the V-A as well as the V+A pieces of the weak isovector neutral current must have the form: $\bar{u}u+\bar{c}c+\bar{t}t-\bar{d}d-\bar{s}s-\bar{b}b$. The full neutral weak current is therefore a pure vector current and parity is predicted to be conserved in all neutral current transitions. In particular, we expect:

$$\sigma(\bar{\nu}+N \rightarrow \bar{\nu}+N) = \sigma(\nu+N \rightarrow \nu+N)$$

$$\sigma(\bar{\nu}+N \rightarrow \bar{\nu}+\text{anything}) = \sigma(\nu+N \rightarrow \nu+\text{anything})$$

Several independent measurements have recently shown that both of these relations disagree with experiment[58], and that the neutral hadronic current is probably parity-violating. Assuming that these experiments are correct the full vector-like theory is ruled out.

6.6 Other Models with V+A Currents

If the full vector-like theory (and, in particular, the vector-like six-quark scheme) are ruled out, what are the remaining possibilities? One possibility is that no V+A currents exist and that the y-anomaly and the $\sigma(\bar{\nu})/\sigma(\nu)$ ratio are due to some kind of scaling violation. If, however, we insist on the V+A explanation of these effects, we have the following minimal set of SU(2) x U(1) doublets[59]:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad ; \quad \begin{pmatrix} u \\ b \end{pmatrix}_R$$

This requires five quarks and suffers from non-cancelling anomalies. We may add only one more right-handed doublet without increasing the number of quarks: $(c, s')_R$. Alternatively, we may introduce a sixth quark, h with charge $Q = -\frac{1}{3}$. The theory then includes:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad ; \quad \begin{pmatrix} u \\ b \end{pmatrix}_R \quad \begin{pmatrix} c \\ h \end{pmatrix}_R$$

This structure emerges in a scheme based on an SU(3) gauge algebra of the weak and electromagnetic interactions[60] and also in the E(7) unified scheme[61] (see Section 8). This type of six-quark scheme was first introduced by Barnett[62].

Many other schemes are possible but they should all avoid the right-handed doublets (u, d); (u, s); (c, d) and lead to a parity-violating neutral current. Some of the features of the vector-like model may be true, but the full scheme as exemplified by the six-quark scheme of sections 6.3 and 6.4 is probably incorrect.

7. The Quark-Lepton Connection

7.1 The First Generation of Fermions (1935)

As early as 1935, a certain degree of quark-lepton analogy could have been established. The known hadrons were p, n (made out of u, d quarks). The known (or predicted) leptons were ν_e, e . With a certain degree of hindsight we can reformulate the 1935 version of elementary particle physics by saying that we have a doublet of quarks and a doublet of leptons, with similar electromagnetic and weak interactions. Taking into account the V-A structure and the three colors of the (u, d) quarks we note that even the anomalies are cancelled ($\sum Q_i = 0$) .

We may therefore formulate a self-consistent gauge theory of weak and electromagnetic interactions based on an SU(2) x U(1) gauge algebra, and including two left-handed doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d \end{pmatrix}_L$$

and three right handed singlets:

$$(e^-)_R \qquad (u)_R \qquad (d)_R$$

This is, essentially, the "old half" of the "standard model", and <u>we will refer to the fermions u, d, ν_e, e as "first generation fermions".</u>

Using these fermions we can already formulate most of the questions which lead us to an investigation of the quark-lepton connection:

(i) The quarks and leptons respond to the weak current in a very similar way. Are they related?

(ii) Why are the electric charges of quarks and leptons quantized in a related way $(Q(d) = \frac{1}{3} Q(e)$, etc.)?

(iii) The required absence of anomalies tells us that we could not have a V-A model of quarks without leptons or leptons without quarks. How do the quarks "know" about leptons and vice versa?

(iv) Both quarks and leptons are "pointlike". Why?

These questions already lead us to suspect a deep connection between quarks and leptons. The suspicion grows when we proceed to the fermions of 1975.

7.2 The Second Generation of Fermions (1975)

The "second half" of the "standard model" is almost identical to the first half. We now have two left-handed quark doublets and two left-handed lepton doublets. All the questions of Section 7.1 remain valid, and the fact that the entire structure repeats itself, while maintaining the close analogy between quarks and leptons, provides further reasons to suspect that we cannot have quarks without leptons or leptons without quarks.

The new features of the 1975 fermions are the presence of s-d mixing and a Cabibbo angle as well as the unexplained large masses and large mass splittings among the "second generation" fermions.

Here we face an extremely puzzling question:

Given that the (ν_e, e^-) and (ν_μ, μ^-) doublets respond in an identical way to all known interactions, what creates the different mass scale of the two lepton doublets? Similarly, given that (u, d) and (c, s) respond in an identical way to all known interactions, *what creates the different mass scale of the two quark doublets?* Both questions are totally unanswered, but it is interesting that, again, the problem appears to be the same for leptons and quarks.

It is, therefore, clear that a striking quark-lepton similarity exists both in the first generation and in the second generation of fundamental fermions. The possible third generation, including (ν_U, U^-) and (t, b) appears to possess the same similarity. What is the source of the quark-lepton connection?

7.3 Grand Unification

If quarks and leptons have the same space-time structure (pointlike $J = \frac{1}{2}$ *fermions*), *perhaps they belong to one large multiplet of fundamental fermions*[61,63,64]. If the weak and electromagnetic interactions are described, by a (Weinberg-Salam) vector gauge theory and the strong interactions are described by a (QCD) vector gauge theory, perhaps there is one large gauge group incorporating both theories, and one large representation including all gauge bosons.

This is the most simple-minded approach to both the problem of quark-lepton connection and the desire to unify all fundamental interactions.

How can it be done technically?

The weak and electromagnetic gauge group is, at least, $SU(2)_W \times U(1)$. The color gauge group is $SU(3)_c$. The weak bosons are colorless. The colored gluons have no weak or electromagnetic couplings. Hence, our starting point is the direct product:

$$SU(3)_c \times SU(2)_W \times U(1)$$

We must search for a group G such that $G \supset SU(3)_c \times SU(2)_W \times U(1)$. The group G must be, at least, of rank four. If we want a true and complete unification of all interactions, leaving no arbitrary parameters, we would prefer to exclude the possibility that G is a direct product of two different groups. (Such a possibility would lead to an arbitrary Weinberg-like angle in the same way that SU(2) x U(1) leaves θ_W undetermined and is not a truly unified scheme.) The smallest possible group obeying these requirements is[64] SU(5), but many other possibilities exist.

The rank of the group G determines the number of conserved additive quantum numbers. The minimal rank (four) includes the two additive quantum numbers of $SU(3)_c$, the electric charge and a "weak" charge which can be chosen as the charge coupled to the Z^o-boson. Of these, the first three are exactly conserved and are coupled to massless vector particles (gluons and photon). The fourth represent a spontaneously broken symmetry and the Z^o is, of course, massive. If G has a rank larger than four, additional quantum numbers are introduced. Since no other massless bosons seem to exist, all of these quantum numbers must correspond to broken symmetries and massive bosons. In particular, we cannot include an exactly conserved baryon number or lepton number operator as a generator of G. On the other hand, the fermion representation of G must include particles of different baryon number and lepton number (quarks and leptons). Hence, these quantum numbers cannot commute with all generators of G.

We therefore conclude that in such a unified scheme, we cannot have exact conservation of both baryon number B and lepton number L[63-66]. We may have exact fermion number conservation, if all particles in a given multiplet are fermions (and the antifermions are in a different multiplet). In that case the fermion number operator F may lie outside G and be exactly conserved. We then have $F = B + L$, where F is conserved and B and L are not.

Alternatively we can invent schemes in which B is conserved, but not F and L. At most one of these three quantum numbers can be exactly conserved in such unification schemes[65].

Many different models have been proposed for the "grand unification scheme". They can be divided into two main classes:

(a) "Minimal" schemes. These are models which do not extend the *weak gauge group beyond* $SU(2)_W \times U(1)$. As a result, the total number of colorless weak bosons remain three: W^+, W^-, Z^0. The full SU(N) group acting on the N quark-flavors is not a subgroup of G. No gauge bosons connect quarks which are in different $SU(2)_W \times U(1)$ doublets and the weak neutral current conserves all flavors. In such models, quarks and leptons are assigned to the same multiplet of G, but not all quarks and all leptons are in one irreducible representation. *A typical example of such a scheme* is the SU(5) model of Georgi and Glashow[64], which we discuss in some detail in Section 8.1. In this model all "first generation fermions" are related to each other; all "second generation fermions" are related to each other; no relation is established between fermions in the two generations. Any number of generations of fermions may exist.

(b) "Maximal" schemes. These are models in which all quarks and all leptons are assigned to one irreducible representaiton of G. The full $SU(N)_f$ flavor group is contained in G. In fact,

$$G \supset SU(N)_f \times SU(3)_c$$

There are many colorless weak bosons, including bosons which connect any given quark to any other quark. Flavor-changing weak neutral currents are allowed and the GIM mechanism is not an integral part of the theory. The suppression of strangeness changing neutral currents is artificial. On the other hand, the scheme achieves the maximal degree of unification by relating all the fundamental fermions to each other. Schemes such as E(7) of Gursey et al.[61] (see Section 8.2) and $SU(4)_f \times SU(4)_c$ of Pati and Salam[63] are typical examples of a maximal scheme.

7.4 Common Features of Unification Schemes: Hopes and Difficulties

All unification schemes which are based on a large gauge group G, and which are constructed along the lines mentioned in the previous section, have many common features:

(i) New gauge bosons which convert quarks into leptons or leptons into quarks are predicted. The adjoint representation of G must include eight colored gluons, at least three (possibly many more) colorless weak bosons, as well as the photon. These are the gauge bosons which are coupled to the generators of $SU(3)_c \times SU(2)_W \times U(1)$ or to $SU(3)_c \times SU(N)_f$. In addition to these, the same representation of G must always include some bosons which carry the quantum numbers of both $SU(3)_c$ and $SU(2)_W$. The simplest set of such bosons would belong to a (3, 2) representation of $SU(3)_c \times SU(2)_W$. These peculiar bosons must possess color, respond to both strong and weak interactions, carry baryon number and lepton number, have third integer charge and be capable of converting a colored quark into a colorless lepton. They are sometime referred to as "leptoquarks". Each unification scheme must have such bosons. Some schemes have large numbers of them, but the minimal number is twelve: A (3, 2) and a $(\bar{3}, 2)$ multiplets. The mass of each "leptoquark" is probably very large. At the same time, they may be confined (if all colored objects are confined).

(ii) All unification schemes lead to a violation of baryon number and/or lepton number conservation. We have explained the reason for this in the previous section. The phenomenological implications of such a violation depend on the model. Whenever we have baryon number nonconservation, the proton becomes unstable. Its allowed decay modes depend on the detailed selection rules of the group G. In some models a second order weak process such as:

$$p \rightarrow e^+ + \pi^0$$

is allowed[64]. In other schemes, only sixth order transitions such as:

$$p \rightarrow \pi^+ + 3\nu$$

are possible[63]. The present upper limit on the proton decay rate dictates, in each case, a lower limit on the masses of the gauge bosons which are responsible for the proton's decay. In models (such as SU(5)) in which a second order decay is allowed, we are led to bosons with masses such as 10^{-9} gram(64). Such a mass scale essentially tells us that the full symmetry limit of the group G is a matter for science fiction rather than science. On the other hand, if the proton decays only via a sixth order transition, bosons of the mass range around 1000 GeV or less, are sufficient. This is only one order of magnitude above the expected masses of $W^{\pm}$ and Z^0.

(iii) The relative strengths of all interactions in the symmetrical, high energy, limit are essentially given by Clebsch-Gordan coefficients. This can happen, of course, only if the order of magnitude of the weak, electromagnetic and strong couplings becomes the same at such energies. The weak interactions are presumably comparable to electromagnetic interactions at energies above the W-masses. The gauge theory of colored quarks and gluons is asymptotically free. At sufficiently high energy its (running) coupling constant may decrease to the level of the weak and electromagnetic couplings. It is difficult to envisage how these explicit high energy relations between the strengths of the different interactions, can be experimentally tested in the foreseeable future.

(iv) Any theory which incorporates the four generators of SU(2) x U(1) into an irreducible representation of a larger group, leads to a determination of the Weinberg angle θ_W. The Weinberg angle is defined by the relation:

$$J_{em} = J_3 \sin\theta_W + J_0 \cos\theta_W$$

where J_{em} is the electromagnetic current and J_3, J_0 are the currents transforming like the third component of $SU(2)_W$ and the U(1) generator, respectively. In a unification scheme based on a group G, the operators J_3, J_o are generators of G. Using the Wigner-Eckart theorem, the matrix elements of J_{em}, J_3 and J_o for a given fermion f_i can be written as:

$$(f_i|J_{em}|f_i) = \frac{Q_i}{\sqrt{\sum Q_i^2}} \cdot (f||J||f)$$

$$(f_i|J_3|f_i) = \frac{I_{3i}}{\sqrt{\sum I_{3i}^2}} \quad (f||J||f)$$

$$(f_i|J_o|f_i) = \frac{Y_i}{\sqrt{\sum Y_i^2}} \quad (f||J||f)$$

where Q_i, I_{3i}, Y_i are the electric charge, third component of weak isospin and weak hypercharge of f_i and $(f||J||f)$ is the reduced matrix element. We may select, for example, a fermion f_i with $Y_i = 0$. We then have:

$$\sin\theta_W = \frac{(f_i|J_{em}|f_i)}{(f_i|J_3|f_i)} = \frac{Q_i}{I_{3i}} \cdot \sqrt{\frac{\sum I_{3i}^2}{\sum Q_i^2}}$$

Since $Y_i = 0$, it follows that $Q_i = I_{3i}$. Hence[66]:

$$\sin^2\theta_W = \frac{\sum I_{3i}^2}{\sum Q_i^2}$$

We see that the Weinberg angle is fully determined if we have a complete list of all the Q and I_3 values of all fermions in one irreducible representation of G. Furthermore, we do not need to know much about the group G itself or about its Clebsch-Gordan coefficients.

If we consider a pure V-A theory based on the first generation fermions (see Section 7.1) we have:

$$\sum I_{3i}^2 = 2$$

$$\sum Q_i^2 = 2 \left\{3\left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] + (-1)^2\right\} = \frac{16}{3}$$

Hence:

$$\sin^2\theta_W = \frac{3}{8} .$$

The standard model or a V-A model with six-quarks and six leptons clearly leads to the same value of θ_W . Consequently, any unification scheme which accommodates such a set of fermions will yield the same value of θ_W .

In a pure vectorlike theory (see Section 6), $\sum Q_i^2$ remains unchanged but $\sum I_{3i}^2$ is doubled. (All right-handed fermions have the same I_3 values as their left-handed counterparts.) We then find:

$$\sin^2\theta_W = \frac{3}{4}$$

This value is common to all vector-like models.

This calculated value of the Weinberg angle may, in principle, be drastically changed by renormalization corrections. The group theoretical calculation presumably applies only to the symmetry limit. But the symmetry is broken in many different steps, corresponding to gauge bosons of widely different masses. There is no guarantee that the above values of θ_W are related to experimentally measured quantities.

It is, however, amusing, that the value $\sin^2\theta_W = \frac{3}{8}$, corresponding to pure V-A theories, is consistent with all present data from neutral current experiments[67]. Whether this is an accident we do not know.

The overall list of "benefits" obtained from the various unification schemes is not very impressive. It includes the presence of unwanted heavy bosons (leptoquarks), the unwanted nonconservation of baryon and lepton number, and an untestable relation between strong and weak coupling constants. The only prediction which may be testable (but only if we ignore renormalization effects or if we learn how to compute them) is the value of θ_W. However, we have showed that this value does not depend in a crucial way on the detailed properties of G, and it is common to many different models based on different groups G.

8. Examples of Grand Unification Schemes

8.1 A "Minimal" Example: SU(5)

The smallest group which contains $SU(3)_c \times SU(2)_W \times U(1)$ and which is not a direct product, is SU(5). Georgi and Glashow[64] have proposed a unification scheme of quarks and leptons and of weak, electromagnetic and strong interactions, based on this algebra. Since it is the simplest unification scheme we will describe it here in some detail.

The fundamental spinor representation of SU(5) is 5-dimensional. Its $SU(3) \times SU(2)$ decomposition is given by:

$$\underline{5} = (3,\ 1) + (1,\ 2)$$

The product of two such quintets gives:

$$\underline{5} \times \underline{5} = \underline{15} + \underline{10}$$

$$\underline{15} = (6,\ 1) + (1,\ 3) + (3,\ 2)$$

$$\underline{10} = (\bar{3},\ 1) + (1,\ 1) + (3,\ 2)$$

The product of a quintet and its conjugate gives:

$$\underline{5} + \underline{\bar{5}} = \underline{24} + \underline{1}$$

$$\underline{24} = (8,\ 1) + (1,\ 3) + (3,\ 2) + (\bar{3},\ 2) + (1,\ 1)$$

The gauge vector bosons in an SU(5) scheme are in the $\underline{24}$ adjoint representation. They include γ, W^+, W^-, Z^0, eight gluons and the minimal set of 12 leptoquark bosons. The $SU(3)_c \times SU(2)_W$ properties of the first generation left-handed fermions and antifermions (see Section 7.1) are:

$(\nu_e, e^-)_L$	in	(1, 2)
e^+_L	in	(1, 1)
$(u, d)_L$	in	(3, 2)
$\bar{u}_L$	in	$(\bar{3}, 1)$
$\bar{d}_L$	in	$(\bar{3}, 1)$

These 15 states fall neatly into a reducible 15-dimensional representation which decomposes into:

$$\underline{10} = (\bar{3}, 1) + (1, 1) + (3, 2) = (\bar{u})_L + (e^+)_L + (u, d)_L$$

$$\underline{\bar{5}} = (\bar{3}, 1) + (1, 2) = (\bar{d})_L + (\nu_e, e^-)_L$$

Baryon and lepton number are, obviously, not conserved. The Weinberg angle obeys (see Section 7.4):

$$\sin^2\theta_W = \frac{3}{8}$$

The decay: $p \to e^+ + \pi$ is allowed. Some of the leptoquarks must be of masses around 10^{-9} gram, in order to sufficiently suppress the proton decay rate.

The second generation fermions and antifermions are, again, assigned to a $\underline{\bar{5}} + \underline{10}$ pair of SU(5) multiplets. No connection between first generation and second generation fermions is established. The GIM mechanism is a natrual part of the theory. The Cabibbo angle mixes different SU(5) multiplets. There is no obvious limit on the number of quarks and leptons. A third or fourth generation of fermions and antifermions can easily be accommodated, as long as they follow the pattern of the first two generations.

An attractive extension of the SU(5) scheme is a model based on the group SO(10). This group has an irreducible 16 dimensional representation which decomposes into $\underline{10} + \bar{\underline{5}} + 1$ when we consider the SU(5) subgroup[66]. This would enable us to accommodate all left-handed fermions and antifermions of a given generation in one irreducible representation.

8.2 A "Maximal" Example: E(7)

The exceptional lie algebra E(7) has also been proposed as a unifying algebra[61]. The E(7) algebra contains $SU(6) \times SU(3)$ as a maximal subgroup. Consequently, it may be viewed as a unification of the $SU(3)_c$ color gauge group and an $SU(6)_f$ gauge group acting on flavors and containing $SU(2)_W \times U(1)$ as a subgroup.

The smallest ("spinor") representation of E(7) is 56-dimensional. Its $SU(6) \times SU(3)_c$ decomposition is given by:

$$\underline{56} = (\underline{6}, \underline{3}) + (\bar{\underline{6}}, \bar{\underline{3}}) + (\underline{20}, \underline{1})$$

The 56 states contain six tricolored quarks in the $(\underline{6}, \underline{3})$ multiplet, six antiquarks in $(\bar{\underline{6}}, \bar{\underline{3}})$ and twenty colorless leptons and antilepton in $(\underline{20}, \underline{1})$. Note that the $\underline{20}$-representation of SU(6) is self-conjugate. It may, therefore, contain a set of leptons together with all their antileptons.

The electric charge operator is a generator of the SU(6) flavor gauge group. Consequently, the sum of all quark charges must vanish. Assuming that four of the six quarks are the usual u, d, s, c, the two remaining quarks must have $Q = -\frac{1}{3}$. This six-quark set is not the one discussed in Sections 4 or 6. It contains the b-quark but the t-quark ($Q = +\frac{2}{3}$) is replaced by another $Q = -\frac{1}{3}$ quark, denoted by h .

The E(7) model necessarily includes equal sets of V-A and V+A currents, since all E(7) representations are self-conjugate. One possible assignment of the six quarks under $SU(2)_W \times U(1)$ is the following:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad (b)_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad (h)_L \;;\; \begin{pmatrix} u \\ b \end{pmatrix}_R \quad (d)_R \quad \begin{pmatrix} c \\ h \end{pmatrix}_R \quad (s)_R \;.$$

The GIM mechanism is "accidental" and there is no guarantee that all neutral currents conserve all flavors. The Weinberg angle is given by $\sin^2\theta_W = \frac{3}{4}$ (see Section 7.4).

The 20 leptons and antileptons include four positively charged, four negatively charged and twelve neutral states. The four $Q = -1$ leptons are, presumably, e^-, μ^-, U^- and one additional charged heavy lepton x^- . The four $Q = +1$ states are e^+, μ^+, U^+, x^+ . The twelve neutral states include the two known neutrinos and their antiparticles, plus eight additional neutral leptons and antileptons. The $SU(2)_W \times U(1)$ assingment of the leptons includes two $SU(2)_W$-triplets, four $SU(2)_W$-doublets and six $SU(2)_W$-singlets.

As in all unified schemes, baryon and lepton numbers are not conserved in E(7).

The dimensionality of the adjoint representation of E(7) is 133. Its $SU(6)_f \times SU(3)_c$ decomposition is given by:

$$\underline{133} = (\underline{35}, \underline{1}) + (\underline{1}, \underline{8}) + (\underline{15}, \underline{3}) + (\overline{\underline{15}}, \overline{\underline{3}})$$

Thus we have 133 gauge bosons. They include eight colored gluons in (1, 8), 35 colorless weak bosons in (35, 1) and 90 leptoquark bosons. The 35 colorless weak bosons include γ, W^+, W^-, Z^0 and 31 additional bosons which connect every quark-flavor to every other flavor. Thus we have a neutral boson which couples to $\bar{d}s$. Such a boson must, of course, be extremely heavy.

The E(7)-scheme is maximal in the sense that it incorporates all quarks and all leptons in one multiplet. Its main predictive power is based on the explicit set of fermions which cannot be extended. The model predicts, for instance, that the t-quark does not exist; it does not allow more than four charged leptons. Such predictions are testable.

Another interesting feature of E(7) is the peculiar relation between the exceptional groups and the octonionic matrices. This relation was studied by Gursey[68] who suggested that the color group $SU(3)_c$ is actually selected by nature as a result of a generalization of quantum mechanics from the domain of complex numbers to that of octonions. The automorphism group of octonions in the exceptional algebra G_2, and the subgroup of G_2 which leaves one octonionic unit invariant is SU(3). According to Gursey, this may be the reason behind the selection of SU(3) as the color group, and this might lead us to consider the five exceptional groups G_2, F_4, E_6, E_7, E_8 as candidates for a unifying symmetry.

8.3 Unanswered Questions

We have discussed some general aspects of the grand unification schemes, and outlined some of the more technical features of two specific models. One cannot escape the feeling that, at present, all unifying schemes gain us very little, at a very expensive price. The testable predictions of each model are very few. They essentially include a predicted list of fundamental fermions (with varying degree of flexibility, depending on the model) and a determination of the Weinberg angle (which is probably subject to major renormalization corrections).

On the other hand, there is a long list of striking questions which are not yet answered, and which must be answered by any satisfactory theory. We would like to list some of these questions:

(i) Why is the neutrino massless? This must come from some symmetry principle, but we do not know of any such principle. It is, perhaps, relevant to note that the neutrinos are the only neutral fermions. Is it true that all fundamental neutral fermions are massless? If so, why? If the neutrino is not exactly massless, or if ν_e is massless and ν_μ is not, we still need a reason for the small masses.

(ii) Are the ν_e-e and u-d mass differences of pure electromagnetic origin? If so, why are the ν_μ-μ and c-s mass differences so much larger? In fact, the mass pattern of the first generation fermions can be qualitatively "understood" if we claim that u, d acquire some mass because of their strong interactions; the u-d difference is small because they only differ by electric charge; the ν_e-e difference is small for the same reason. This naive "explanation" sounds good until we note that all interactions of the second generation fermions are identical to those of the first generation, while the mass pattern is completely different.

(iii) What is the origin of isospin symmetry? Why are two of the quarks approximately degenerate, while all others have different masses?

(iv) What determines the Cabibbo angle and all additional Cabibbo-like angles and phases which are needed in all extensions of the standard model?

(v) How many mass scales exist among the fundamental fermions? In the standard model one might argue that there is one mass scale which somehow determines the masses of the second generation fermions, while the first generation fermions are approximately massless. This approach would be in trouble if a third generation of fermions is found and if its mass scale is substantially higher. Is there an additional new interaction which is responsible only for the different mass ranges of different generations of fermions?

These questions (and others) must be answered by a convincing unifying theory. It follows that all present theories are far from convincing. However, the search for such a theory should be regarded as one of the most exciting subjects in physics today.

9. What Next?

The incredible sequence of experimental discoveries in the last two years transformed several respectable theories (such as charm) into facts; it transformed some wild speculations (such as color) into respectable theories; it transformed unthinkable topics (such as grand unification) into a subject o wild speculation.

What next?

We first have to establish some additional facts about charm: Complete the spectrum of low-lying charmed mesons and baryons; understand the open experimental and phenomenological features of the ,-family; establish the space-time nature of charm-changing weak transitions.

The possible existence of a heavy lepton is the most immediate and direct element which goes beyond the standard model. This issue must be settled soon. If the U-lepton is there, we must learn whether it is a sequential lepton, whether it has a massless neutrino, and whether it decays via a V-A interaction.

The possible V+A currents in $\bar{\nu}N$ scattering is the next item on the "beyond charm" list. Here both new currents and a new quark (b) are involved. The issue can be resolved by more $\bar{\nu}N$ experiments, as well as by new deep inelastic eN or μN experiments which can help determine whether the y-anomaly is a "harmless" scaling violation or a new chapter in weak interaction physics.

The theoretical problems of QCD, including the cardinal question of confinement, remain among the most important topics in particle physics. However, these problems are only marginally related to the new physics which we discussed in these lectures. The only items which have some bearing on QCD are the low energy value of R which provides us with a new argument for color, and the phenomenology of the Zweig-Iizuka rule which touches on the question of asymptotic freedom. The confinement issue itself remains unrelated to the number of quark flavors or to their weak and electromagnetic properties.

Finally, the convincing evidence for the existence of at least sixteen fundamental fermions, (e, ν_e, μ, ν_μ, tricolored u,d,s,c) and the possible existence of more (U?ν_U?, b?, t??, h??) tell us that it is extremely unlikely that these are indeed the ultimate fundamental building blocks of matter. There are simply too many of them! We believe that the chapter of physics which goes beyond quarks and leptons is *not* far ahead.

The simplest attitude would be to construct quarks and leptons from yet another set of more fundamental objects. This idea was suggested by several authors[69], but it appears to be too naive to be true.

The next simplest approach is to assign quarks and leptons to one multiplet of a unifying gauge theory. We have discussed here some of the *pioneering* attempts *in* this direction, but, again, we feel that nature is more subtle and more rich than implied by these models.

How the quarks and leptons are related is one question which must be answered in the next few years. We believe that the answer will come and will be more profound than all present attempts.

In many respects, the situation reminds us of the early 19th century. Electric charges and magnetic charges were known. They possessed similar (Coulomb) interaction. They were "pointlike". Electric charges were free. Magnetic charges were confined and always came in pairs. In most daily experiences, magnetic charges were "stronger". It was clear that the two may be related.

Electric and magnetic charges <u>are</u> related. They are not made out of more fundamental charges. They are not simply related by residing together in some multiplet. The relation is more profound, and yet it is elegant and simple. In fact, the relation tells us that the magnetic charges do not really exist, but that certain manifestations of electric charge phenomena appear to behave like pairs of magnetic charges.

Will the understanding of the quark-lepton relation solve the mystery of the nonexistence of quarks, in the same way that the understanding of the connection between electricity and magnetism explained the mystery of the missing magnetic poles?

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