

THE RISHON MODEL*

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We present a detailed analysis of the rishon model of composite quarks, leptons, scalar particles and weak bosons. The fundamental lagrangian is assumed to be gauge invariant under $SU(3)_H \times SU(3)_C \times U(1)_{EM}$. The fundamental particles are the two types of rishons (T and V), hypergluons, gluons and the photon. No fundamental scalars exist. Below the hypercolor scale Λ_H , only $SU(3)_H$ singlets exist. The simplest composite fermions are made of three rishons or three antirishons and reproduce the observed properties of one generation of quarks and leptons. A new approximate $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry emerges at the composite level. The weak interactions appear only at the composite level as residual short-range interactions among hypercolor singlets. If composite W and Z bosons are formed, the effective lagrangian at low energies is likely to be gauge invariant and renormalizable, except for terms proportional to inverse powers of Λ_H . Two elusive Goldstone bosons are predicted by the model. The 't Hooft consistency requirement is simply obeyed in a manner similar to the situation in QCD. The generation problem and the proton's decay as well as some theoretical difficulties and experimental signatures are briefly discussed. Two main difficulties are noted: an unusual pattern of chiral symmetry breaking and the existence of light composite vector bosons.

1. Introduction

There are several reasons to consider the possibility that quarks and leptons are composite objects [1]. The probable existence of six types of leptons and six types of quarks, the repetitive pattern of generations, the analogy and similarities between the properties of quarks and leptons, the connection between the electric charges of quarks and leptons and the existence of more than 20 free parameters in the "standard model", all point towards a common underlying structure.

We also have convincing arguments [2] for the compositeness of the scalar particles in the spontaneously broken electroweak theory. These arguments are not directly related to a possible substructure of quarks and leptons. However, in the

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absence of such a substructure, the composite scalars inevitably lead to many additional quark-like objects, intensifying the feeling that one faces too many “fundamental” particles and too many independent parameters.

It is attractive to postulate that all quarks and leptons as well as all scalar particles are composites of a small set of new fundamental fermions. All relations between quarks and leptons should be traceable to their construction from the same set of building blocks. The generation pattern should be understood in terms of excitations of the composite systems. The “standard model” would then become a low-energy approximation of the “true” theory, based on an “effective” lagrangian involving composite objects.

The experimental limits on the point-like character of quarks and leptons indicate a compositeness scale Λ of the order of, at least, a few hundred GeV [1, 3]. Hence, for the first time, we encounter composite objects whose Compton wavelength exceeds their actual radius by many orders of magnitude. Alternatively, we may say that the composite quarks and leptons are approximately massless in comparison with the scale parameter Λ .

The only known “natural” scenario [4] which may yield approximately massless composite objects starts with a local gauge theory with massless fundamental fermions. Such a theory would normally possess a certain degree of chiral symmetry. If the chiral symmetry is not broken (or if it is broken and a chiral subsymmetry remains conserved), it may “protect” some of the composite fermions from acquiring a mass [4]. The binding force among the fundamental massless fermions is, presumably, a new color-like force called “hypercolor” with a scale Λ_H . The new fundamental massless fermions are hypercolor confined and cannot be directly observed experimentally. The hypercolor scale is substantially larger than the ordinary color scale: $\Lambda_H \gg \Lambda_C$. We identify Λ_H with the compositeness scale mentioned in the previous paragraph. All composite quarks, leptons and scalar particles are hypercolor singlets and are therefore “observable” below Λ_H .

Using the above reasoning, one may consider a variety of composite models of quarks and leptons. Sometime ago, we proposed such a model [5] (the “rishon model”), based on an earlier “counting scheme” [6, 7]. The present paper is devoted to a detailed analysis of the rishon model.

The standard model contains several types of gauge bosons: photons, gluons, W^\pm and Z . To these we must now add the hypergluons of the hypercolor force. Do the gauge bosons appear as fundamental fields in the original “underlying” lagrangian, or are they composite?

The photon, gluon and hypergluon are massless. They represent exact local gauge symmetries which are not broken in any stage of the theory. At least the hypergluon must appear in the underlying lagrangian, since it is responsible for the binding of the fundamental fermions. In spite of several attempts [8], no one has, so far, succeeded in constructing a theory with massless composite vector gauge particles. We therefore assume that the photon, gluon and hypergluon are fundamental and

that $SU(N)_H \times SU(3)_C \times U(1)_{EM}$ is a local gauge symmetry of the underlying theory.

The situation concerning the weak bosons W and Z is quite different. They are not massless and the corresponding symmetry is spontaneously broken. In the usual Higgs mechanism, the longitudinal components of W and Z are “born” from the scalar particles which are now assumed to be composite. Furthermore, W^\pm is the only charged gauge boson and its charge equals three times the fundamental electric charge of any set of new fundamental fermions. It is, therefore, not unlikely that the massive W and Z are actually composite objects [6]. If so, the full electroweak group will not appear as a local gauge symmetry of the underlying theory. It will only be an approximate gauge symmetry of the low-energy effective lagrangian which contains the composite fields of W , Z , quarks, leptons and scalars. Such a lagrangian will differ from that of the standard model by terms which are proportional to inverse powers of the compositeness scale Λ_H .

In the rishon model, we postulate that the ordinary W^\pm and Z , as well as an additional set of W^\pm and Z belonging to an $SU(2)_L \times SU(2)_R \times U(1)$ electroweak group, are composite particles. This assumption leads to interesting consequences and to difficult open problems, which we discuss in the following sections.

The plan of this paper is as follows: In sect. 2 we describe a search for the most economic composite model which is consistent with the framework outlined above. We show that the rishon model is the minimal possible scheme. Sect. 3 is devoted to a detailed analysis of the underlying lagrangian, and its local, global and discrete symmetries. The spectrum of first-generation quarks and leptons is constructed in sect. 4. The weak interactions are introduced in sect. 5, as residual short-range forces operating between hypercolor singlet composite fermions. The effective lagrangian containing the composite fermions is shown to possess a larger global symmetry. In sect. 6 we discuss the effective lagrangian, the composite W and Z bosons and the question of “approximate renormalizability”. Sect. 7 is devoted to a study of the patterns of symmetry breaking, beginning with the underlying theory and proceeding towards the spectrum of physical composite particles. We encounter Goldstone bosons and discuss the ‘t Hooft consistency conditions. Sect. 8 includes a brief discussion of some open questions including the generation problem, proton decay and the $g_c \rightarrow 0$ limit of the theory. In the last section we present a summary and outlook.

2. Searching for a minimal scheme

Starting from the theoretical framework of the preceding section, we now try to find the minimal realistic scheme which obeys our various requirements. The line of reasoning presented in this section cannot be considered as a “derivation” of the model. It represents, however, a systematic search for the most economic candidate

for a composite model. We proceed with the following steps:

(i) We assume that the full local gauge group is $SU(N)_H \times SU(3)_C \times U(1)_{EM}$. If the hypergluons, gluons and photon are fundamental particles, nothing less will do. However, we do not allow any additional gauge symmetries.

(ii) We assume that all fundamental massless fermions are in the N -dimensional multiplet of $SU(N)_H$. They cannot be hypercolor singlets, since that would make them observable. The fundamental N representation is clearly the simplest and the most economic. Fermions and antifermions are, of course, in N and \bar{N} representations, respectively.

(iii) In order to form a composite $SU(N)_H$ singlet fermion, we now need at least N fundamental fermions, where N is odd. The smallest and most economic N is three. Hence, we take the full gauge group to be [5] $SU(3)_H \times SU(3)_C \times U(1)_{EM}$.

(iv) In order to construct neutral and charged composite fermions, we need at least two fundamental fermions with different electric charges. The fundamental unit of charge is clearly $\frac{1}{3}e$. Hence, the simplest model will involve one fermion with charge $\frac{1}{3}$ (denoted by T) and one neutral fermion (denoted by V), both in hypercolor triplets. We refer to both fermions as "rishons" [6]. The antirishons are hypercolor antitriplets with charges $-\frac{1}{3}, 0$.

(v) The only assignments which remain to be determined are the ordinary $SU(3)_C$ colors of T and V. The simplest color representations for each one of them could be 1, 3 or $\bar{3}$. We must form composite quarks and composite leptons belonging, respectively, to (1, 3) and (1, 1) representations of $SU(3)_H \times SU(3)_C$. Hence, we need hypercolor singlets with different color trialities. Consequently, the two types of rishons must have different colors. Since the definition of 3 and $\bar{3}$ is arbitrary, we are left with only two possibilities: (a) The two types of rishons are in the 1 and 3 color representations, respectively. (b) The two types of rishons are in 3 and $\bar{3}$ color representations. The first possibility leads to a difficulty with Fermi statistics in the construction of leptons (see sect. 4). The only remaining possibility is, therefore [5]: T in $(3, 3)_{1/3}$ and V in $(3, \bar{3})_0$.

The starting point of our model is, therefore, the following: We postulate a local gauge symmetry based on $SU(3)_H \times SU(3)_C \times U(1)_{EM}$, with three independent parameters $\Lambda_H, \Lambda_C, \alpha$. There are two types of massless left-handed and right-handed rishons. We also have the corresponding antirishons in the conjugate representations (see table 1).

The complete list of fundamental particles includes rishons and antirishons, hypergluons, gluons and the photon. All particles are massless. No mass parameters exist. No fundamental scalar particles exist.

The basic model suggested here is clearly extremely economic. The important question is whether, starting from our minimal set of particles, we can construct a realistic self-consistent theory. We try to do that in the following sections. The theory we obtain has many attractive features but it is still far from satisfactory, as many important questions remain open.

TABLE I
The assignments of the rishons and antirishons

Rishons	Antirishons
$T(3, 3)_{1/3}$	$\bar{V}(\bar{3}, 3)_0$
$V(3, \bar{3})_0$	$\bar{T}(\bar{3}, \bar{3})_{-1/3}$

3. The underlying theory

We have now defined the ingredients of our model. The full fundamental lagrangian is given by:

$$\begin{aligned}
 L = & \bar{T}_{jj'} \left(i \delta_{kk'}^j \delta_{kk'}^{j'} \not{\partial} + g_H \delta_{kk'}^j (\lambda^a)_{k'}^{j'} A_H^a + g_C \delta_{kk'}^j (\lambda^a)_{k'}^{j'} A_C^a + \frac{1}{3} e \delta_{kk'}^j \delta_{kk'}^{j'} A \right) T^{kk'} \\
 & + \bar{V}_{jj'} \left(i \delta_{kk'}^j \delta_{kk'}^{j'} \not{\partial} + g_H \delta_{jj'}^k (\lambda^a)_{k'}^{j'} A_H^a + g_C \delta_{jj'}^k (\lambda^a)_{k'}^{j'} A_C^a \right) V_k^{k'} \\
 & - \frac{1}{4} (F_{EM})_{\mu\nu} (F_{EM})^{\mu\nu} - \frac{1}{4} (F_C^a)_{\mu\nu} (F_C^a)^{\mu\nu} - \frac{1}{4} (F_H^a)_{\mu\nu} (F_H^a)^{\mu\nu}.
 \end{aligned}$$

Here, T and V are Dirac four-spinors, each representing a right-handed and a left-handed massless fermion; $a = 1, \dots, 8$ is an index specifying an $SU(3)$ generator; λ^a is the corresponding 3×3 matrix; $j, k = 1, 2, 3$ are color indices; $j', k' = 1, 2, 3$ are hypercolor indices; $A_{H\mu}^a, A_{C\mu}^a, A_\mu$ are the hypergluon, gluon and photon fields, respectively; $(F_H^a)_{\mu\nu} = \partial_\mu A_{H\nu}^a - \partial_\nu A_{H\mu}^a + g_H f_{abd} A_{H\mu}^b A_{H\nu}^d$; $(F_{EM})_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; upper and lower color indices correspond to the 3 and $\bar{3}$ representations, respectively.

The basic lagrangian may include, in addition, terms of the form $\theta F\tilde{F}$, potentially yielding strong and hyperstrong CP violation. These terms do not pose any special problem in our model [9] and we do not discuss them any further in this paper.

Note that our lagrangian contains no mass parameters. The only free parameters are g_H, g_C and e (or $\Lambda_H, \Lambda_C, \alpha$). Note also that for Λ_H values between 10^3 and 10^8 GeV, g_C/g_H at Λ_H is around 0.15–0.25 while $e \sim 0.3 (e^2/4\pi = \alpha)$.

The model involves three different local gauge symmetries: hypercolor, color, electromagnetism. These symmetries will remain exact at all levels of the theory. They are also symmetries of the vacuum. No other exact symmetry will exist. *Hence, in our model, all exact symmetries are local gauge symmetries.*

The underlying lagrangian possesses, by construction, additional global and discrete symmetries. These symmetries are not necessarily shared by the vacuum, and we will see in the following sections that all of them must be spontaneously broken. We now turn to study these symmetries.

The discrete symmetries of the lagrangian are very simple. It is clearly invariant under parity and charge conjugation. All fermions appear as both left-handed and right-handed and we have only vector interactions.

The global symmetries are somewhat more complicated. First we note that although we have two types (or "flavors") of rishons, we do not have any global SU(2) symmetry. T and V have identical hypercolor but opposite color (see table 1). T and \bar{V} have identical color but opposite hypercolor. The two rishons and the two antirishons represent the four possible combinations of hypercolor and color triplets and antitriplets. Each combination appears once and only once.

Superficially, the lagrangian appears to conserve separately the number of left-handed and right-handed T and V rishons. Hence, we must consider four possible U(1) symmetries for which we may define the following four currents:

$$\begin{aligned} (J_{\text{vector}}^T)_\mu &= \bar{T}\gamma_\mu T, & (J_{\text{axial}}^T)_\mu &= \bar{T}\gamma_\mu\gamma_5 T, \\ (J_{\text{vector}}^V)_\mu &= \bar{V}\gamma_\mu V, & (J_{\text{axial}}^V)_\mu &= \bar{V}\gamma_\mu\gamma_5 V. \end{aligned}$$

The first current, J_{vector}^T , is actually identical to the electromagnetic current. It therefore represents the local gauge symmetry of U(1)_{EM} and no extra global symmetry is involved.

The net number of V-rishons is conserved by our lagrangian and the corresponding current J_{vector}^V is a conserved vector current, representing a global U(1)_V symmetry of the lagrangian.

It will be convenient to define two linear combinations of the net numbers of T-rishons and V-rishons. We therefore define the currents [5]:

$$J_\mu^R = \frac{1}{3}(\bar{T}\gamma_\mu T + \bar{V}\gamma_\mu V), \quad J_\mu^{B-L} = \frac{1}{3}(\bar{T}\gamma_\mu T - \bar{V}\gamma_\mu V).$$

The quantum number R ("rishon number") counts the total number of rishons minus antirishons. The second quantum number counts $[(n_T - n_{\bar{T}}) - (n_V - n_{\bar{V}})]$ and will turn out to be equal to baryon minus lepton number. We may replace the invariance under U(1)_{EM} × U(1)_V by global invariance under U(1)_R × U(1)_{B-L} where U(1)_{EM} is contained in the product.

The two axial currents J_{axial}^T and J_{axial}^V are not conserved. Their divergences do not vanish because of the well-known anomaly terms:

$$\begin{aligned} \partial_\mu (J_{\text{axial}}^T)_\mu &= \frac{3g_H^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_H^a)_{\mu\nu} (F_H^a)_{\rho\sigma} + \frac{3g_C^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_C^a)_{\mu\nu} (F_C^a)_{\rho\sigma} \\ &\quad + \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_{\text{EM}})_{\mu\nu} (F_{\text{EM}})_{\rho\sigma}, \\ \partial_\mu (J_{\text{axial}}^V)_\mu &= \frac{3g_H^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_H^a)_{\mu\nu} (F_H^a)_{\rho\sigma} + \frac{3g_C^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_C^a)_{\mu\nu} (F_C^a)_{\rho\sigma}. \end{aligned}$$

Note that the coefficients of the $F_H \tilde{F}_H$ ($F_C \tilde{F}_C$) terms in both divergences are equal. This follows from the fact that T and V are in hypercolor (color) representa-

tion of the same dimensionality. Again, it is useful to define the sum and difference of the two currents:

$$X_\mu = \bar{T}\gamma_\mu\gamma_5 T + \bar{V}\gamma_\mu\gamma_5 V,$$

$$Y_\mu = \bar{T}\gamma_\mu\gamma_5 T - \bar{V}\gamma_\mu\gamma_5 V.$$

We immediately observe that while X_μ is not conserved, Y_μ is essentially conserved [10]:

$$\begin{aligned}\partial_\mu X_\mu &= \frac{3g_H^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_H^a)_{\mu\nu} (F_H^a)_{\rho\sigma} + \frac{3g_C^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_C^a)_{\mu\nu} (F_C^a)_{\rho\sigma} \\ &\quad + \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_{EM})_{\mu\nu} (F_{EM})_{\rho\sigma}, \\ \partial_\mu Y_\mu &= \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} (F_{EM})_{\mu\nu} (F_{EM})_{\rho\sigma}.\end{aligned}$$

The current Y_μ is not conserved only as a result of the electromagnetic anomaly term. We can always define a conserved charge \tilde{Y} and our lagrangian is invariant under the corresponding global axial $U(1)_{\tilde{Y}}$ symmetry [10]. We therefore have a chiral $U(1) \times U(1)$ invariance, which is closely related to our hypercolor and color assignments of the rishons.

The $U(1)_X$ symmetry is broken by color and hypercolor instantons. The lagrangian does not possess a global $U(1)_X$ symmetry. However, the breaking of the X -charge is always done in “lumps” of 12 units. The instanton term (for color or for hypercolor) must always involve $3T + 3V + 3\bar{T} + 3\bar{V}$ of the same “handedness” for a total $|\Delta X| = 12$. Consequently, a discrete Z_{12} subsymmetry of $U(1)_X$ remains conserved [11] (or, in other words, the axial charge X is conserved modulo 12).

The complete list of $U(1)$ symmetries and the corresponding currents is summarized in table 2. The complete list of quantum numbers for all left and right-handed rishons and antirishons is given in table 3.

The overall symmetry of our lagrangian is given by $SU(3)_H \times SU(3)_C \times U(1)_R \times U(1)_{B-L} \times U(1)_{\tilde{Y}}$ with parity, charge conjugation and Z_{12} as additional discrete symmetries and a local gauge group $U(1)_{EM}$ contained in $U(1)_R \times U(1)_{B-L}$. The global and discrete symmetries need not be respected by the vacuum and may eventually be spontaneously broken.

In addition to the above exact symmetries of the lagrangian, one should also consider the approximate symmetry obtained by sending g_c and/or e to zero. In the limit $g_c = e = 0$, our lagrangian contains only hypercolor interactions and is invariant under a global $SU(6)_L \times SU(6)_R \times U(1)$ symmetry. Such a theory is isomor-

TABLE 2
A summary of all vector and axial U(1) symmetries of the rishon model lagrangian

	U(1) symmetry	Name of current	Current	Remarks
A complete set of currents	vector $\begin{cases} U(1)_{EM} \equiv U(1)_T \\ U(1)_V \end{cases}$	$(J_{EM})_\mu \equiv (J_{vector}^T)_\mu$ $(J_{vector}^V)_\mu$	$\bar{T}\gamma_\mu T$ $\bar{V}\gamma_\mu V$	Gauged. Conserved. Global. Conserved.
	axial $\begin{cases} U(1)_{AT} \\ U(1)_{AV} \end{cases}$	$(J_{axial}^T)_\mu$ $(J_{axial}^V)_\mu$	$\bar{T}\gamma_\mu\gamma_5 T$ $\bar{V}\gamma_\mu\gamma_5 V$	Global. Not conserved due to color and hypercolor anomalies.
An alternative choice for the complete set of currents	vector $\begin{cases} U(1)_R \\ U(1)_{B-L} \end{cases}$	J_μ^R J_μ^{B-L}	$\frac{1}{3}(\bar{T}\gamma_\mu T + \bar{V}\gamma_\mu V)$ $\frac{1}{3}(\bar{T}\gamma_\mu T - \bar{V}\gamma_\mu V)$	Global. Conserved. Global. Conserved.
	axial $\begin{cases} U(1)_Y \\ U(1)_X \end{cases}$	Y_μ X_μ	$\bar{T}\gamma_\mu\gamma_5 T - \bar{V}\gamma_\mu\gamma_5 V$ $\bar{T}\gamma_\mu\gamma_5 T + \bar{V}\gamma_\mu\gamma_5 V$	Global. Current not conserved due to EM anomalies Conserved charge can be defined. Global. Current not conserved due to color and hypercolor anomalies. A discrete Z_{12} subsymmetry remains The charge X is conserved modulo 12.

TABLE 3
Summary of all quantum numbers for rishons and antirishons

Particle	Vector charges						Axial charges			
	SU(3) _H	SU(3) _C	V(1) _T	U(1) _V	U(1) _R	U(1) _{B-L}	U(1) _{AT}	U(1) _{AV}	U(1) _Y	U(1) _X
T_L T_R	3	3	1	0	$\frac{1}{3}$	$\frac{1}{3}$	1 -1	0	1 -1	1 -1
\bar{T}_L \bar{T}_R	$\bar{3}$	$\bar{3}$	-1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	1 -1	0	1 -1	1 -1
V_L V_R	3	$\bar{3}$	0	1	$\frac{1}{3}$	$-\frac{1}{3}$	0	1 -1	-1 1	1 -1
\bar{V}_L \bar{V}_R	$\bar{3}$	3	0	-1	$-\frac{1}{3}$	$\frac{1}{3}$	0	1 -1	-1 1	1 -1

All fermions and antifermions with subscript L [R] transform according to the $(\frac{1}{2}, 0)$ representation $[(0, \frac{1}{2})$ representation] of the Lorentz group.

phic to a *six-flavor* massless QCD model. We know that in *two-flavor* massless QCD the $SU(2)_L \times SU(2)_R$ chiral symmetry is spontaneously broken, leaving the diagonal vectorial $SU(2)$ intact. In spite of several plausibility arguments, we do not really know why the chiral symmetry breaks in this way in QCD.

One may wish to make the following three “sensible” assumptions:

- (i) Chiral $SU(6)_L \times SU(6)_R$ must be spontaneously broken.
- (ii) In analogy with the two-flavor QCD, the surviving subgroup must be the “diagonal” $SU(6)$.
- (iii) The above situation (valid for $g_c = e = 0$) remains unchanged for the physical non-vanishing physical values of g_c and e .

If all three assumptions are correct, the final spectrum of composite fermions will contain only heavy fermions and will possess a vectorial $SU(6)$ symmetry. Since no such symmetry is observed in nature, we can proceed with our discussion only if we postulate that at least one of the above three assumptions is untrue. The reader who considers all three assumptions to be necessarily valid need not continue reading beyond this point. In his or her opinion our model has no validity.

There are several possible ways in which one or more of the above assumptions may be invalid. It is possible that for higher numbers of flavors ($N=6$) the chiral symmetry breaking follows a different pattern. It is possible that because of that, $SU(6)_L \times SU(6)_R$ breaks directly into $SU(3)_C$ or, perhaps, into $SU(3)_L \times SU(3)_R$. It is possible that for $g_c = e = 0$, several operators may obtain vacuum expectation values (v.e.v.) but for non-vanishing g_c or e , a specific direction of symmetry breaking is chosen. It is possible that, for some reason, the rishon bilinear $r\bar{r}$ does not acquire a v.e.v. but, say, $rr\bar{r}\bar{r}$ or $rrr\bar{r}\bar{r}$ obtains a v.e.v. It is even possible that the physics of $g_c = e = 0$ is quite different from that of the physical values of g_c and e .

Each of the above proposals are unusual, but none is *a priori* ruled out, in our opinion. In order to proceed we must clearly state our conjecture that at least one of the three “sensible” assumptions (i) (ii) (iii) is invalid and for physical g_c and e no $SU(6)$ symmetry remains unbroken.

We return to this issue in sect. 8.

4. Composite fermions

At energies well above Λ_H , our theory is asymptotically free. As we decrease the energy, all rishons and hypergluons become confined. Only hypercolor-singlets remain as “free” particles. Composite bosons and fermions are, respectively, constructed from even and odd numbers of fundamental fermions. All possible combinations with less than eight constituents are listed in tables 4 and 5. The tables contain the R -values of the composites. It is clear that bosons (fermions) must have even (odd) R -values.

What can we say about the masses and radii of such composites? The effective radius of the confining hypercolor force is, presumably, Λ_H^{-1} . Hence, all combina-

TABLE 4
All hypercolor-singlet composite bosons with less than eight constituents

No. of constituents	Rishon content		
	$\frac{1}{2}R = 1$	$\frac{1}{2}R = 0$	$\frac{1}{2}R = -1$
2		$r + \bar{r}$	
4		$2r + 2\bar{r}$	
6	$6r$	$3r + 3\bar{r}$	$6\bar{r}$

Here r is any rishon (T or V) and \bar{r} any antirishon.

tions which actually form composite states are likely to have effective radii of order Λ_H^{-1} . The underlying lagrangian had a chiral symmetry. If it is not broken spontaneously, or if it is broken but a chiral subgroup remains conserved, the chiral symmetry can be realized, in the composite level, in two ways: We may have massive parity doublets (probably with $M \sim \Lambda_H$) and we may have massless composite fermions. As long as we cannot fully calculate the confining hypercolor forces, we cannot decide whether or not the chiral symmetry is broken, and – in case it is not broken – which composite fermions will emerge massless. At most we may have consistency conditions which should be obeyed by any candidate theory [4].

The only exercise we can perform is to check whether the simplest set of composite fermions resembles the observed spectrum of “approximately massless” quarks and leptons. The simplest composite hypercolor-singlet fermions are formed from three rishons or three antirishons. The R and $B - L$ values are determined by the explicit rishon content. For each combination we consider the simplest allowed color representation. The complete list of such three-constituent composites is given in table 6.

We immediately notice that the table displays the precise content of one generation of quarks, leptons and their antiparticles. Each $B - L$ value appears with the correct corresponding color and the only allowed combinations are the ones observed in nature. (No integer-charge color triplets; no fractional-charge color singlets; the only quark charges are $\pm \frac{1}{3}, \pm \frac{2}{3}$; all charges are one or less; the only allowed $B - L$ values are $\pm 1, \pm \frac{1}{3}$ and not, e.g. $\pm \frac{2}{3}$).

TABLE 5
All hypercolor-singlet composite fermions with less than eight constituents

No. of constituents	Rishon content	
	$\frac{1}{2}R = \frac{1}{2}$	$\frac{1}{2}R = -\frac{1}{2}$
3	$3r$	$3\bar{r}$
5	$4r + \bar{r}$	$4\bar{r} + r$
7	$5r + 2\bar{r}$	$5\bar{r} + 2r$

TABLE 6
All hypercolor-singlet, three-constituent composite fermions,
assuming the minimal color for each configuration

Minimal allowed color	$B - L$	$\frac{1}{2}R$	
		$\frac{1}{2}$	$-\frac{1}{2}$
1	1	TTT (e^+)	$\bar{V}\bar{V}\bar{V}(\bar{\nu}_e)$
3	$\frac{1}{3}$	TTV (u)	$\bar{V}\bar{V}\bar{T}(d)$
$\bar{3}$	$-\frac{1}{3}$	TVV (\bar{d})	$\bar{V}\bar{T}\bar{T}(\bar{u})$
1	-1	VVV (ν_e)	$\bar{T}\bar{T}\bar{T}(e^-)$

The relation between the quantization of quark and lepton charges is trivially understood, since all charges come from one fundamental charged particle—the T-rishon. A proton contains $4T + \bar{T} + 2V + 2\bar{V}$ (“net content” of $3T$). Hence, a hydrogen atom is not only neutral ($4T + 4\bar{T} + 2V + 2\bar{V}$) but contains equal numbers of rishons and antirishons and has $Q = B - L = R = 0$. This immediately raises the question of proton stability (see sect. 8) and of the matter-antimatter symmetry [6].

Another puzzle that is trivially solved here is the famous vanishing sum of all quark and lepton charges in one generation. It is well-known that the renormalizability of the standard model requires the absence of anomalies, leading to the constraint

$$Q(e^-) + Q(\nu) + 3Q(u) + 3Q(d) = 0.$$

This constraint is indeed obeyed by a “miraculous” cancellation between quark and lepton charges. In fact, it is the only ingredient of the standard model which tells us that the observed leptons could not have existed without the observed quarks, and vice versa. In our model, the vanishing of the above sum is extremely natural and simple [6]. The total rishon-content of e^-, ν, u, d is $6T + 6\bar{T} + 6V + 6\bar{V}$. Hence, the sum of all electric charges as well as the sum of all $B - L$ values automatically vanish.

Our next concern is to verify that the necessary combinations of rishons in table 6 obey Fermi statistics. We assume that the effective field of the composite quark or lepton is always constructed from the rishon fields without any derivative coupling. Consider first the leptons and antileptons. Their wave function contains three identical rishons or antirishons. It is totally antisymmetric under both hypercolor and color (three SU(3)-triplets forming a singlet). The only remaining degrees of freedom are the Lorentz properties. Each rishon can be described by a Dirac four-spinor transforming according to a $(\frac{1}{2}, 0) + (0, \frac{1}{2})$ representation of the Lorentz group. The required overall antisymmetry of the wave function forces the Lorentz

part to be totally antisymmetric. The totally antisymmetric product of three $(\frac{1}{2}, 0) + (0, \frac{1}{2})$ representations is given by one $(\frac{1}{2}, 0) + (0, \frac{1}{2})$ representation. Consequently, our composite leptons can be "legally" constructed, they must have $J = \frac{1}{2}$, there are no $J = \frac{3}{2}$ leptons and there is only one lepton state for each set of three rishons [5].

The above discussion tells us that we could not have chosen T and V to be in 3 and 1 representations of $SU(3)_C$ respectively. Such a choice would not have enabled us to form a VVV lepton with $J = \frac{1}{2}$.

An alternative way to look at the lepton wave functions would describe a left-handed (or a right-handed) rishon as a two-component spinor. In that case T_L and T_R are different particles. We can, at most, antisymmetrize two T_L spinors or two T_R 's. Hence, we find

$$e_L^+ = T_R T_R T_L, \quad e_R^+ = T_L T_L T_R,$$

where the two identical rishons always form a Lorentz scalar. The masslessness of the rishons is an important part of the argument here. Note that for massless rishons we cannot consider concepts such as orbital angular momentum and spin at rest. Had we considered a non-relativistic approximation, we could not have constructed a totally antisymmetric three-rishon state without orbital angular momenta. However, this is not the case for the extreme relativistic case of massless fermions, where the orbital angular momentum cannot even be defined.

Note that all leptons and antileptons have $Y = \pm 1$. Note also that in each lepton we find a Lorentz-scalar two-rishon set $(T_L T_L)$ or $(T_R T_R)$ or $(V_L V_L)$, etc. This "dirishon" is always antisymmetric in the hypercolor, color and Lorentz degrees of freedom. We will later encounter such "dirishons" again and again in the wave functions of the quarks and the W^\pm bosons.

The analysis of the quark wave function is more complicated. There are several possible ways of forming a $J = \frac{1}{2}$, color-triplet, TTV composite with a given handedness. Table 7 contains all the possible ways of constructing a three-rishon composite with the quantum numbers of the u_L quark. Unlike the lepton case, $J = \frac{3}{2}$ quarks cannot be *a priori* excluded. However, there are general arguments against the existence of massless $J = \frac{3}{2}$ fermions [12]. We will also see later, in sect. 6, that

TABLE 7
Candidate wave functions for a u_L quark

Rishon content	Properties of (TT) system		Total Y
	color	Lorentz	
$T_R T_R V_L$	$\bar{3}$	(0,0)	-3
$T_L T_L V_L$	$\bar{3}$	(0,0)	1
$T_L T_L V_L$	6	(1,0)	1
$T_L T_R V_R$	$\bar{3}$ or 6	$(\frac{1}{2}, \frac{1}{2})$	1

the electroweak anomalies vanish only if each lepton is accompanied by only one tricolored $J = \frac{1}{2}$ quark.

At this point of our discussion we cannot definitely choose the appropriate wave-function for the u_L quark. We remark, however, that if we want the quarks to have the same Y -values as the leptons (i.e. $Y = \pm 1$) and to include a “dirishon” system in a color antisymmetric, Lorentz scalar (as the leptons do), the only allowed solution is given by the second entry of table 7:

$$u_L = (T_L T_L) V_L, \quad u_R = (T_R T_R) V_R.$$

However, we must postpone our definite choice of the quark wave function till the end of the next section, after we have discussed the weak interactions.

5. The weak interactions

Up to this point we have not discussed the weak interactions. The only fundamental interactions in our model are hypercolor, color and electromagnetism. The massive W and Z do not exist in the fundamental level of the theory. The short-range weak interaction is not introduced in the underlying lagrangian and does not operate between single-rishon states.

Consider the forces between two hypercolor-singlet composite objects such as two leptons. At large distances there will be no net hypercolor forces between two $SU(3)_H$ singlets. However, at short distances of the order of Λ_H^{-1} we expect complicated short-range residual forces, reflecting the color and hypercolor interactions between the rishons inside the two composite objects. These forces are analogous to the residual color forces operating between two colorless hadrons. The residual color forces between hadrons are identified with the “strong” or hadronic or nuclear force. We now conjecture that the residual forces among hypercolor-singlet composites of rishons be identified with the conventional weak interactions [5, 13]. The “tail” of the hadronic forces is determined by the masses of the lightest composite colorless mesons which can be exchanged (i.e. pions, ρ -mesons, etc.). The “tail” of the weak force would now be determined by the masses of the lightest composite hypercolor-singlet bosons which can be exchanged (presumably W and Z).

If our conjecture is correct, the weak interactions should not be considered as one of the fundamental forces of nature. In order to prove such a conjecture, we should be able to derive the observed weak interaction phenomena from our fundamental hypercolor and color forces. This we cannot do, at present. However, we are able to study the symmetry properties of the forces among hypercolor-singlet composite fermions.

Let us consider the low-energy effective lagrangian involving the composite quark and lepton fields. It contains the kinetic terms for the composite fermions as well as

the QCD and QED terms describing their interactions with gluons and photons. Consider two composite fermions with the same $B - L$ values, e.g. TTT and $\bar{V}\bar{V}\bar{V}$ (see table 6). Except for the electric charges, all their quantum numbers are identical. It is clear that all the effective lagrangian terms mentioned above (except for the photon couplings) are invariant under a global $SU(2)$ symmetry, under which the two composite fermions form a doublet [5]. If these fermions are massless, the global symmetry is actually $SU(2)_L \times SU(2)_R$.

Note that the symmetry here is not discrete, since we can separately rotate $(TTT)_L$ into any linear combination of $(TTT)_L$ and $(\bar{V}\bar{V}\bar{V})_L$ without changing the lagrangian.

It is interesting to note that at the underlying level T and \bar{V} have the same color and $B - L$. Nevertheless, we did not have a global $SU(2)$ symmetry connecting them, because of their different hypercolors. In the composite level, the $SU(3)_H$ singlets TTT and $\bar{V}\bar{V}\bar{V}$ do not, anymore, differ by their hypercolor. The hypercolor is "integrated away" and a new approximate global $SU(2)$ symmetry, which is absent at the fundamental level, appears in the effective lagrangian.

The quantum number which distinguished between, say, TTT and $\bar{V}\bar{V}\bar{V}$ is the R -number. It is easy to see that [5]

$$\frac{1}{2}R = I_{3L} + I_{3R}.$$

We also have a neutral axial charge, given by $I_{3L} - I_{3R}$. We identify it with the axial Y charge of the underlying theory:

$$\frac{1}{2}Y = I_{3L} - I_{3R}.$$

The Y -values of the left-handed and right-handed leptons and antileptons were determined in sect. 4, where the lepton wave functions have been introduced. Thus

$$Y(e_R^-) = Y(e_R^+) = -Y(e_L^-) = -Y(e_L^+) = 1.$$

If we now identify $\frac{1}{2}Y$ with $(I_{3L} - I_{3R})$, we can easily determine the $SU(2)_L \times SU(2)_R$ classification of the leptons. We find that the left-handed leptons $(\nu_e, e^-)_L$ and the right-handed antileptons $(e^+, \bar{\nu}_e)_R$ are in $(\frac{1}{2}, 0)$ representation while $(\nu_e, e^-)_R$ and $(e^+, \bar{\nu}_e)_L$ are in $(0, \frac{1}{2})$.

The Y -values of the quarks depend on the choice of their wave functions (see sect. 4 and table 7). If Y is indeed identified as the neutral axial charge of the new global $SU(2)_L \times SU(2)_R$, the Y -values of all quarks and antiquarks must be ± 1 , rejecting the first entry of table 7. With these Y -values, we find that $(u, d)_L$ and $(\bar{d}, \bar{u})_R$ are in $(\frac{1}{2}, 0)$ while $(u, d)_R$ and $(\bar{d}, \bar{u})_L$ are in $(0, \frac{1}{2})$. There is no candidate quark wave function which would be consistent with any other assignment.

What is the overall continuous symmetry of the effective low-energy lagrangian? At the underlying level we had an $SU(3)_H \times SU(3)_C \times U(1)_R \times U(1)_{B-L} \times U(1)_Y$

symmetry. In the effective lagrangian, the following changes occur:

(i) Only $SU(3)_H$ singlets appear. Hence, the $SU(3)_H$ symmetry is trivial and meaningless.

(ii) A new approximate global $SU(2)_L \times SU(2)_R$ appears, containing $U(1)_R$ and $U(1)_Y$ as the two neutral generators.

It is then easy to see that the full approximate continuous symmetry of the effective lagrangian is [5] $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. This is precisely the symmetry group of the left-right symmetric extension of the standard model, except that in our case, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is a global symmetry. It is significant that all our composite fermions are necessarily assigned to the correct representations of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. All left-handed quarks and leptons and all right-handed antiquarks and antileptons are in $(\frac{1}{2}, 0)$, and all other fermion states are in $(0, \frac{1}{2})$, precisely as required. Under $SU(2)_L$ (the standard model) all left-handed (right-handed) quarks and leptons are in doublets (singlets).

It is remarkable that we are able to find our initial Y -charge inside the global $SU(2)_L \times SU(2)_R$ symmetry. It is even more surprising that all quarks and leptons have the correct Y , I_{3L} and I_{3R} values and that the three neutral currents of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ are identical to the three neutral currents of our original lagrangian corresponding to $U(1)_R \times U(1)_{B-L} \times U(1)_Y$ or $U(1)_{EM} \times U(1)_V \times U(1)_Y$.

We conclude this section with a determination of the quark wave function. Considering the possibilities listed in table 7, we have already eliminated the first entry which has the wrong Y -value. We have already noted in sect. 4 that it would be attractive to conjecture that the quark contains a dirishon state identical to the one found in the lepton wave function, i.e. a Lorentz scalar $(T_L T_L)$ state in a $(\bar{3}, \bar{3})$ representation of $SU(3)_H \times SU(3)_C$. The only entry in table 7 which obeys this requirement and has the correct Y -value is the second entry $(T_L T_L) V_L$.

It turns out that this wave function is also consistent with the requirement that the *same* operator I^+ performs the transformations $\bar{\nu}_e \rightarrow e^+$, $d \rightarrow u$, $\bar{u} \rightarrow \bar{d}$, $e^- \rightarrow \nu_e$. Describing such an operator in terms of rishons we find that the state $\nu_{eL} e_R^+$ (corresponding to I_L^+) is given by $(T_L T_L) T_R (V_R V_R) V_L$ where the parentheses denote a Lorentz-scalar dirishon. The same combination could describe $u_L \bar{d}_R$ if $u_L \equiv (T_L T_L) V_L$ and $\bar{d}_R \equiv (V_R V_R) T_R$, supporting the choice of the quark wave function as the second entry in table 7.

Table 8 summarizes the wave-functions and the full $SU(2)_L \times SU(2)_R$ assignments of the first-generation fermions.

We summarize: We have conjectured that the weak interactions are residual forces which appear only in the effective lagrangian. We have shown that, at that level, a new continuous symmetry group exists. It is identical to the left-right symmetric extension of the standard model and the quark and lepton classification is correct. However, the symmetry is global, we do not know whether W and Z bosons exist and we have not yet discussed the symmetry-breaking mechanism.

TABLE 8
Quark and lepton wave functions and their $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum numbers

	$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$	$\frac{1}{2}R$	$\frac{1}{2}Y$	$I_{3L} = \frac{1}{4}(R+Y)$	$I_{3R} = \frac{1}{4}(R-Y)$
$e_L^+ = (\bar{T}_R \bar{T}_R) \bar{T}_L$ $\bar{\nu}_L = (\bar{V}_R \bar{V}_R) \bar{V}_L$	$(0, \frac{1}{2})_1$	$\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$ $\frac{1}{2}$	0 0	$\frac{1}{2}$ $-\frac{1}{2}$
$u_L = (\bar{T}_L \bar{T}_L) V_L$ $d_L = (\bar{V}_L \bar{V}_L) \bar{T}_L$	$(\frac{1}{2}, 0)_{1/3}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	0 0
$\bar{d}_L = (V_L V_L) \bar{T}_L$ $\bar{u}_L = (\bar{T}_L \bar{T}_L) \bar{V}_L$	$(0, \frac{1}{2})_{-1/3}$	$\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$ $\frac{1}{2}$	0 0	$\frac{1}{2}$ $-\frac{1}{2}$
$\nu_L = (V_R V_R) V_L$ $e_L^- = (\bar{T}_R \bar{T}_R) \bar{T}_L$	$(\frac{1}{2}, 0)_{-1}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	0 0

6. The low-energy effective lagrangian

Our composite theory involves two different lagrangians. The fundamental “underlying” lagrangian (see sect. 3) is renormalizable. It includes rishons, hypergluons, gluons and the photon. Its continuous symmetry is $SU(3)_H \times SU(3)_C \times U(1)_R \times U(1)_{B-L} \times U(1)_Y$, and it describes physics at energies well above Λ_H . In principle, the same lagrangian should also enable us to calculate low-energy phenomena. The low-energy “effective” lagrangian should be a direct consequence of the basic lagrangian of the theory.

The “effective” lagrangian provides us with a phenomenological description of physics well below Λ_H . No hypercolored objects can appear. Hence, the only fields which participate in both lagrangians, are the gluons and the photon. To these we must now add all hypercolor-singlet composites of mass smaller than Λ_H , such as quarks, leptons and, possibly, additional particles. The continuous symmetry of the effective lagrangian (see sect. 5) is $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

In order to provide us with a realistic theory, the low-energy effective lagrangian should presumably resemble, as much as possible, the lagrangian of the standard QCD electroweak model. Since we already have a global $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry, we would actually expect the effective lagrangian to approximate the left-right symmetric extension of the standard model, based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, rather than the basic $SU(2) \times U(1)$ model.

What are the ingredients of the lagrangian of the left-right symmetric extension of the standard model?

(i) It includes quarks, leptons, gluons, photon, their kinetic terms and all their interactions (i.e. photon couplings to charged quarks and leptons, gluon-quark couplings and gluon self-couplings).

(ii) It includes six weak bosons ($W_L^\pm, W_R^\pm, Z_1, Z_2$), their kinetic terms, their self-interactions and their interactions with quarks and leptons. Only two coupling constants [g_2 for SU(2) and g_1 for U(1)] describe all of these couplings.

(iii) It includes an unknown number of Higgs fields which provide masses to weak bosons, quarks and leptons. At least some Higgs fields couple to each of these particles.

Experimentally, we have good evidence for part (i), indirect evidence for (ii) and no evidence for (iii). With parts (i) and (ii) the theory would be renormalizable but all particles remain massless. With parts (i), (ii) and (iii) we regain the full standard renormalizable model.

Following the discussion of sects. 4 and 5, the effective lagrangian of the rishon model already contains all the terms of part (i). We have already noted that we must also have residual short-range forces operating between hypercolor-singlet fermions. These may be described in terms of effective four-fermion interaction terms. Alternatively, it is possible that composite color-singlet, hypercolor-singlet, bosons are formed at masses below Λ_H . Such bosons would then be the carriers of the residual force and would play the role of W's and Z's. We have no specific dynamical reason to expect the creation of such composite bosons, but they may, nevertheless, exist.

If they do, we already know their rishon-content. For instance, W_L^+ should have the rishon-content of the operator I_L^+ , already discussed in sect. 5. Hence, we have

$$\begin{aligned} W_L^+ &\equiv (T_L T_L) T_R (V_R V_R) V_L, & W_R^+ &\equiv (T_R T_R) T_L (V_L V_L) V_R, \\ W_L^- &\equiv (\bar{T}_R \bar{T}_R) \bar{T}_L (\bar{V}_L \bar{V}_L) \bar{V}_R, & W_R^- &\equiv (\bar{T}_L \bar{T}_L) \bar{T}_R (\bar{V}_R \bar{V}_R) \bar{V}_L. \end{aligned}$$

The parentheses denote, again, a Lorentz-scalar pair of rishons, in an antisymmetric color and hypercolor state. We may also have neutral vector particles with $B - L = R = Y = 0$. They may mix, and we do not *a priori* know which are the physical eigenstates. Their rishon content can be viewed in terms of $T\bar{T}$ and $V\bar{V}$ combinations, but also in terms of $2r + 2\bar{r}$ or $3r + 3\bar{r}$ combinations as long as all quantum numbers of all combinations are identical.

Our weak bosons are the analogs of the vector mesons ρ, ω, ϕ, A_1 , etc. The latter are quark composites and can be viewed as effective mediators of the strong nuclear force which is a residual color interaction. Our W's and Z's are rishon composites mediating the weak force which is also a residual interaction. However, there must be an important difference which we cannot explain: Hadronic vector mesons have masses of order Λ_C . Our composite W and Z bosons must be light compared with Λ_H ($M_{W,Z} \ll \Lambda_H$).

What do we know about the couplings of these "weak bosons"? Since the effective interactions of the weak bosons are residual, their effective couplings should be, in principle, given in terms of complicated functions of the fundamental color and

hypercolor coupling constants g_C and g_H (in the same way that the effective ρNN coupling should, in principle, be calculable from QCD).

Our effective lagrangian, prior to the introduction of the weak bosons, possessed a global $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry. If we have only one weak boson, transforming like each generator of the global group, and if all weak boson couplings respect the *global* symmetry, it follows that all the weak boson couplings obey the same relationships as the corresponding couplings in an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ *local gauge theory* (i.e. universal coupling of bosons to quarks, leptons and weak bosons). This is an old well-known result [14] which can be derived, using the vector-dominance idea or the field-current identity approach.

Until now we have not discussed the mass terms in the effective lagrangian. Quark and lepton masses could be due to a small non-vanishing mass of the rishons themselves, or they could be due to dynamical symmetry breaking (or both). The dynamical symmetry breaking could be due to scalar condensates which form as a result of the short-range residual interactions between leptons or quarks. We have discussed this mechanism in detail elsewhere [15] and showed that it could provide us with the correct spectrum of scalar condensates needed in order to reproduce all the phenomenological requirements of an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory. We essentially have two types of condensates:

- (i) A scalar (or scalars) transforming according to the $(0, 1)_{-2}$ representation of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. This condensate is responsible for parity violation, W_R masses, $B-L$ violation and Majorana masses for neutrinos [15]. All of these phenomena are due only to this scalar field (or fields).
- (ii) Scalars transforming like a $(\frac{1}{2}, \frac{1}{2})_0$ representation. These contribute to fermion and weak boson masses.

The weak bosons may have masses induced by the same dynamical symmetry breaking mechanism. They may also have masses due to the binding forces of the rishons. The chiral symmetry which “protected” the composite fermions from acquiring a mass (prior to the dynamical symmetry breaking) does not protect the weak bosons.

An extremely interesting situation occurs if the elementary photon and the composite W and Z bosons belong to the *same multiplet*. In that case there are only seven vector particles, six of them are composite and the seventh is the photon. The situation is even more interesting if the masses of all W and Z bosons are entirely due to a dynamical symmetry breaking mechanism which respects the $U(1)_{EM}$ subgroup of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, e.g. the mechanism described above. In such a case the neutral boson which couples to $U(1)_{EM}$ the photon does not acquire a mass, but the other six weak bosons become massive.

If we neglect all terms which are proportional to inverse powers of Λ_H , our effective lagrangian becomes renormalizable and gauge invariant under a local $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group. Thus, the low-energy effective lagrangian is “approximately renormalizable” and “approximately gauge invariant”.

It will be convenient for us to define these concepts to mean as follows:

(i) An “approximately gauge invariant” effective lagrangian is an effective lagrangian which becomes gauge invariant when we remove all terms which are proportional to inverse powers of Λ_H .

(ii) An “approximately renormalizable” effective lagrangian is an effective lagrangian which becomes renormalizable when we remove all terms which are proportional to inverse powers of Λ_H .

If the W and Z masses are entirely due to the scalar condensates, the effective lagrangian, *including* the mass terms will be “approximately renormalizable”. If additional W and Z mass terms come from other mechanisms, this “approximate renormalizability” cannot be guaranteed.

Let us summarize what we have done up to this point. In addition to the quarks and leptons of sects. 4 and 5, we *assumed* the existence of composite W and Z bosons whose couplings obey the same global symmetry which we have found earlier. We have also introduced dynamical symmetry breaking through scalar condensates, contributing masses to quarks, leptons and weak bosons. If the elementary photon and the composite W and Z belong to the same multiplet and if all masses are only due to the effective Higgs mechanism, the emerging low-energy effective lagrangian is renormalizable and $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge invariant, except for terms which behave like inverse powers of Λ_H . If additional mass contributions exist, they may be non-renormalizable.

Can we say, at this point, anything about the relationship between g_2 and g_1 , the effective weak coupling constants of $SU(2)_L \times SU(2)_R$ and $U(1)_{B-L}$, respectively? The underlying lagrangian is invariant under a global $U(1)_R \times U(1)_{B-L}$, which turns out to be a subgroup of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ (since $\frac{1}{2}R = I_{3L} + I_{3R}$). The $U(1)_R$ coupling constant is $\sqrt{\frac{1}{2}} g_2$ while the $U(1)_{B-L}$ coupling constant is g_1 . The global symmetry group $U(1)_R \times U(1)_{B-L}$ is equivalent to the global $U(1)_T \times U(1)_V$ (see sect. 3). However, if $U(1)_R \times U(1)_{B-L}$ becomes an approximate *local* gauge symmetry at the low-energy world of the effective lagrangian, it is equivalent to a *local* $U(1)_T \times U(1)_V$ only if all of these U(1) factors have the same coupling. Hence, the coupling constants of $U(1)_R$ and of $U(1)_{B-L}$ must be equal at the energy of the effective lagrangian (or, equivalently, the couplings to T and V must be identical). We then obtain:

$$\sqrt{\frac{1}{2}} g_2 = g_1.$$

The usual definition of $\sin^2 \theta_w$ is

$$\sin^2 \theta_w = \frac{g_1^2}{g_2^2 + 2g_1^2}.$$

Hence [5, 6],

$$\sin^2 \theta_w = \frac{1}{4}.$$

This relation is valid at the level of the effective lagrangian. There are several corrections to this approximate result, depending somewhat on the masses of the W bosons, and on possible additional contributions proportional to Λ_H^{-N} terms.

We may actually find ourselves with an effective lagrangian which is completely equivalent to the standard gauge lagrangian of $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ (except for terms of order $1/\Lambda_H$).

This, however, would require some “miracles” such as an explicit relation between α and the effective weak couplings (which are complicated functions of g_C and g_H). Such an equivalence between the standard model lagrangian and the low-energy effective lagrangian of a composite model, must emerge if we accept the ansatz known as “Veltman’s theorem” [16]. If the full low-energy effective lagrangian is to be realistic and “approximately renormalizable”, it must coincide with the standard model lagrangian (or with its left-right symmetric extension).

The “approximate renormalizability” of the effective lagrangian requires vanishing anomalies for all triple products of currents within $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. This requirement is obeyed only if, for each pair of composite leptons we have exactly one pair of tricolored composite quarks. This provides us with extra motivation for accepting only one of the quark candidate wave functions as a valid description of a composite fermion of mass below Λ_H (see sects. 4 and 5).

Theoretically, we are not sure that “approximate renormalizability” of the effective lagrangian is a necessary requirement. The lower Λ_H , the less we would be inclined to assume it.

Experimentally, we must remember that the W, Z and scalar particles have not been seen yet, and only a few terms in the standard model lagrangian have actually been directly tested.

Consequently, it is not clear whether the effective lagrangian should indeed obey the “Veltman theorem” or whether it should exhibit only a few terms which resemble the standard-model lagrangian [17], and be allowed to contain additional non-renormalizable terms. This is an important open question, common to many composite models, which should be further investigated.

We remind ourselves, however, that regardless of the resolution of the above question, the effective lagrangian should, in principle, follow from the fundamental lagrangian, and the latter is, of course, renormalizable.

7. Symmetry breaking, Goldstone bosons and the ‘t Hooft condition

We are now in a position to review our various symmetry groups and their symmetry breaking.

It is interesting that, in our model, all exact symmetries are gauge symmetries. The only exact symmetries are, in fact, the three local gauge symmetries of the fundamental theory: hypercolor, color and electromagnetism.

All other symmetries of the original lagrangian are dynamically broken. The discrete symmetries are parity, charge conjugation and the Z_{12} subgroup of $U(1)_X$ (see sect. 3). Our scalar condensates break these discrete symmetries, yielding $W_R - W_L$ mass differences, neutrino Majorana masses [15] and Cabibbo mixing which breaks Z_{12} [11]. Clearly, no Goldstone bosons are involved in the breaking of discrete symmetries.

The situation concerning the global symmetries is, again, more complicated. Consider the axial $U(1)_Y$ symmetry of the fundamental lagrangian. At the underlying level, $U(1)_Y$ is a global symmetry group. At the level of the effective lagrangian we have postulated an "approximate gauge invariance" under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ which contains $U(1)_Y$ as a subgroup. Hence, $U(1)_Y$ becomes an "approximate local gauge symmetry" of the effective lagrangian. The "approximate $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry" is then dynamically broken by scalar condensates, breaking the $U(1)_Y$ subgroup as well. We know that the dynamical symmetry breaking of a local gauge symmetry (Higgs mechanism) does not require physical massless Goldstone bosons. Thus, in the approximation that $U(1)_Y$ is a local gauge group, we do not expect a Y -breaking Goldstone boson. However, $U(1)_Y$ is *not* an *exact* local gauge symmetry of the effective lagrangian. There will be terms proportional to Λ_H^{-1} which violate the $U(1)_Y$ local gauge invariance but respect the original global $U(1)_Y$ symmetry. Thus we *may* have a massless Goldstone boson χ , reflecting the breaking of $U(1)_Y$ and coupling to light fermions, with coupling constants of order Λ_H^{-1} . In fact, we *must* have such a Goldstone boson, since the original theory had an *exact* global $U(1)_Y$ symmetry and the final outcome shows a breaking of the same $U(1)_Y$.

We can check the consistency of the result in a different way. Define the parameter F_χ as $\langle 0 | Y_\mu(x) | \chi(q) \rangle = i F_\chi q_\mu e^{-iq \cdot x}$. Clearly, F_χ must be of order Λ_H (what else?). We have the Goldberger-Treiman relation $m_f = g_{\chi ff} F_\chi$, where f is a composite fermion and $g_{\chi ff}$ is the coupling constant of the χ boson to the same fermion. Since in our case $F_\chi \sim \Lambda_H$, $m_f \ll \Lambda_H$, we must indeed have $g_{\chi ff} \sim m_f / \Lambda_H$.

A similar argument follows for the global $U(1)_V$ symmetry of our fundamental lagrangian. Here, again, we expect a massless V -breaking Goldstone boson ζ with a small coupling to composite fermions.

If the rishons have a small mass, the boson χ becomes a massive pseudo-Goldstone boson but ζ remains massless.

Can we observe such Goldstone bosons experimentally? The ζ -boson is very similar to the "Majoron" [18] and it couples directly only to neutrinos, with a coupling proportional to Λ_H^{-1} (in fact, the "Majoron" is a linear combination of ζ and χ). It is many orders of magnitude away from experimental observation [18].

The χ -boson is more similar to a massless or a low-mass axion with a very small coupling to fermions. It seems that the strongest limit on its existence comes [19]

from the process:

$$\gamma + e^- \rightarrow \chi + e^-,$$

which could frequently take place in the sun (or in stars) and cause energy losses by the emission of the χ -particles [20].

The obtained limits on Λ_H depend on the χ -mass and on assumptions concerning models of stellar evolution. For instance, for $m(\chi) \lesssim 200$ keV one gets $\Lambda_H \gtrsim 10^4 - 10^5$ GeV. Analysis of the expected properties of χ deserves a more detailed study.

We therefore conclude that, at present, the two Goldstone particles do not necessarily pose a major experimental threat to the model. On the other hand, it should be interesting to investigate the theoretical and experimental implications of the χ -boson, with the hope that they may yield a practical experimental test of the theory.

Our effective lagrangian and the standard-model lagrangian are supposed to be equivalent, except for terms of order Λ_H^{-N} . We have now learned that these terms may include not only terms involving particles of mass Λ_H or effective four-fermion and six-fermion interactions, but also Yukawa coupling of massless Goldstone bosons with coupling constants $g \sim m_f/\Lambda_H$.

We should add, at this point, that the appearance of a weakly coupled Goldstone boson is a general feature which must appear whenever a composite model exhibits an underlying global symmetry which becomes a dynamically broken "approximate local gauge symmetry" in the composite level.

In his remarkable paper, 't Hooft has derived [4] necessary consistency conditions relating an underlying theory to its composite companion. In our fundamental lagrangian, we should consider four anomaly terms, described by the triple products of the currents $Y_\mu Y_\mu Y_\mu$, $Y_\mu J_\mu^R J_\mu^R$, $Y_\mu J_\mu^{B-L} J_\mu^{B-L}$ and $Y_\mu J_\mu^R J_\mu^{B-L}$ (for notations see sect. 3, table 2). Of these, the first three show a vanishing anomaly at the rishon level. Only the last product, $Y_\mu J_\mu^R J_\mu^{B-L}$, has a non-vanishing anomaly.

We must then have [4, 21] composite massless particles which, at the composite level will enable us to obtain the same results. There are two possible ways in which such composite massless particles can account for the anomaly. The first is the existence of massless composite fermions whose contribution exactly provides the needed anomaly. This is the possibility considered (and essentially rejected) by 't Hooft [4]. The second way is the existence of a massless composite Goldstone particle which couples directly to the relevant global current and provides the necessary contribution. This is the way in which QCD obeys the anomaly constraint. In our case, a very similar situation exists. Our χ -boson plays the role of the pion in QCD. It couples directly to the current Y_μ and the condition is automatically and trivially obeyed. The composite quarks and leptons are almost, but not exactly, massless. It is debatable [4, 21] whether their contribution to the anomaly should also be included in the consistency equation. However, since all the relevant currents Y , R and $B-L$, are included in the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, we know that

the total contribution of each generation of quarks and leptons to any anomaly involving these currents, must vanish. Hence, we have no difficulty with the consistency requirements, with or without the contributions of the quarks and the leptons. If the rishons have a small mass, so will the χ -bosons and the consistency remains valid.

8. Open problems

We now turn to the difficulties and open problems of the model.

(i) *Pattern of chiral symmetry breaking and the limit $g_C \rightarrow 0$.* In ordinary QCD with approximately massless u and d quarks, the $SU(2) \times SU(2)$ “flavor” chiral symmetry is spontaneously broken and an approximately massless Goldstone pion exists. No residual chiral symmetry remains intact and all composite fermions acquire masses of order Λ_C or more. In contrast, any realistic composite model of quarks and leptons, based on hypercolor dynamics, must exhibit some degree of unbroken chiral symmetry yielding massless (or almost massless) composite fermions well below Λ_H . Regardless of the detailed pattern, one should be able to understand the reason for the difference between the chiral symmetry breaking in QCD and the situation in the composite model. We do not have a satisfactory answer to this question, except to note that even in QCD the reason for chiral symmetry breaking is not fully understood.

As we have already explained in sect. 3, the full axial symmetry of our model (for physical values of g_C and e) is $U(1)_Y \times Z_{12}$ where Z_{12} is a discrete subgroup of $U(1)_X$. In addition, we must remember that in the limit $g_C = e = 0$ we have a larger chiral symmetry of $SU(6)_L \times SU(6)_R \times U(1)$.

We have explained in sect. 7 that $U(1)_Y$ must be broken, leading to the χ Goldstone particle. We must therefore conclude that the axial symmetry which is preserved after the first stage of symmetry breaking is a Z_n discrete subgroup of $U(1)_Y$. This can happen if $r\bar{r}$ does not acquire a vacuum expectation value of order Λ_H , while a more complicated quadrilinear or hexalinear fermion operator obtains a v.e.v. In such a case, $Y = \pm 1$ composite fermions do not get a mass of order Λ_H and the χ -boson decouples from them.

The existence of a large v.e.v. for $rr\bar{r}\bar{r}$ or $rrr\bar{r}\bar{r}\bar{r}$ without a large v.e.v. for $r\bar{r}$ is an unusual dynamical assumption which we cannot justify, at the present time. We consider it to be an important difficulty of our model.

In sect. 3 we have already explained that we cannot accept a description according to which the chiral $SU(6)_L \times SU(6)_R$ symmetry of the $g_C = e = 0$ limit breaks into a diagonal $SU(6)$ which remains intact at physical g_C and e values. The pattern of symmetry breaking must be different, in our model. We may consider at least two alternatives:

(a) At $g_C = e = 0$, $SU(6)_L \times SU(6)_R \times U(1)$ breaks directly into $SU(3)_C \times U(1)$, presumably by a v.e.v. of a multi-fermion operator of the type discussed above. The

residual axial symmetry for $g_C = e = 0$ as well as for the physical g_C and e , is a discrete axial subgroup.

In this scenario, the unusual dynamical assumption is, as before, the symmetry breaking by a v.e.v. of a complicated operator, while $\langle r\bar{r} \rangle$ is small or zero.

(b) The situation for physical g_C and e is quite different from that of $g_C = e = 0$ due to some discontinuity or some interplay between the color and hypercolor groups, which we do not understand, at present. In such a case the physical spectrum of composite quarks and leptons may show no resemblance to the theoretical composite spectrum which is obtained for $g_C = e = 0$. Needless to say, the possibility of a strong g_C (or e) dependence is also unusual.

We summarize this point by repeating that our model may remain a realistic candidate for a correct theory only if the pattern of chiral symmetry is quite different from that of ordinary QCD.

(ii) *Composite W and Z bosons.* As discussed in sect. 6, our second major difficulty is the need for light (but not massless) composite vector bosons, corresponding to an approximate gauge symmetry of the effective low-energy lagrangian. We will not repeat here the discussion of sect. 6, except to note that the possibility of light composite vector particles is an extremely interesting issue, independent of the present model.

(iii) *The generation structure.* The quarks and leptons described in section IV account for one generation. Additional generations are presumably obtained by excitations of the first generation. Orbital and radial excitations clearly have the wrong energy scale. The most likely type of excitation is the addition of rishon-anti-rishon pairs. Elsewhere [11], we have speculated that the discrete Z_{12} subsymmetry of $U(1)_X$ (see sect. 3) provides us with a suitable generation-labelling scheme. We do not repeat here the full discussion of this idea, but the main point is the following: Higher generations can be formed by adding to the first generation a $(T_L \bar{T}_L V_L \bar{V}_L)$ or a $(T_R \bar{T}_R V_R \bar{V}_R)$ Lorentz scalar. All quantum numbers of this Lorentz scalar vanish, except for the X -charge which is changed by ± 4 and is conserved modulo 12. In fact, if we insist that each two generations possess *different* X -values, we can only have three generations. More generally, the number of generations is dictated by the number of colors. The Z_{12} symmetry is dynamically broken [11] by the same type of scalar condensates which produce the fermion masses, yielding Cabibbo mixing and fermion mass matrices.

(iv) *Proton decay.* In a separate publication [22] we discuss the question of proton decay in the rishon model. Again, we shall not repeat here the full argument. To lowest order in Λ_H (i.e. $\tau_p \sim \Lambda_H^4$), proton decay is forbidden*. It is allowed in higher orders and we obtain:

$$\tau_p \sim \Lambda_H^8 / M^9,$$

* For a criticism of our model see ref. [23]. We explain in the present paper how the question of $SU(6)_L \times SU(6)_R$ symmetry may be solved. The problem of proton decay is discussed in detail in ref. [22].

where the M^9 mass factor may involve quark, lepton, proton or W masses. For different proton decay processes we find that the experimental limit on τ_p yields [22] Λ_H values around 10^8 GeV.

9. Summary and outlook

We have followed here an extremely simple-minded approach. There are too many species of quarks, leptons and scalar particles. There are too many free parameters in the standard model. We, therefore, try to describe a model in which there are only two types of fundamental fermions, no fundamental scalars and only three parameters. Since the goal is so ambitious, it is not surprising that we encounter difficulties.

Our rishon model is obtained as the minimal scheme obeying a certain reasonable set of assumptions (see sect. 2). It is surprising that such a simple scheme develops the right amount of complexity needed to allow for a realistic spectrum of composite quarks and leptons and for a realistic symmetry of the effective lagrangian. We have to make several unusual bold assumptions on the way. We believe that they are perfectly self-consistent, but we certainly cannot prove them. A complete understanding of the combined non-perturbative physics of QCD and of its hypercolor analogue, should enable us to prove or disprove every one of these assumptions. (Do we have almost massless composite fermions? Scalar condensates? Composite W and Z bosons? Is the chiral symmetry approximately conserved and, if so, why? What happens at the limit $g_c \rightarrow 0$? Do generations differ by the X-charge?) Such an understanding is far away, even for QCD alone.

Our approach is significantly different from that of the grand unified theories. We do not attempt to unify the strong and weak interactions into one gauge group. Instead, we pursue an analogy between the weak interactions and the hadronic forces, demoting both into the role of residual forces. We have three fundamental forces (in addition to gravity). Each of them is described in terms of an exact local gauge symmetry and is mediated by massless gauge bosons. Whether one can further unify these three forces, only time will tell.

There are three types of short-range forces in physics: Van der Waals forces, hadronic forces and weak forces. In our model, each short-range force becomes the residual force of a fundamental interaction originating from an exact local gauge symmetry: Electromagnetism, color and hypercolor.

The weak interactions, being a residual force, obey only approximate, dynamically broken symmetries. It is clear in our approach why the weak interactions, and only them, violate parity [15]. We also explain why the relevant weak symmetry is SU(2) and not any other SU(N). It is SU(2) because there are only two ways to construct composite hypercolor singlet fermions with identical color and $B - L$: one from rrr , the other from $\bar{r}\bar{r}\bar{r}$.

Finally, we are disappointed by the absence of decisive experimental tests of our model. We feel, however, that at the present stage, it is most important to pursue the

theoretical questions of self-consistency (especially those discussed in sects. 6–8) and to consider only sufficiently general experimental aspects of the model.

The following experimental results could teach us something about the validity of the rishon model:

(i) A discovery of integer-charge quarks or fractional-charge leptons would immediately destroy the model. A discovery of additional “bottom” quarks without corresponding “top” quarks, would also destroy it.

(ii) A discovery of right-handed currents or a W_R boson would encourage us, although it will not prove compositeness. On the other hand, if an $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry is experimentally confirmed, our model predicts that all bosons (scalar and vector) must have integer $I_{3L} + I_{3R}$ and all fermions must have odd $I_{3L} + I_{3R}$. A scalar boson in a $(\frac{1}{2}, 0)$ representation or a fermion in $(0, 0)$ would rule out the model.

(iii) A proton lifetime of 10^{31} or 10^{32} years is consistent with our model as well as with the standard grand unified theories. However, unlike those theories, we can also live with $\tau_p \sim 10^{36}$ or 10^{40} years. As long as we do not have a strong reason for choosing a specific value of Λ_H , it is not very probable that Λ_H happens to be precisely in the right “window” which enables detection of proton decay in the next few years.

(iv) We cannot avoid neutrino masses [15] of order $m_f^2/m(W_R)$.

All of these features are very general and none of them are specific to our model. More specific tests such as a possible enhancement of $(B - L)$ -violating proton decays [22], properties of the Goldstone boson χ and the prediction for the mass ratio [15] between the right-handed W^\pm and the right-handed Z , depend on explicit details of the model. These could be wrong, even if the model itself turns out to be correct. One should therefore handle such tests with great care. We feel that only a combined slow evolution of theoretical understanding and experimental discoveries will enable us to tell whether this model or any other composite scheme shows any resemblance to the real world.

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